ECE317 : Feedback and Control

Lecture :
Bode plots

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Course roadmap

Modeling

- Laplace transform
- Transfer function
- Block Diagram
- Linearization
- Models for systems
  - electrical
  - mechanical
  - example system

Analysis

- Stability
  - Pole locations
  - Routh-Hurwitz
- Time response
  - Transient
  - Steady state (error)
- Frequency response
  - Bode plot

Design

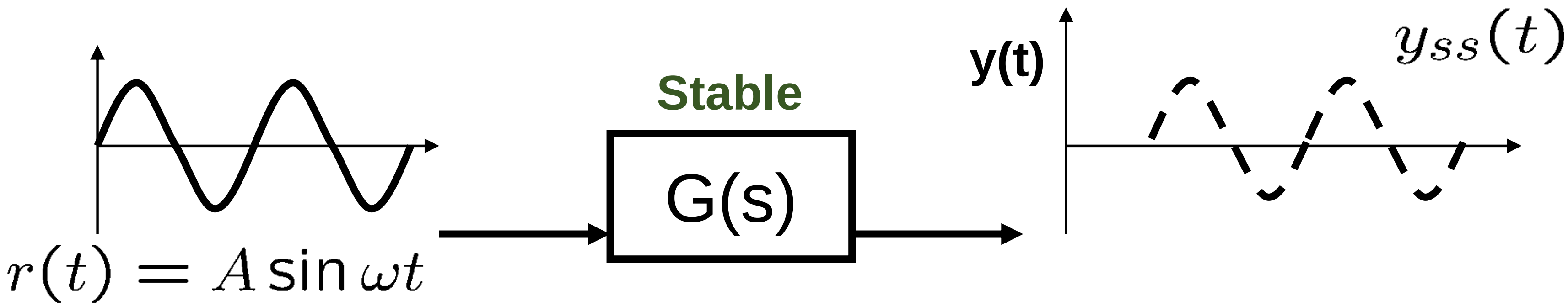
- Design specs
- Frequency domain
- Bode plot
- Compensation
- Design examples

Matlab & PECS simulations & laboratories

Frequency response (review)

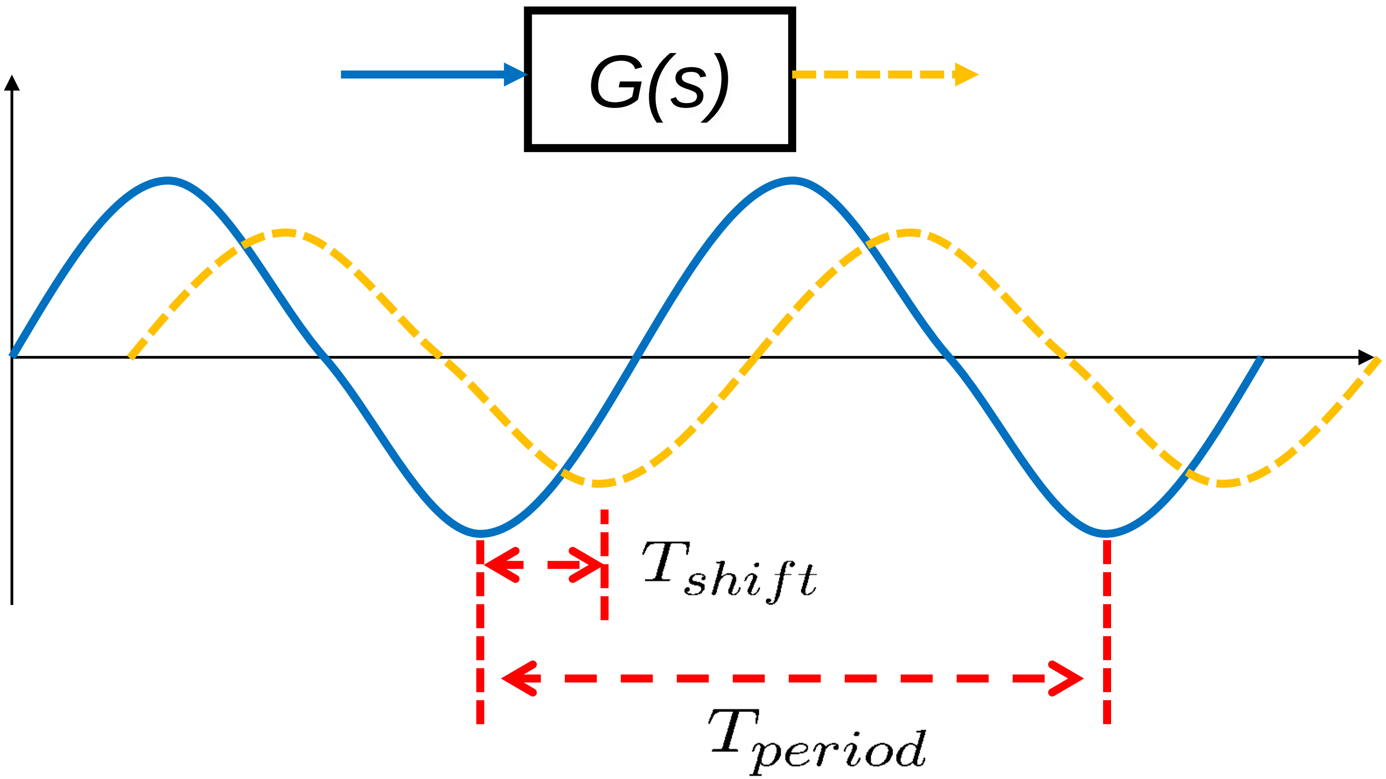
- Steady state output $y_{ss}(t) = A|G(j\omega)|\sin(\omega t + \angle G(j\omega))$
  - Frequency is same as the input frequency ω
  - Amplitude is that of input (A) multiplied by $|G(j\omega)|$
  - Phase shifts $\angle G(j\omega)$

Gain



- Frequency response function (FRF): *G(jω)*
- Bode plot: Graphical representation of *G(jω)*

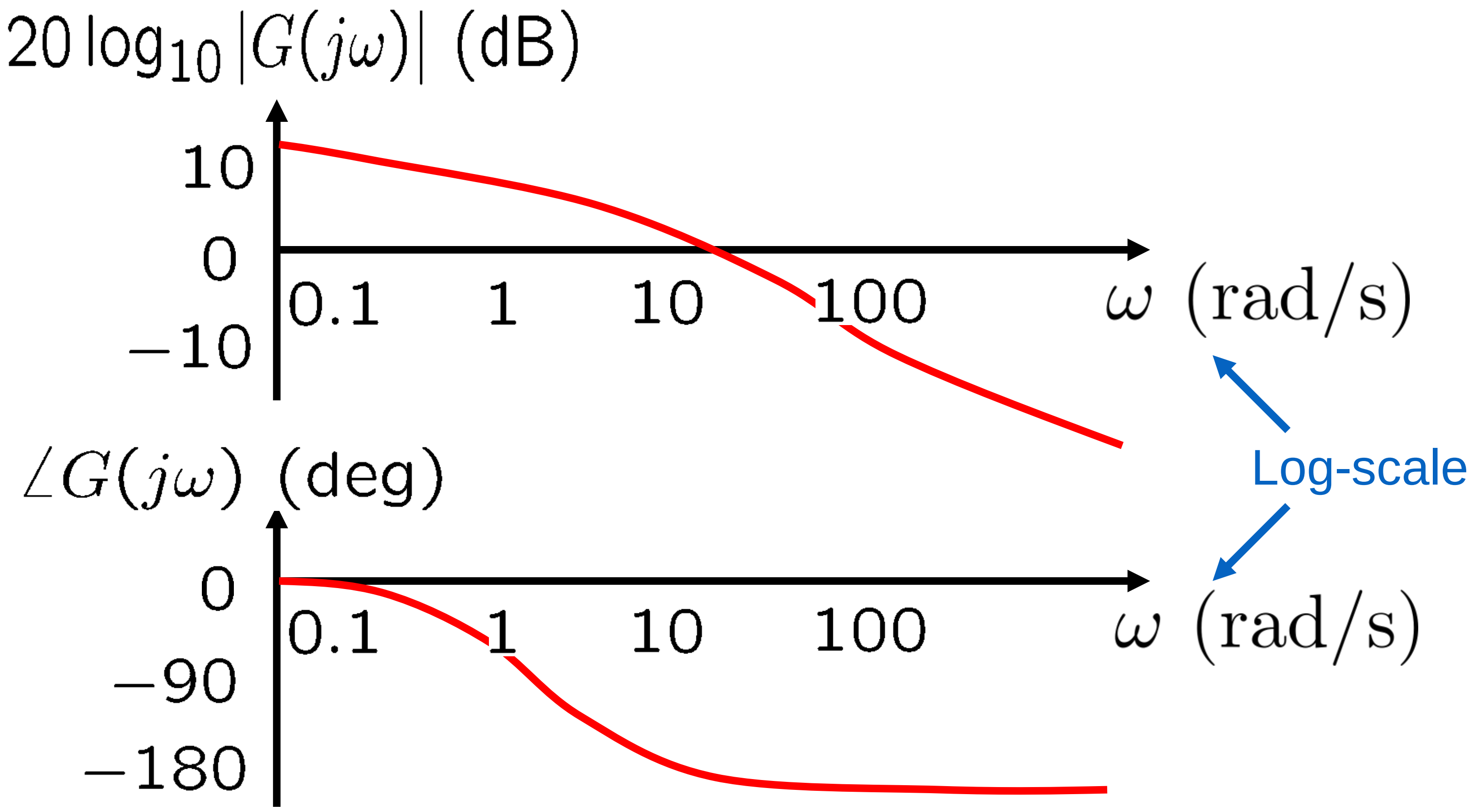
Phase shift (review)



