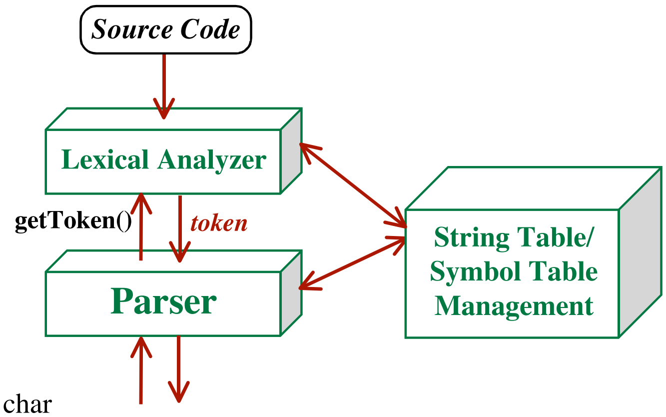
Lexical Analysis



- Must be efficient
- Looks at every input char
- Textbook, Chapter 2

Tokens

Token Type

Examples: ID, NUM, IF, EQUALS, ...

Lexeme

The characters actually matched.
Example:

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... if x == -12.30 then ...
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How to describe/specify tokens?

Formal:

Regular Expressions

`Letter ( Letter | Digit )*`

Informal:

"// through end of line"

Lexical Analysis - Part 1

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Informal:

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Tokens will appear as TERMINALS in the grammar.

Stmt → **while** Expr **do** StmtList **endWhile**
→ ID "=" Expr ";"
→ ...

Lexical Errors

Most errors tend to be "typos"

Not noticed by the programmer

```
return 1.23;
retunn 1,23;
```

... Still results in sequence of legal tokens

```
<ID,"retunn"> <INT,1> <COMMA> <INT,23> <SEMICOLON>
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No lexical error, but problems during parsing!

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Errors caught by lexer:

- EOF within a String / missing "
- Invalid ASCII character in file
- String / ID exceeds maximum length
- Numerical overflow

  etc...

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etc...

Lexer must keep going!

Always return a valid token.

Skip characters, if necessary.

May confuse the parser

The parser will detect syntax errors and get straightened out (hopefully!)

Managing Input Buffers

Option 1: Read one char from OS at a time.

Option 2: Read N characters per system call

e.g., N = 4096

Manage input buffers in Lexer

More efficient

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`... 1 2 3 4 ? ...`

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Convert to FLOAT or INT?

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Token could overlap / span buffer boundaries.
$\Rightarrow$ need 2 buffers

Code:
```
if (ptr at end of buffer1) or (ptr at end of buffer2) then ...
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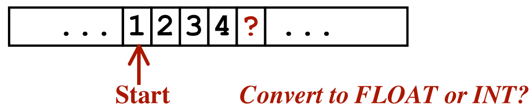

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Code:
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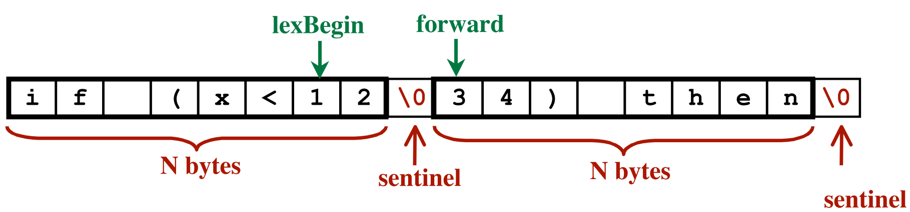
Technique: Use “Sentinels” to reduce testing

Choose some character that occurs rarely in most inputs

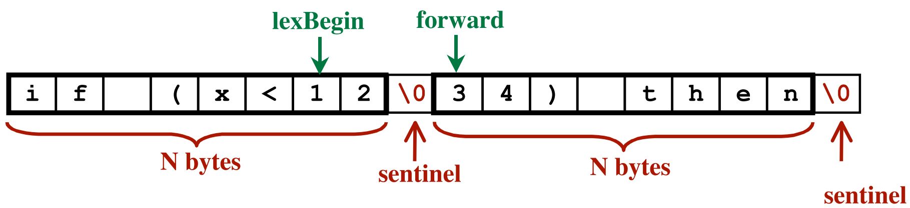
`'\0'`

Lexical Analysis - Part 1



Goal: Advance **`forward`** pointer to next character

...and reload buffer if necessary.



