CS 581: Theory of Computation
Final exam

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This is a closed-book, closed-notes exam. All problems have equal weight.

1. Briefly justify or refute the following claims. You may cite without proof any relevant results from lecture or the text.

   (a) $CFL \subseteq P$

   (b) $CFL \subseteq REG$

   (c) $NP \subseteq EXPTIME = \bigcup_k TIME(2^{n^k})$

   (d) $TR \cap coTR = DEC$

   (e) $TR \cup coTR = DEC$

   Abbreviations: TR Turing-recognizable, $coTR$ co-Turing recognizable, DEC the decidable languages, REG the regular languages, P polynomial time computable, NP nondeterministic polynomial time computable, and CFL Context Free Languages.

2. Rice's theorem for recursive index sets proves that all non-trivial properties of the languages accepted by Turing machines are undecidable.

   In homework you proved a statement of Rice's theorem:

   > Let P be a language consisting of Turing machine descriptions where P fulfills two conditions. First, P is nontrivial —it contains some, but not all, TM descriptions. Second, P is a property of the TM's language—whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$. Here M_1 and M_2 are any TMs. P is an undecidable language.

   Which of the following properties satisfy the conditions of Rice's theorem:

   (a) TM has an odd number of states

   (b) TM accepts an odd number of strings

   (c) TM decides A_{TM}

   (d) TM accepts Σ^*

   (e) TM accepts $a^n b^n c^n$

3. Show that Context Free Languages are closed under intersection with a regular set.

4. Prove that A_{TM} is undecidable by diagonalization. Do not assume the undecidability of any other set to complete this proof.

5. In the proof of the Cook-Levin theorem an arbitrary polynomial time computation of a nondeterministic TM M is compiled into an instance of SAT. To describe the construction of the formula, the proof builds a tableau (table). The formula is satisfiable if the tableau can be filled in according to a set of rules.

   (a) What does the tableau represent? How is it related to M?

   (b) How big is the tableau (table)?

6. For each set below please say if it is decidable, Turing-recognizable, or not Turing-recognizable. Briefly justify your answers. You can refer to any results presented in the text or in lecture without proof.

   (a) Syntactically correct sentences in the language of number theory (using operators $+$ and $\times$).

   (b) Sentences in number theory provable in a reasonable proof system. (Recall that a reasonable proof system is one in which (a) the "proves" relation is decidable and (b) all provable sentences are true.)

   (c) All true sentences in number theory ($\text{Th}(\mathcal{N}, +, \times)$).