$$\frac{T_{shift}}{T_{period}} = \frac{-\angle G(j\omega)}{360^\circ} \Rightarrow \angle G(j\omega) = -\frac{T_{shift}}{T_{period}} \times 360^\circ$$

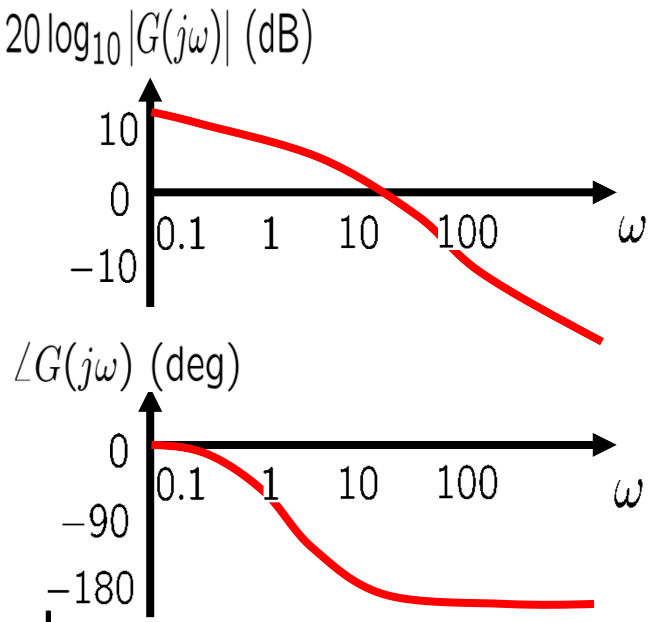
Bode plot of *G(jω)* (review)

- Bode diagram consists of gain plot & phase plot



Sketching Bode plot

- Basic functions
  - Constant gain
  - Differentiator and Integrator
  - First order system and its inverse
  - Second order system



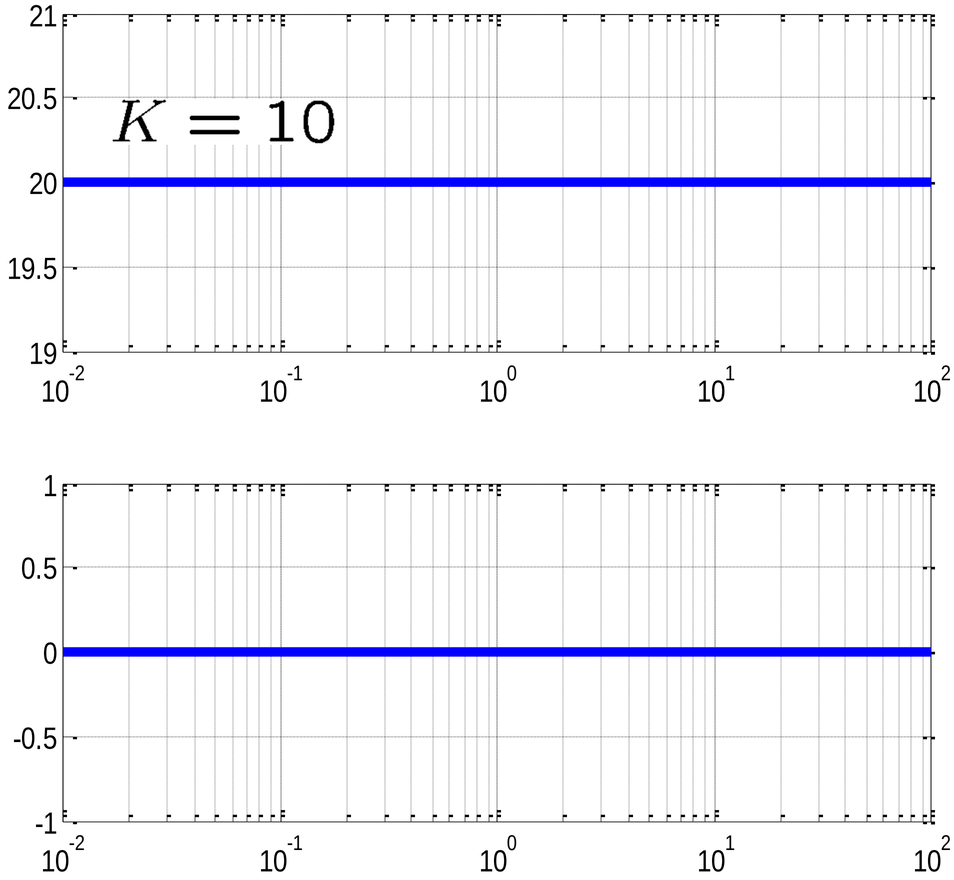
- Product of basic functions
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.

Old method:

Bode plot of a constant gain

$$G(s) = K \Rightarrow |G(j\omega)| = K, \ \angle G(j\omega) = 0^\circ, \ \forall\omega$$

(for all)



K	$20\log_{10} K$
100	40 dB
10	20 dB
2	≈ 6 dB
1	0 dB
0.1	-20 dB
0.01	-40 dB

New method:

Bode plot of a constant gain

$$H(s) = A$$

$$\left|H(s)\right| \quad \underline{\qquad\qquad A \qquad\qquad}$$

$$\angle H(s) \quad \underline{\qquad\qquad 0^o \qquad\qquad}$$

Sketching Bode plot

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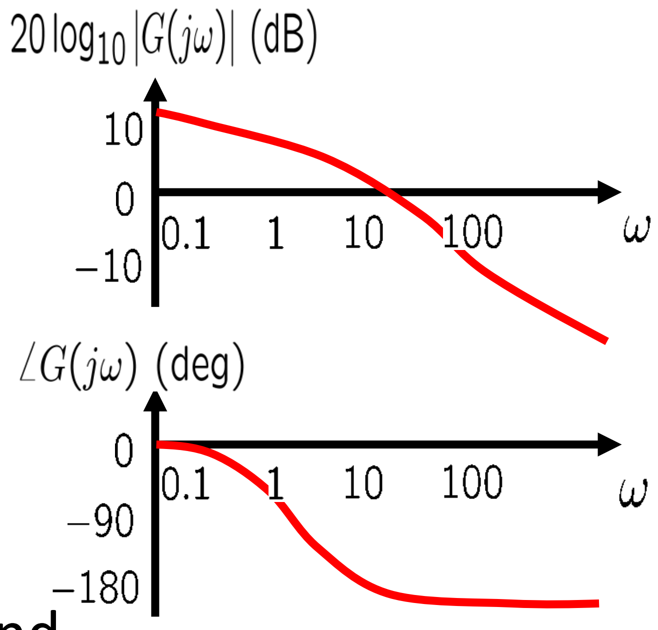
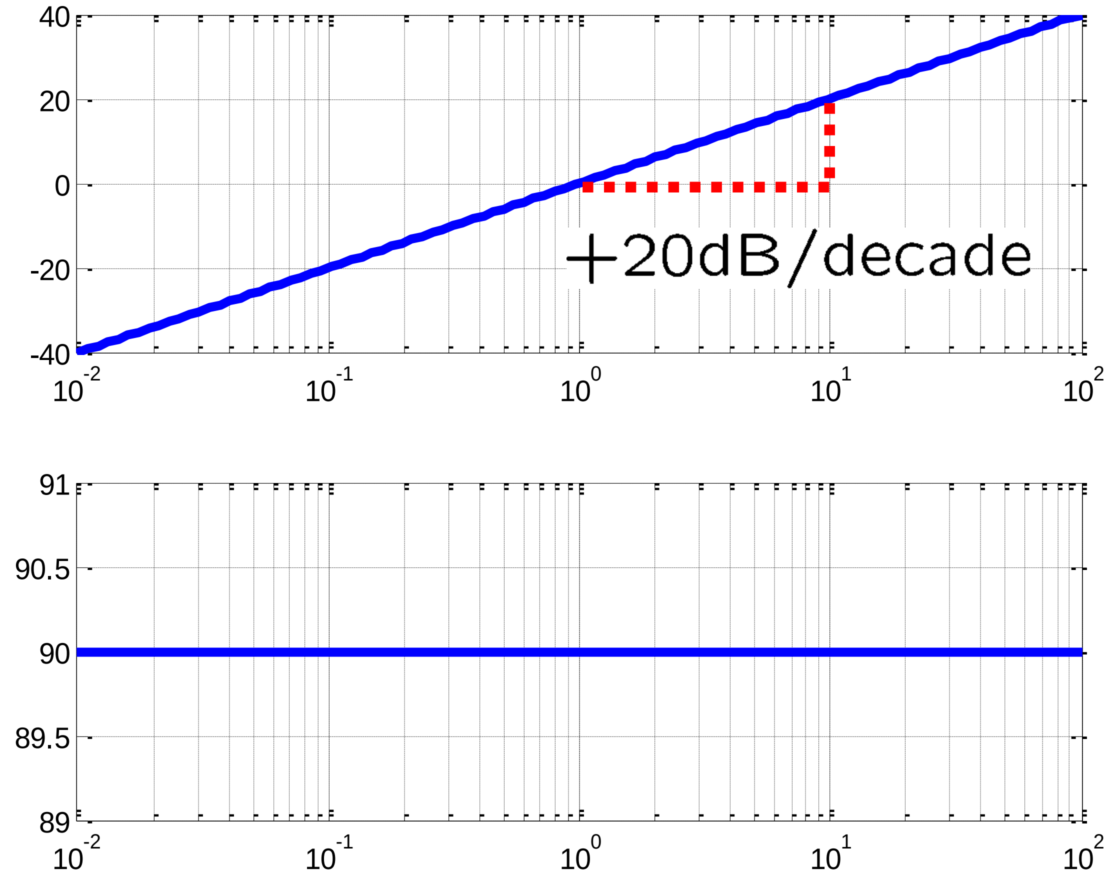


- Product of basic functions
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.

Old method:
Bode plot of a differentiator

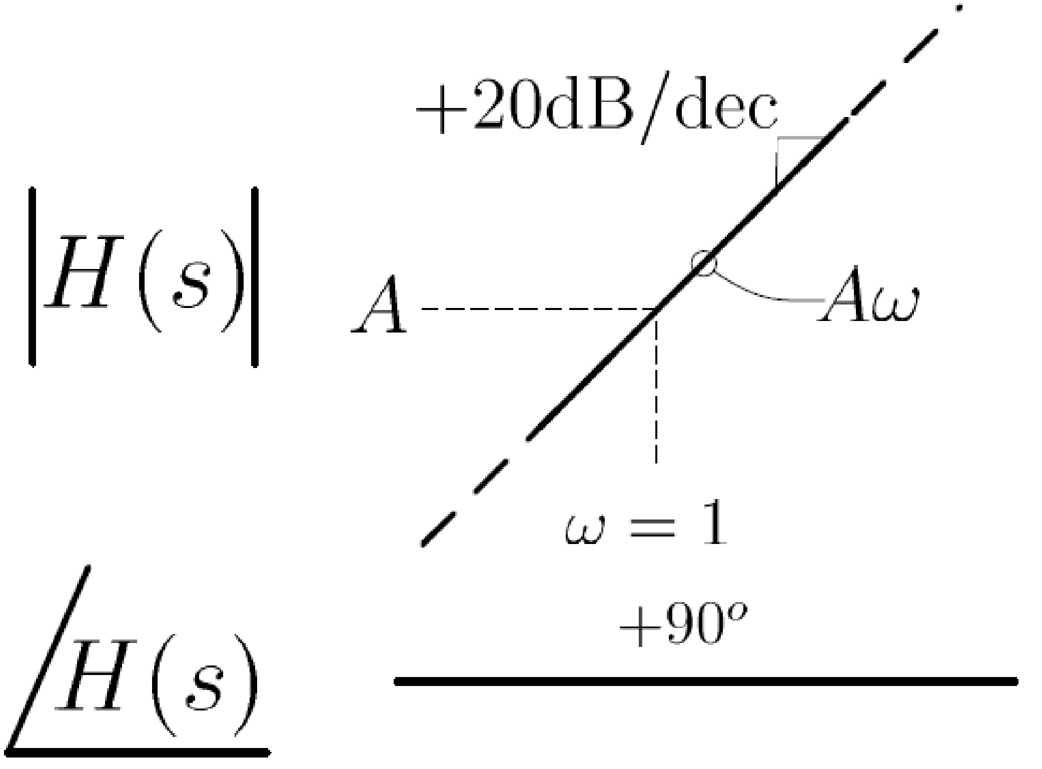
$$G(s) = s \Rightarrow |G(j\omega)| = \omega, \ \angle G(j\omega) = \angle j\omega = 90^\circ, \ \forall\omega$$



New method:

Bode plot of a differentiator

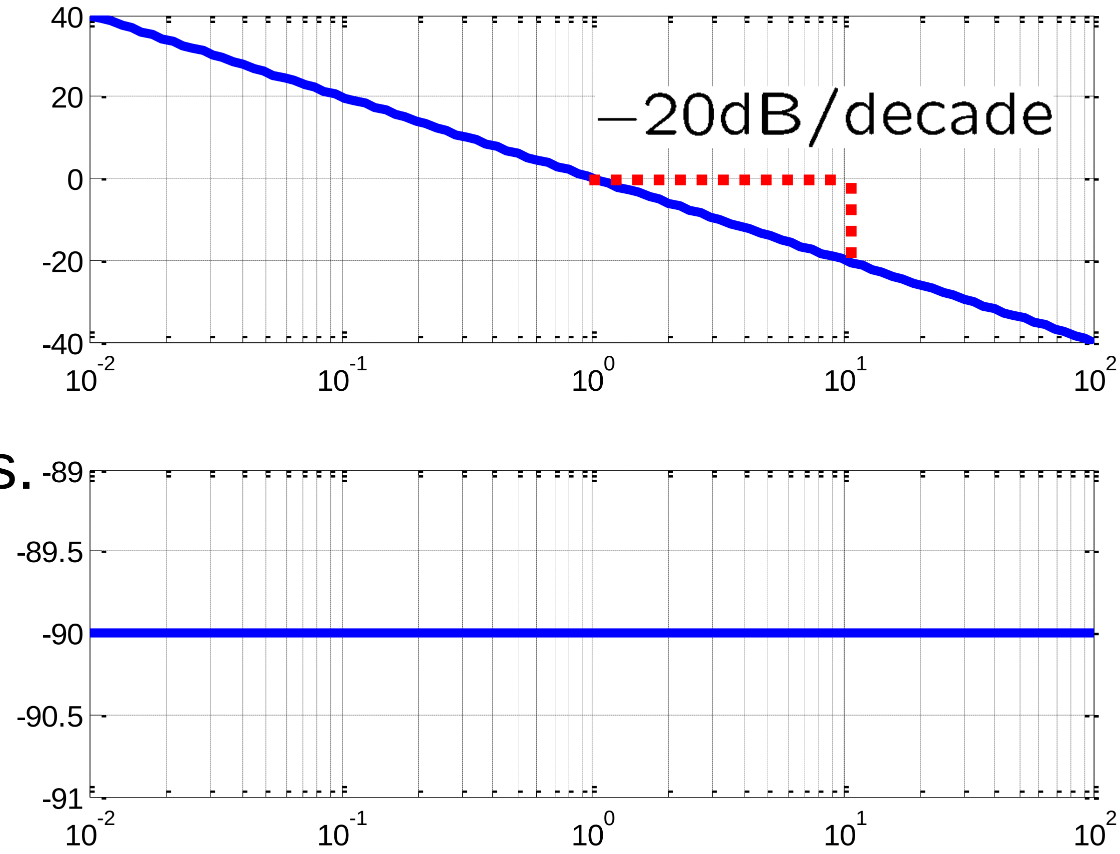
$$H(s) = As$$



Old method:
Bode plot of an integrator

$$G(s) = \frac{1}{s} \Rightarrow |G(j\omega)| = \frac{1}{\omega},\ \angle G(j\omega) = \angle\frac{1}{j\omega} = -90^\circ,\ \forall\omega$$