Goal: Advance **`forward`** pointer to next character
...and reload buffer if necessary.

Code :

One fast test ...which usually fails

```
forward++;
if *forward == '\0' then
  if forward at end of buffer #1 then
    Read next N bytes into buffer #2;
    forward = address of first char of buffer #2;
  elseIf forward at end of buffer #2 then
    Read next N bytes into buffer #1;
    forward = address of first char of buffer #1;
  else
    // do nothing; a real \0 occurs in the input
  endIf
endIf
```


"Alphabet" (Σ)

A set of symbols ("characters")

Examples: Σ = { **a**, **b**, **c**, **d** }
Σ = ASCII character set

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Sequence of symbols

Finite in length

Example: `abbadc` Length of s = |s|

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"Empty String" (ε, "epsilon")

It is a string

$|\varepsilon| = 0$

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Example: `abbadc`   Length of s = |s|

“Empty String” (ε, “epsilon”)

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|ε| = 0

“Language”

A set of strings

Examples: L_1 = { `a`, `baa`, `bccb` }

L_2 = { }

L_3 = { ε }

L_4 = { ε, `ab`, `abab`, `ababab`, `abababab`, . . . }

L_5 = { s | s can be interpreted as an English sentence making a true statement about mathematics}

Each string is finite in length, but the set may have an infinite number of elements.

“Prefix” ...of string s

s = `hello`   *Prefixes:* `he`

`hello`

ε

"Prefix" ...of string s

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"Suffix" ...of string s

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"Substring" ...of string s

Remove a prefix and a suffix

s = `hello` *Substrings:* `ell`
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"Prefix" ...of string s

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hello

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"Proper" prefix / suffix / substring ... of s

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hello
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"Proper" prefix / suffix / substring ... of s

≠ s and ≠ ε

"Subsequence" ...of string s,

s = **compilers** *Subsequences:* **opilr**
cors
compilers
ε

"Concatenation"

Strings: x, y

Concatenation: xy

Example:

x = **abb**

y = **cdc**

xy = **abbcdc**

yx = **cdcabb**

Other notations: **x || y**

x + y

x ++ y

x · y

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What is the "identity" for concatenation?

$\varepsilon x = x\varepsilon = x$

Multiplication ⇔ Concatenation

Exponentiation ⇔ ?

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Define $s^0 = \varepsilon$

$s^N = s^{N-1}s$

Example x = **ab**

$x^0 = \varepsilon$

$x^1 = x =$ **ab**

$x^2 = xx =$ **abab**

$x^3 = xxx =$ **ababab**

...etc...

"Concatenation"

Strings: x, y

Concatenation: xy

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...etc...

$x^* = x^\infty =$ **abababababab...**

Infinite sequence of symbols!

Technically, this is not a "string"

"Language"
A set of strings
L = { ... }
M = { ... }

Generally, these are infinite sets.

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"Union" of two languages
$L \cup M$ = { s | s is in L or is in M }

Example:
L = { `a`, `ab` }
M = { `c`, `dd` }
$L \cup M$ = { `a`, `ab`, `c`, `dd` }

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"Concatenation" of two languages
$L\,M = \{ st \mid s \in L \text{ and } t \in M \}$

Example:
L = { a, ab }
M = { c, dd }
L M = { ac, add, abc, abdd }

Repeated Concatenation

Let: L = { a, bc }

Example: $L^0 = \{ \varepsilon \}$
L^1 = L = { a, bc }
L^2 = LL = { aa, abc, bca, bcbc }
L^3 = LLL = { aaa, aabc, abca, abcbc, bcaa, bcabc, bcbca, bcbcbc }
...etc...
$L^N = L^{N-1}L = LL^{N-1}$