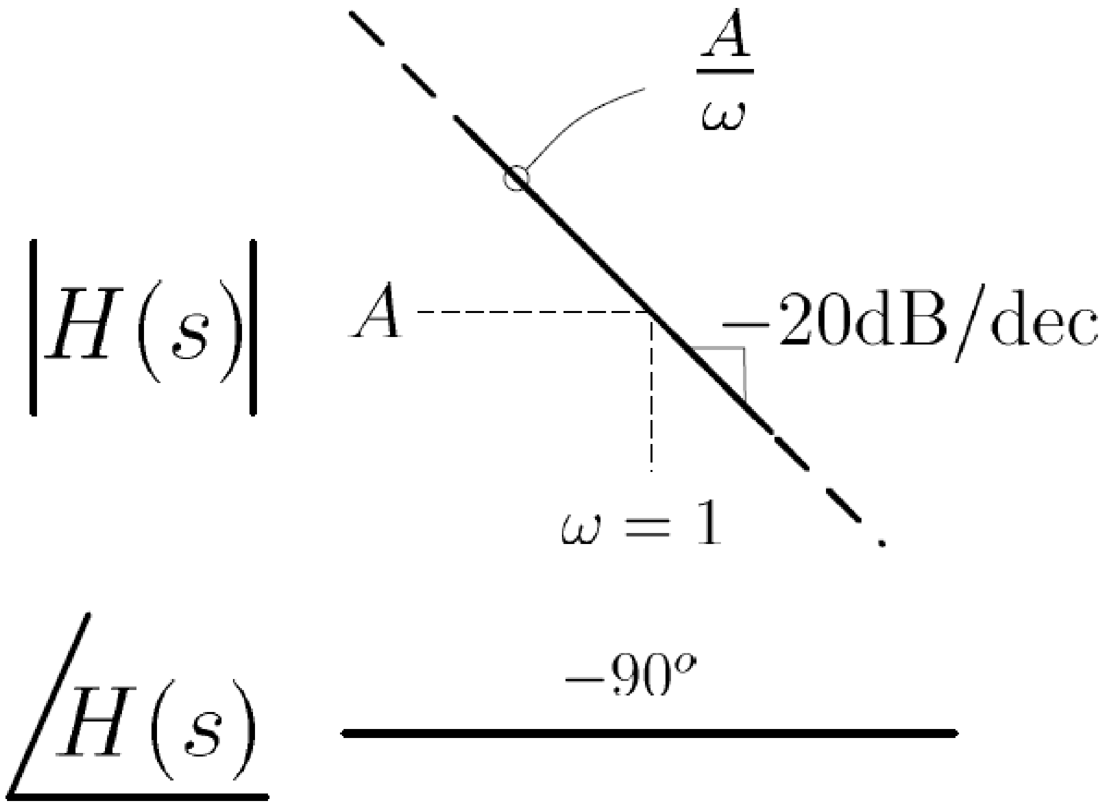
Mirror image of the Bode plot of *G(s)=s* with respect to ω-axis.



New method:

Bode plot of an integrator

$$H(s) = \frac{A}{s}$$



Sketching Bode plot

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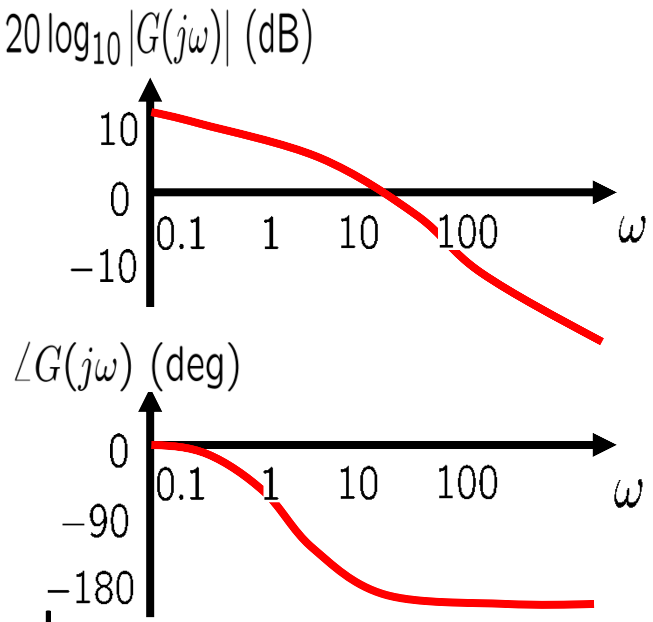


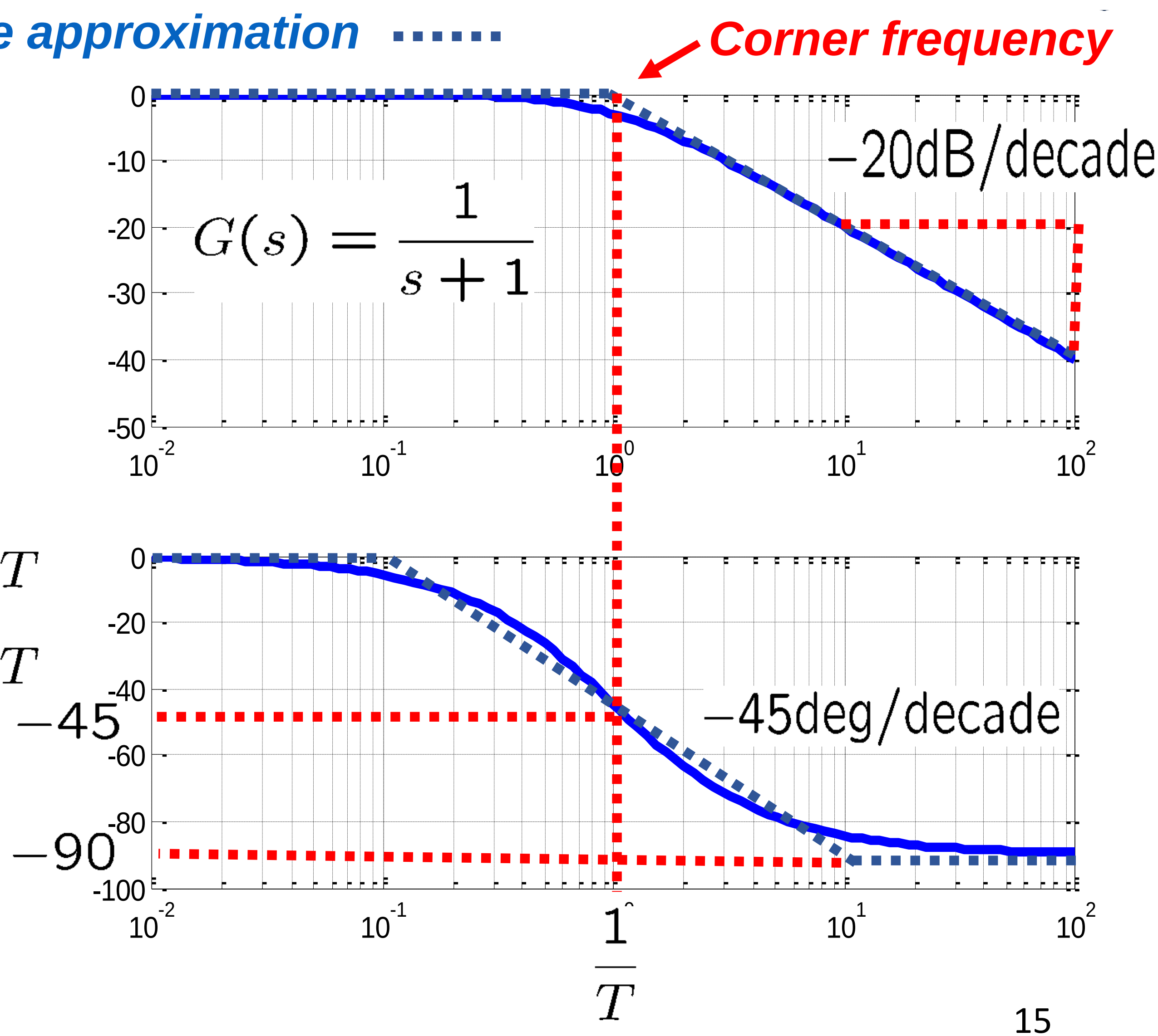
- Product of basic functions
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.

Old method:
Bode plot of a 1st order system

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{j\omega T + 1}$$

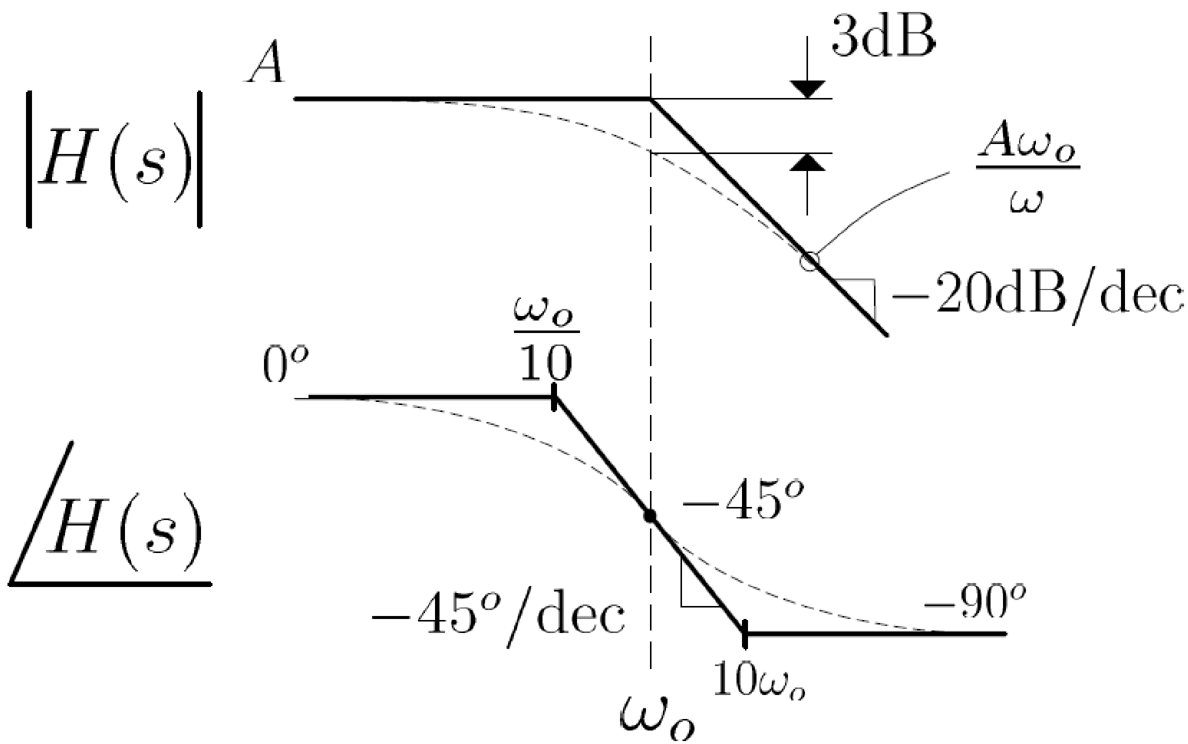
$$\approx \begin{cases} 1 & \text{if } 1 \gg \omega T \\ \frac{1}{j\omega T} & \text{if } 1 \ll \omega T \end{cases}$$



New method:

Bode plot of a 1st order system

$$H(s) = \frac{A}{1 + \frac{s}{\omega_o}}$$



Maximum Error @ $\omega_o = 3\text{dB}$

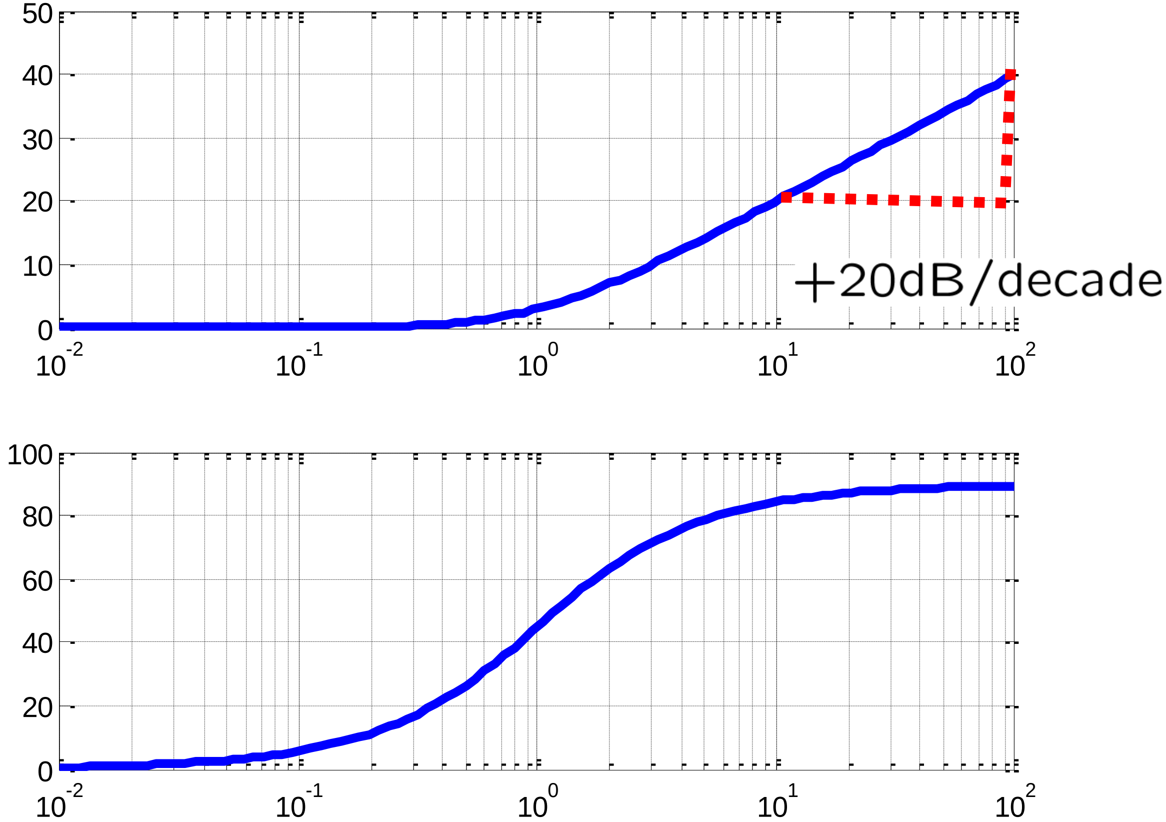
Maximum Error @ $\frac{\omega_o}{10}$ & $10\omega_o = 5.7^o$