Kleene Closure

Let: $L = \{ \texttt{a}, \texttt{bc} \}$

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$$\sum_{i=0}^{\infty} a^i = a^0 \cup a^1 \cup a^2 \cup \ldots$$

The "Kleene Closure" of a language:

$$L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots$$

Example:

$$L^* = \{ \underbrace{\varepsilon,}_{L^0} \underbrace{\texttt{a}, \texttt{bc},}_{L^1} \underbrace{\texttt{aa}, \texttt{abc}, \texttt{bca}, \texttt{bcbc},}_{L^2} \underbrace{\texttt{aaa}, \texttt{aabc}, \texttt{abca}, \texttt{abcbc}, \ldots}_{L^3} \}$$

Positive Closure

Let: $L = \{ \texttt{a}, \texttt{bc} \}$

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The "Positive Closure" of a language:

$$L^+ = \bigcup_{i=1}^{\infty} L^i = L^1 \cup L^2 \cup L^3 \cup \ldots$$

Example:

$$L^+ = \{ \underbrace{\texttt{a}, \texttt{bc},}_{L^1} \underbrace{\texttt{aa}, \texttt{abc}, \texttt{bca}, \texttt{bcbc},}_{L^2} \underbrace{\texttt{aaa}, \texttt{aabc}, \texttt{abca}, \texttt{abcbc}, \ldots}_{L^3} \}$$

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Note that ε is not included UNLESS it is in L to start with

Examples

Let: $L = \{ \texttt{a}, \texttt{b}, \texttt{c}, \ldots, \texttt{z} \}$
$D = \{ \texttt{0}, \texttt{1}, \texttt{2}, \ldots, \texttt{9} \}$

$D^+ =$

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"Sequences of zero or more letters and digits"

$L ((L \cup D)^*)$ =
"Set of strings that start with a letter, followed by zero or more letters and and digits."

Regular Expressions

Assume the alphabet is given... *e.g.,* Σ = { a, b, c, ... z }

Example: a (b | c) d* e

A regular expression describes a language.

Meta Symbols:
( )
|
*
ε

Notation:
r = regular expression
L(r) = the corresponding language

Example:
r = a (b | c) d* e
L(r) = { abe,
abde,
abdde,
abddde,
...,
ace,
acde,
acdde,
acddde,
...}

How to "Parse" Regular Expressions

* has highest precedence.
Concatenation as middle precedence.
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Use parentheses to override these rules.

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Concatenation and | are associative.

```
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(a | b) | c  =  a | (b | c)  =  a | b | c
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Example:

`b d | e f * | g a =`

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Example:

`b d | e f * | g a  =  ((b d) | (e (f *))) | (g a)`

Fully parenthesized

Definition: Regular Expressions

(Over alphabet Σ)

- ε is a regular expression.
- If **a** is a symbol (i.e., if **a**$\in\Sigma$), then **a** is a regular expression.
- If **R** and **S** are regular expressions, then **R|S** is a regular expression.
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(Over alphabet Σ)

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  $L(\mathbf{(R)}) = L(\mathbf{R})$

Regular Languages

Definition: "Regular Language" (or "Regular Set")
... A language that can be described by a regular expression.

- Any finite language (i.e., finite set of strings) is a regular language.
- Regular languages are (usually) infinite.
- Regular languages are, in some sense, simple languages.

Regular Langauges $\subset$ Context-Free Languages

Examples:

```
a | b | cab      {a, b, cab}
b*               {ε, b, bb, bbb, ...}
a | b*           {a, ε, b, bb, bbb, ...}
(a | b)*         {ε, a, b, aa, ab, ba, bb, aaa, ...}
```
"Set of all strings of a's and b's, including ε."

Equality v. Equivalence

Are these regular expressions equal?

R = `a a* (b | c)`
S = `a* a (c | b)`

... No!

Yet, they describe the same language.
L(R) = L(S)

"Equivalence" of regular expressions
If L(R) = L(S) then we say R ≈ S
"R is equivalent to S"

"Syntactic" equality versus "deeper" equality...
Algebra:
Does... x(3+b) = 3x+bx ?

From now on, we'll just say R = S to mean R ≈ S

Notation:

Equality	Equivalence
=	=
	==
	≈
	≡
	⇔

Algebraic Laws of Regular Expressions

Let R, S, T be regular expressions...

| is commutative

R | S = S | R

| is associative

R | (S | T) = (R | S) | T = $\underbrace{\text{R | S | T}}_{\text{Preferred}}$

Concatenation is associative

R (S T) = (R S) T = $\underbrace{\text{R S T}}_{\text{Preferred}}$

Concatenation distributes over |

R (S | T) = RS | RT

(R | S) T = RT | ST

ε is the identity for concatenation

ε R = R ε = R

* is idempotent

(R*)* = R*

Relation between * and ε

R* = (R | ε)*

Regular Definitions

Letter = a | b | c | ... | z

Digit = 0 | 1 | 2 | ... | 9

ID = Letter (Letter | Digit)*

Regular Definitions

<u>Letter</u> = `a | b | c | ... | z`
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Names (e.g., <u>Letter</u>) are underlined to distinguish from a sequence of symbols.

`Letter ( Letter | Digit )*`
= {"`Letter`", "`LetterLetter`", "`LetterDigit`", ... }

Lexical Analysis - Part 1

Regular Definitions

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<u>Digit</u> = `0 | 1 | 2 | ... | 9`
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`Letter ( Letter | Digit )*`
= {"`Letter`", "`LetterLetter`", "`LetterDigit`", ... }

Each definition may only use names *previously* defined.

⇒ No recursion

Regular Sets = no recursion

CFG = recursion

Addition Notation / Shorthand

One-or-more: $+$

$X^+ = X(X^*)$

Digit$^+$ = Digit Digit* = Digits

Lexical Analysis - Part 1

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Optional (zero-or-one): **?**

$X? = (X \mid \varepsilon)$

Num = Digit$^+$ (. Digit$^+$)?

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Character Classes: [*FirstChar*-*LastChar*]

Assumption: The underlying alphabet is known ...and is ordered.

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Variations:

Zero-or-more: ab*c = a{b}c = a{b}*c