Exact Phase: $-tan^{-1}\left(\frac{\omega}{\omega_o}\right), \forall\omega$

Old method:
Bode plot of an inverse system

$$G(s) = Ts + 1 = \left(\frac{1}{Ts+1}\right)^{-1}$$

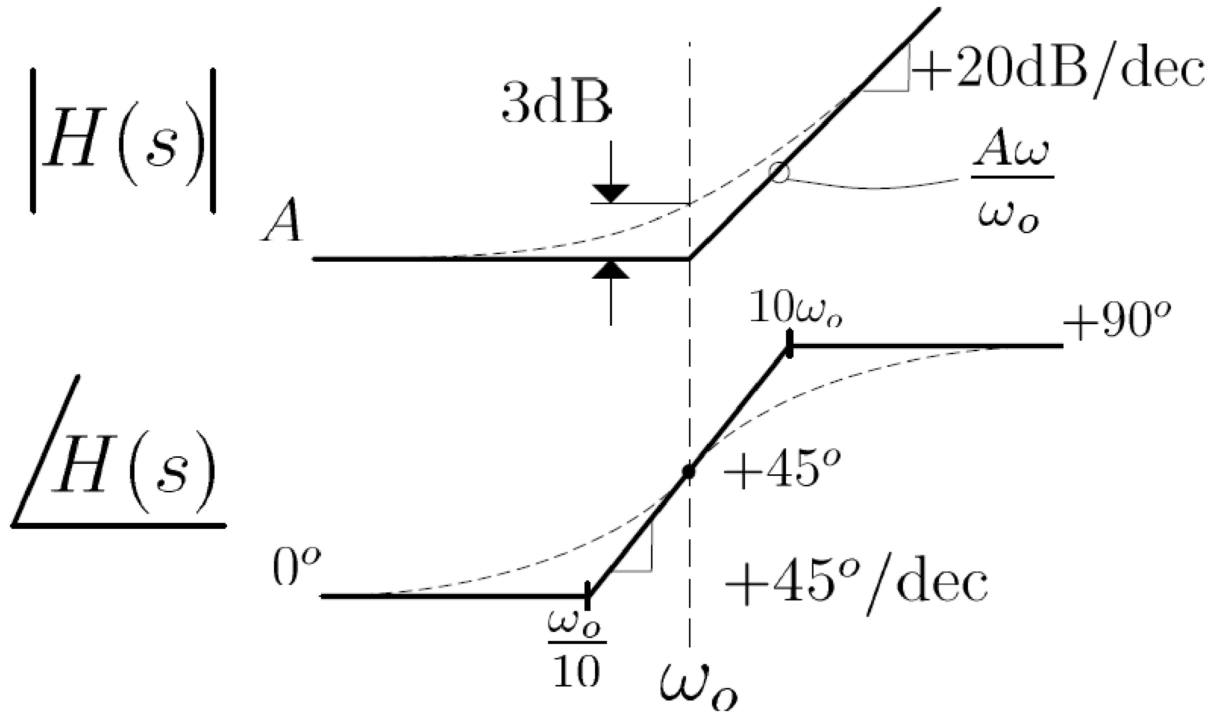
Mirror image of the original Bode plot with respect to ω-axis.



New method:

Bode plot: Zero at ω_o

$$H(s) = A\left(1 + \frac{s}{\omega_o}\right)$$



Maximum Error @ $\omega_o = 3\text{dB}$

Maximum Error @ $\frac{\omega_o}{10}$ & $10\omega_o = 5.7^o$

Exact Phase: $tan^{-1}\left(\frac{\omega}{\omega_o}\right), \forall\omega$

Sketching Bode plot

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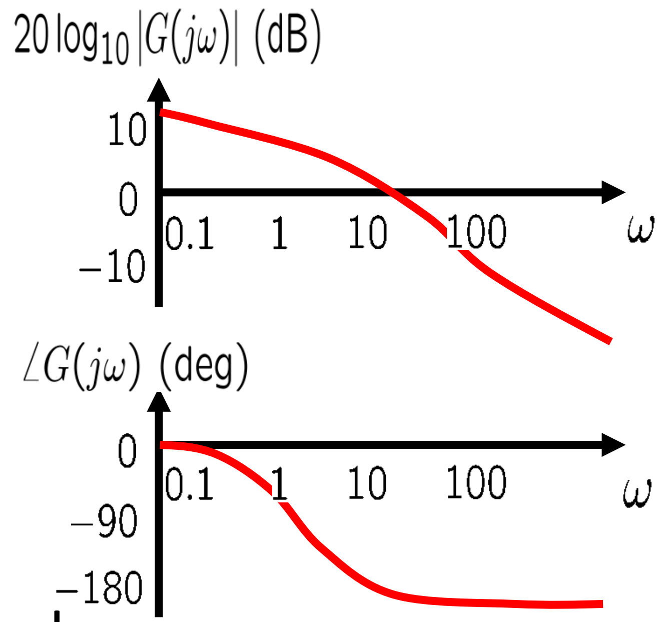
  - Second order system



- Product of basic functions
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.

Old method:
Bode plot of a 2nd order system

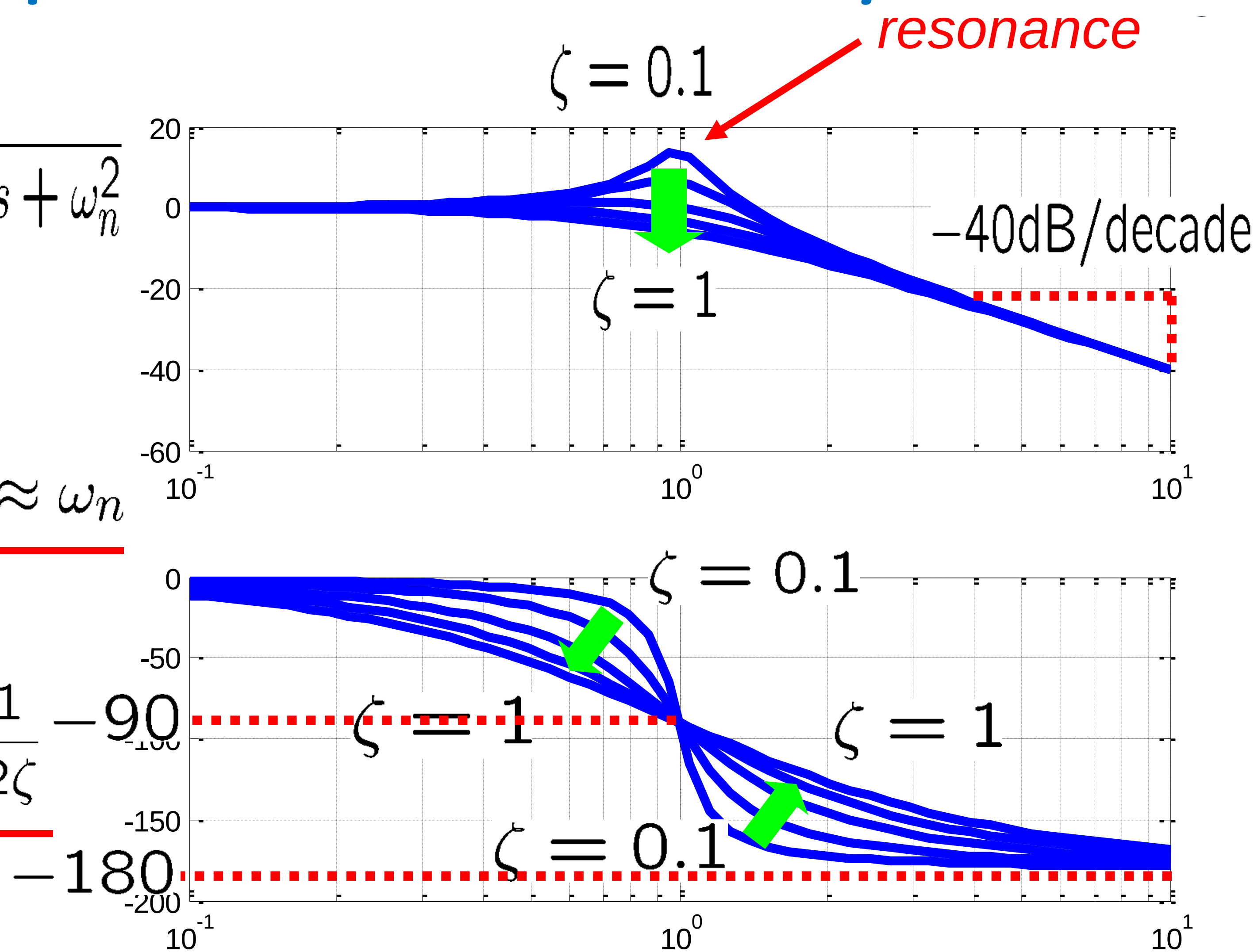
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Resonant freq.

$$\omega_n\sqrt{1-2\zeta^2} \approx \omega_n$$

Peak gain

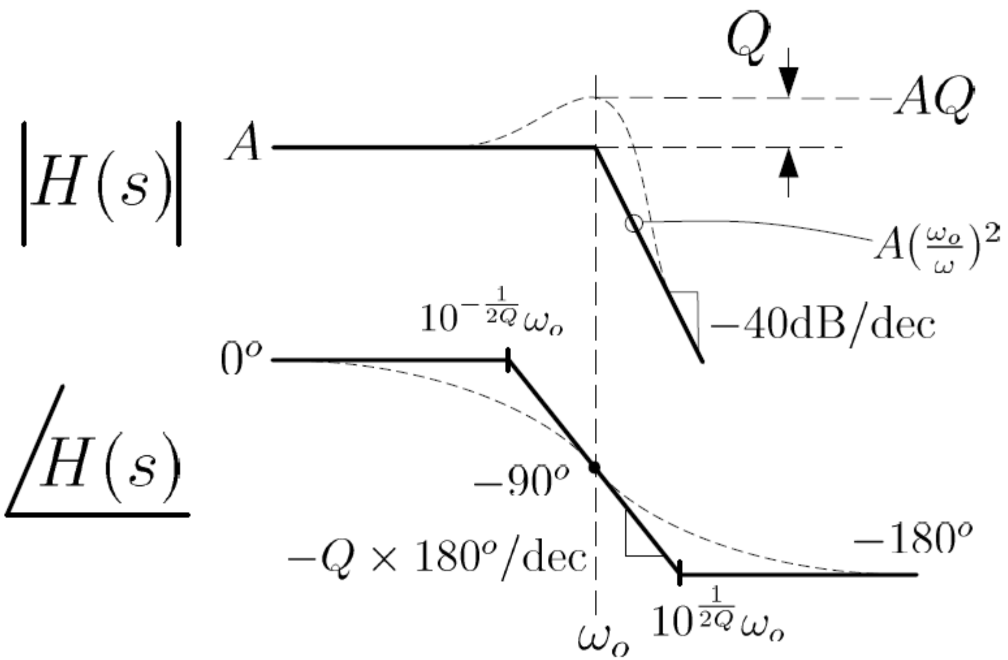
$$\frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta}$$



New method:
Bode plot of a 2nd order system

$$H(s) = \frac{A}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

$$Q = \frac{1}{2\varsigma}$$



ω_o=Corner Frequency

$Q > \frac{1}{2} \implies$ Complex Roots

Q=Quality Factor: Exact Gain @ ω_o
Approximate Maximum Value

Exact Phase: $-\,tan^{-1}\left[\frac{\frac{1}{Q}\frac{\omega}{\omega_o}}{1-(\frac{\omega}{\omega_o})^2}\right], \forall\omega$

Sketching Bode plot

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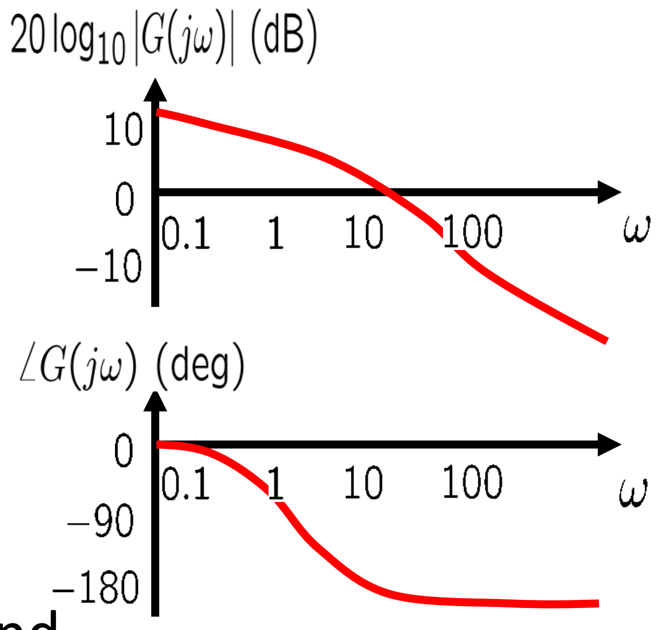


- Product of basic functions
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.

Main advantage of Bode plot!

An advantage of Bode plot

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2.
  - Gain

$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

  - Phase

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- Later, we use this property to design $C(s)$ so that $G(s)C(s)$ has a “desired” shape of Bode plot.

Short proofs

- Use polar representation

$$G_1(j\omega) = |G_1(j\omega)|e^{j\angle G_1(j\omega)} \qquad G_2(j\omega) = |G_2(j\omega)|e^{j\angle G_2(j\omega)}$$

Then,
$$\begin{aligned} G_1(j\omega)G_2(j\omega) &= |G_1(j\omega)||G_2(j\omega)|e^{j\angle G_1(j\omega)}e^{j\angle G_2(j\omega)} \\ &= |G_1(j\omega)||G_2(j\omega)|e^{j\{\angle G_1(j\omega)+\angle G_2(j\omega)\}} \end{aligned}$$

Therefore,

$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)|\cdot|G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

Example 1

- Sketch the Bode plot of a transfer function

$$G(s) = \frac{10}{s}$$

1. Decompose *G(s)* into a product form:

$$G(s) = 10 \cdot \frac{1}{s}$$

2. Sketch a Bode plot for each component on the same graph.

3. Add them all on both gain and phase plots.

Example 1 (cont'd)