One-or-more: ab$^+$c = a{b}$^+$c

Optional: ab?c = a[b]c

What does ab...bc mean?

Many sets of strings are not regular.
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{ X**c**X | X is a string of **a**'s and **b**'s }

$\underbrace{\texttt{abbbab}}\texttt{c}\underbrace{\texttt{abbbab}}$

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The Problem?

In order to recognize a string,
these languages require *memory*!

Problem: How to describe tokens?

Solution: Regular Expressions

Problem: How to recognize tokens?

Approaches:

- Hand-coded routines

  Examples: E-Language, PCAT-Lexer
- Finite State Automata
- Scanner Generators (Java: JLex, C: Lex)

Scanner Generators

Input: Sequence of regular definitions

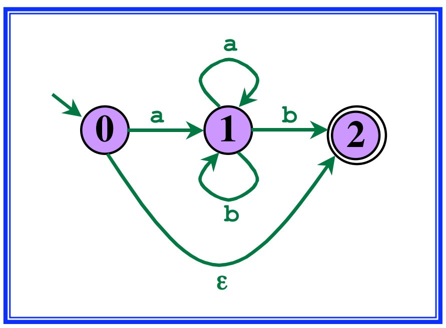
Output: A lexer (e.g., a program in Java or "C")

Approach:

- Read in regular expressions
- Convert into a Finite State Automaton (FSA)
- Optimize the FSA
- Represent the FSA with tables / arrays
- Generate a table-driven lexer (Combine "canned" code with tables.)

Finite State Automata (FSAs)

("Finite State Machines","Finite Automata", "FA")



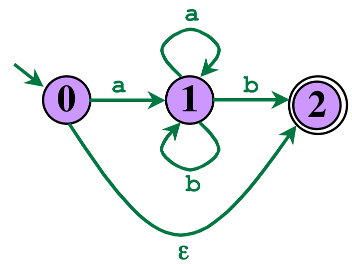
- One start state
- Many final states
- Each state is labeled with a state name
- Directed edges, labeled with symbols
  - Deterministic (DFA)

    No ε-edges

    Each outgoing edge has different symbol
  - Non-deterministic (NFA)

Finite State Automata (FSAs)

Formalism: $< S, \Sigma, \delta, S_0, S_F >$



S = Set of states
  $S = \{s_0, s_1, ..., s_N\}$

Σ = Input Alphabet
  Σ = ASCII Characters

δ = Transition Function
  $S \times \Sigma \rightarrow$ States (deterministic)
  $S \times \Sigma \rightarrow$ Sets of States (non-deterministic)

s_0 = Start State
  "Initial state"
  $s_0 \in S$

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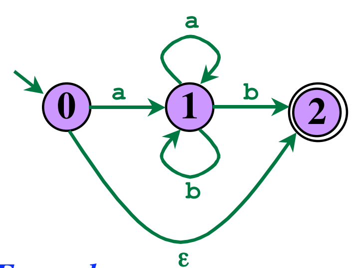


Example:

$S = \{0, 1, 2\}$
$\Sigma = \{$`a`, `b`$\}$
$s_0 = 0$
$S_F = \{ 2 \}$
δ =

Input Symbols

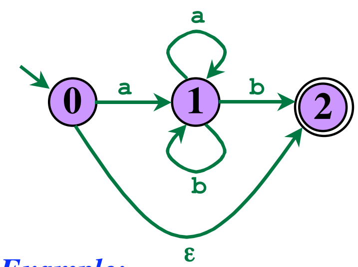
States	a	b	ε
0	{1}	{}	{2}
1	{1}	{1,2}	{}
2	{}	{}	{}

Finite State Automata (FSAs)

A string is "**accepted**"...
(a string is "**recognized**"...)

by a FSA if there is a path
from Start to any accepting state
where edge labels match the string.

Example:
This FSA accepts:
`ε`
`aaab`
`abbb`



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Deterministic Finite Automata (DFAs)

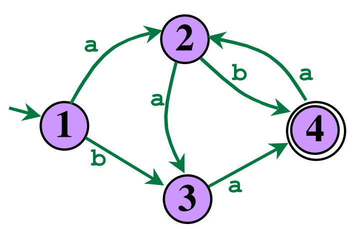
No ε-moves
The transition function returns a single state
$\delta: S \times \Sigma \rightarrow S$