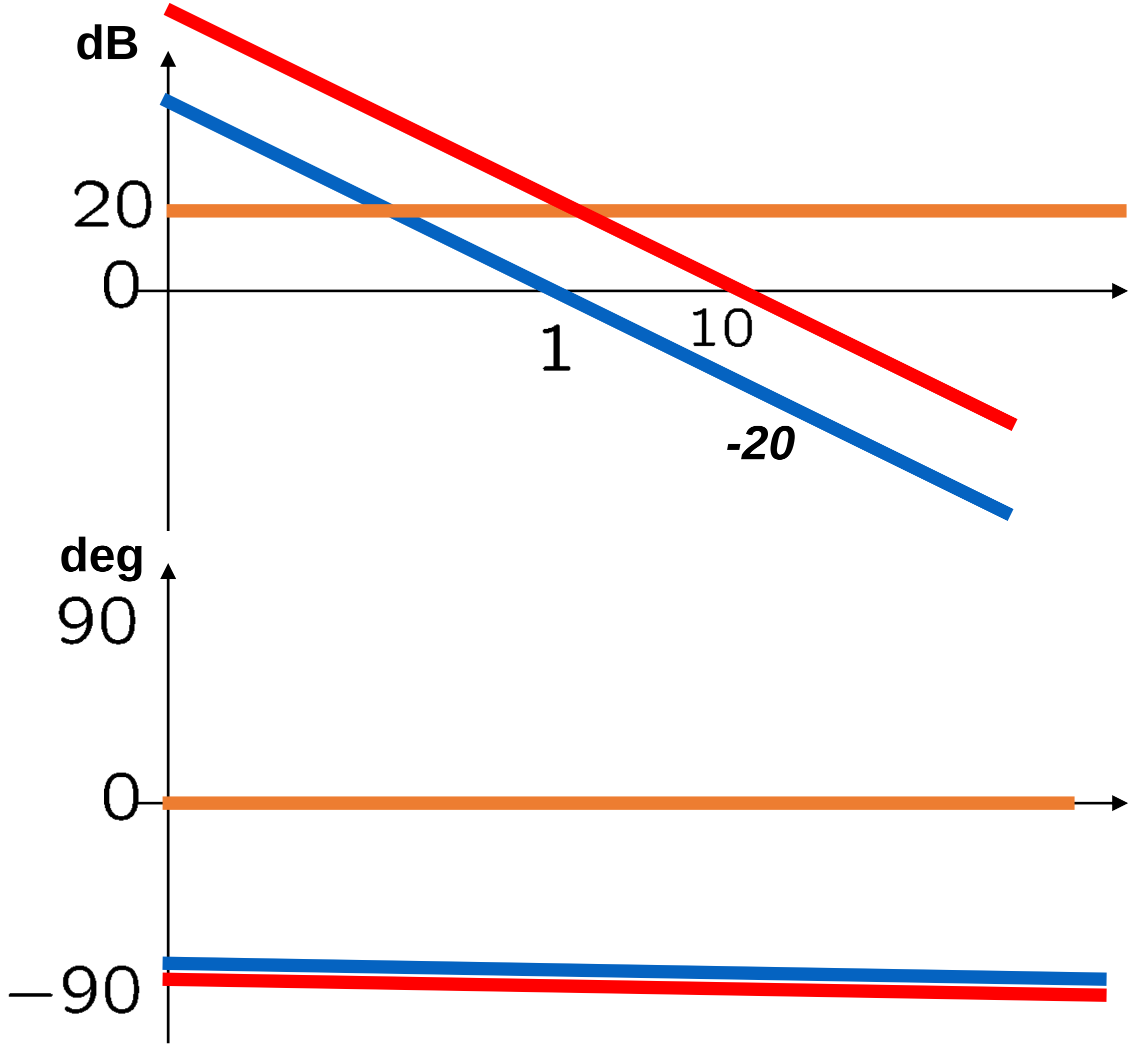
$$G(s) = 10$$

$\times$

$$G(s) = \frac{1}{s}$$

$\Downarrow$

$$G(s) = \frac{10}{s}$$



Example 2

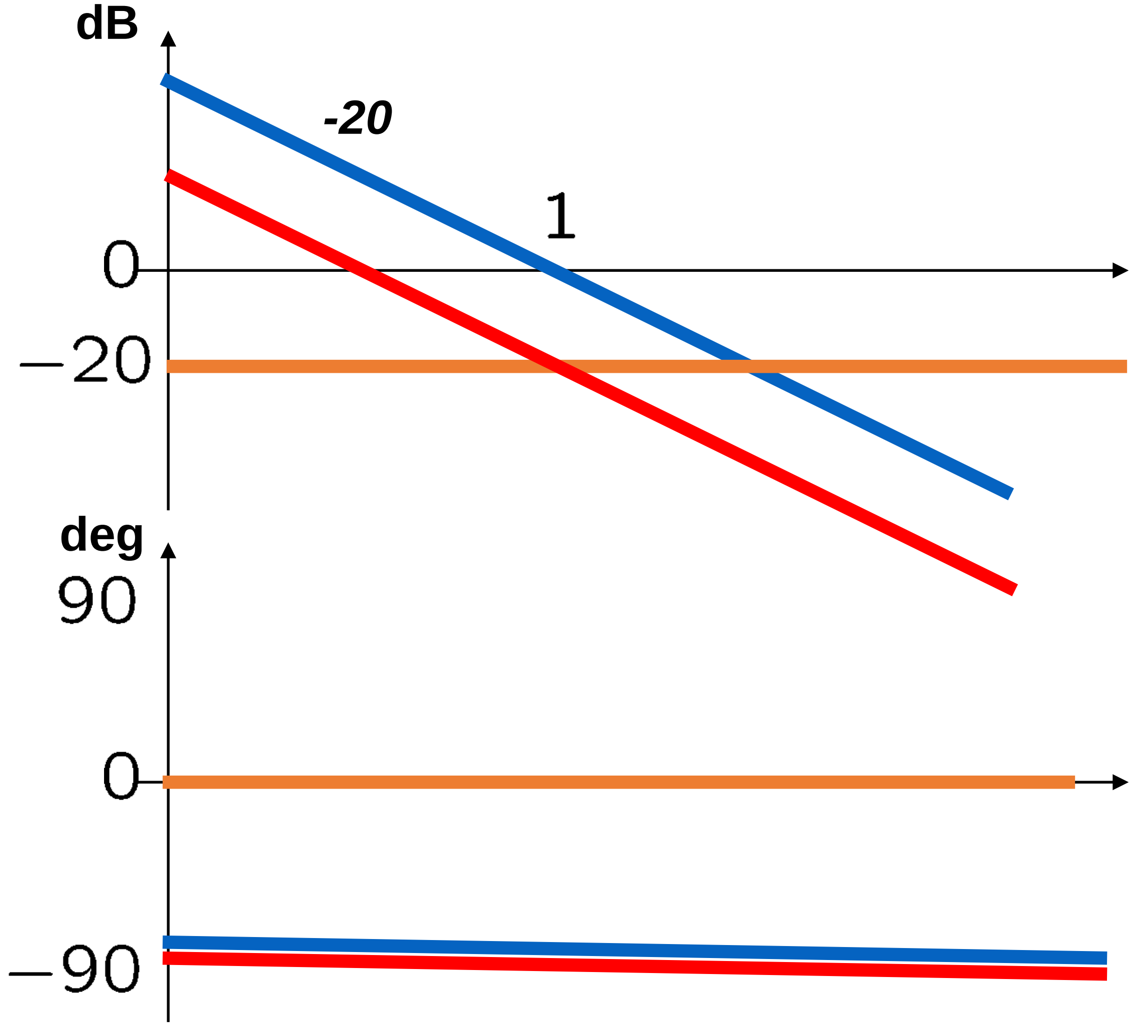
$$G(s) = 0.1$$

$\times$

$$G(s) = \frac{1}{s}$$

$\Downarrow$

$$G(s) = \frac{0.1}{s}$$



Example 3

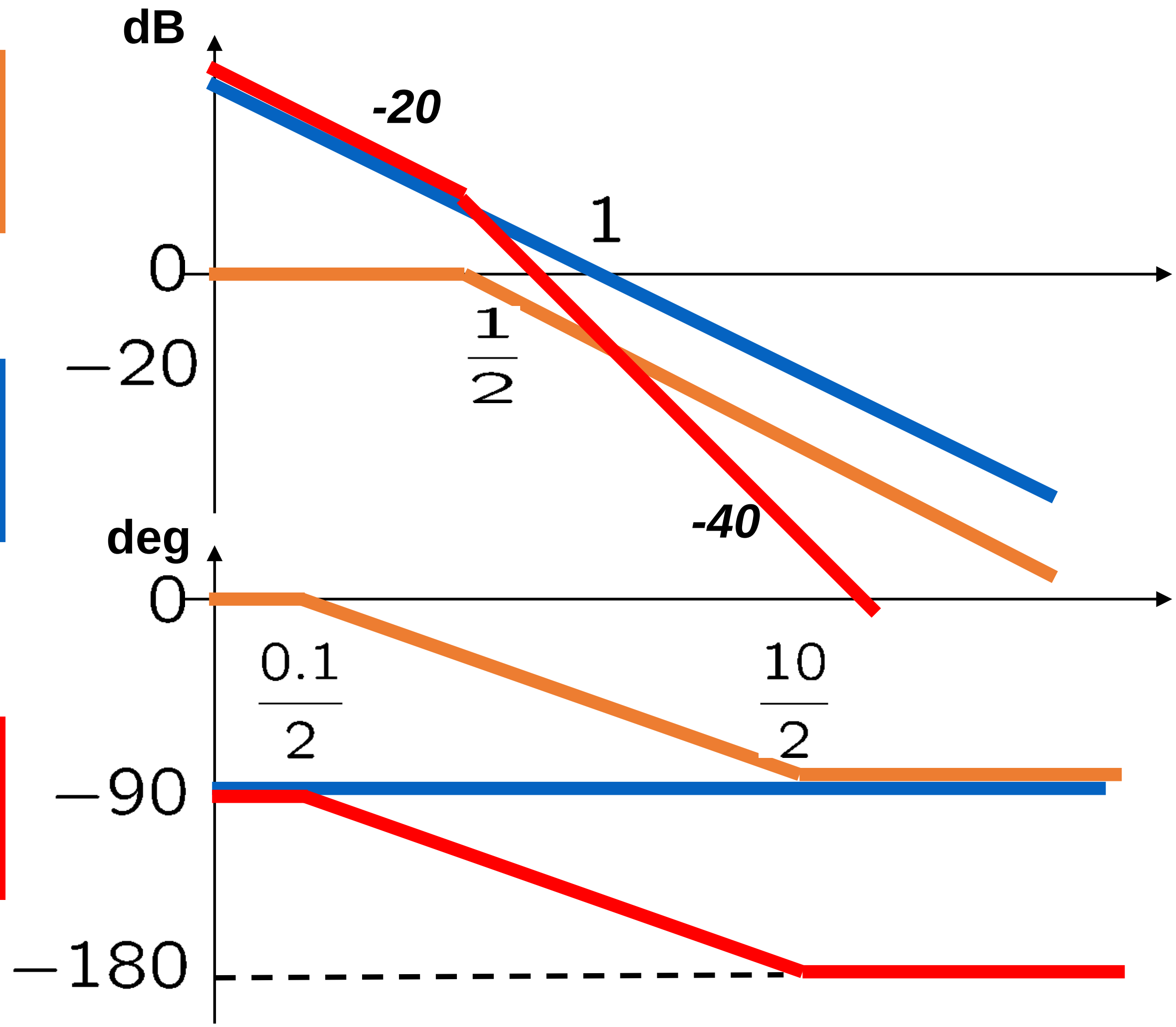
$$G(s) = \frac{1}{2s+1}$$

$$\times$$

$$G(s) = \frac{1}{s}$$

$$\Downarrow$$

$$G(s) = \frac{1}{s(2s+1)}$$



Remark

- You can use MATLAB command “bode” to obtain the precise magnitude and phase responses.

Summary

- Sketches of Bode plots
  - Basic transfer functions
  - Products of basic transfer functions
- The new approach to sketching Bode plots is useful:
  - With simplified annotations we're able to quickly obtain good approximations to magnitude and phase values.
  - This approach will be particularly useful in the process of compensator design where simplified design expressions will be derived from the sketched plots. (After a design is completed, it may be verified with MATLAB using the exact transfer function expressions).
- Next, *practice sketching Bode plots*