```
function Move (s:State, a:Symbol) returns State
```

$\delta =$

Input Symbols

States	a	b
1	2	3
2	3	4
3	4	---
4	2	---



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Input Symbols

States

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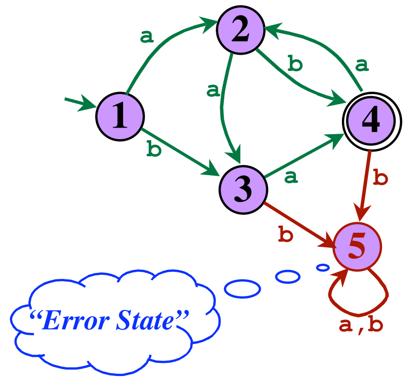
4	2	5
5	5	5



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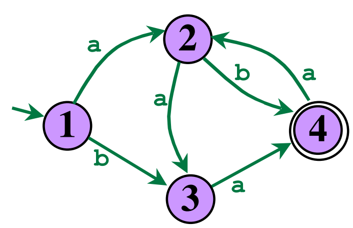


Non-Deterministic Finite Automata (NFAs)

Allow **ε**-moves

The transition function returns a *set of* states

$\delta: S \times \Sigma \rightarrow \text{Powerset}(S)$

$\delta: S \times \Sigma \rightarrow P\ (S)$

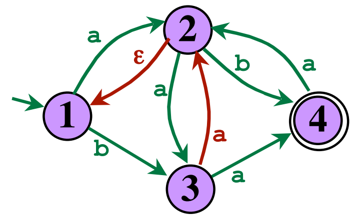
```
function Move (s:State, a:Symbol) returns set of State
```

$\delta =$

Input Symbols (a, b)

States

	a	b	ε
1	{2}	{3}	{}
2	{3}	{4}	{1}
3	{4,2}	{}	{}
4	{2}	{}	{}



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- DFAs, NFAs, and Regular Expressions all have the same "power".
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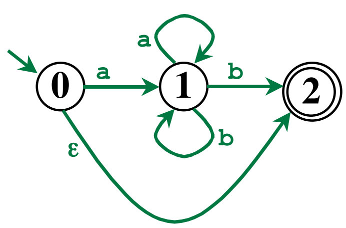
- The DFA may have a lot more states than the NFA.
  (May have exponentially as many states, but...)



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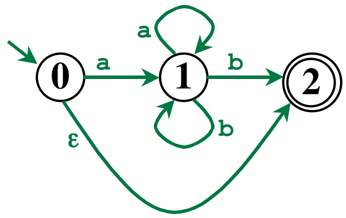
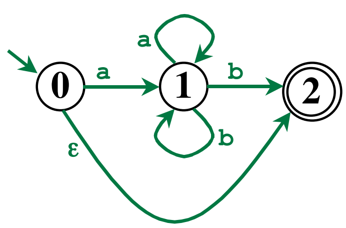


What is the regular expression?

ε | `a (a|b)* b`

What is an equivalent DFA?



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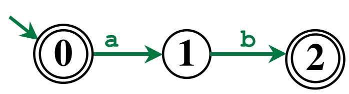

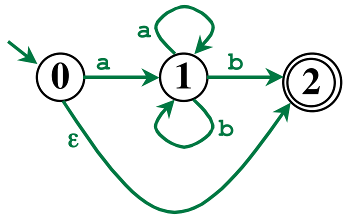

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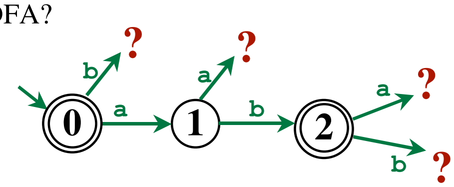

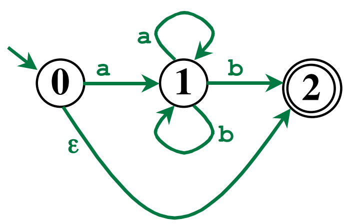

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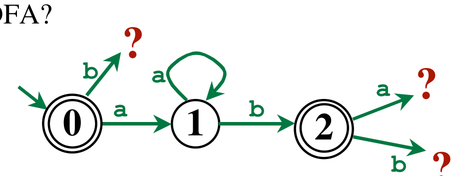

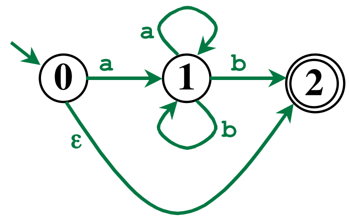

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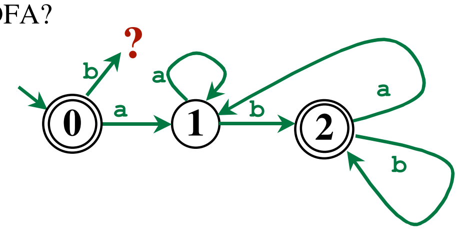
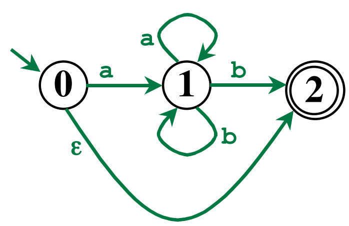


Lexical Analysis - Part 1



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