You can work alone or with one other person to complete this project. If you work with another person, create only one report and one set of m-files. Be sure that both of your names are on the first page of your report.

Background

Figure 1 depicts the water supply components for an orchard irrigation system. Station 1 is the surface of a supply of surface water, e.g., an irrigation ditch or pond. Station 4 is at the supply to the sprinkler system.

For a given orchard topology, the elevation difference between the surface water supply and the pump, Δz_{12}, and the elevation between the pump discharge and the sprinkler supply, Δz_{34}, are known. The supply pressure to the sprinklers, p_4 is also known from the design requirements of the sprinklers.

The goal of this project is to analyze the head loss in the system and find an appropriate pump to meet the flow requirements for the system.

Head Loss Model

The energy conservation equation between station 1 and station 4 is

$$\left[\frac{p}{\gamma} + \frac{V^2}{2g} + z\right]_4 = \left[\frac{p}{\gamma} + \frac{V^2}{2g} + z\right]_1 + h_p - h_L \tag{1}$$

where p is the static pressure, γ is the specific weight of the fluid, V is the average velocity at any section in the pipe, z is the elevation, h_p is the head gain due to the pump, and h_L is the sum of

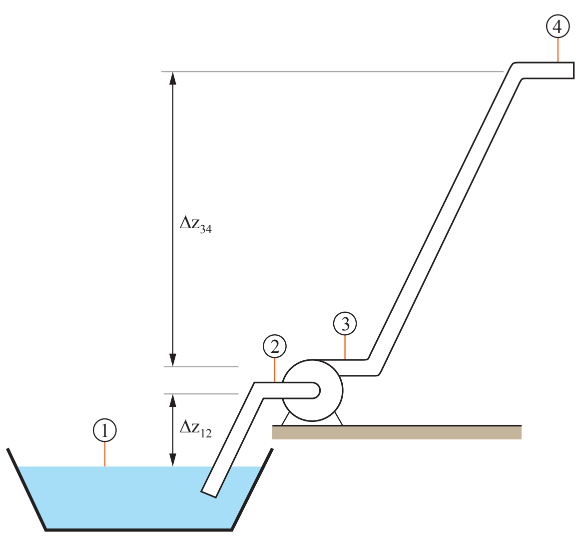

Figure 1: Schematic of irrigation supply system.

viscous and minor head losses between stations 1 and 4. In gage units of pressure, $p_1 = 0$. Also at the free surface, $V_1 = 0$. Taking advantage of those simplifications and rearranging yields

$$\frac{p_4}{\gamma} + \frac{V_4^2}{2g} + h_L + z_4 - z_1 = h_p \tag{2}$$

In terms of the volumetric flow rate in the (round) pipe

$$V_4 = \frac{Q}{A_4} = \frac{4Q}{\pi D_4^2}. \tag{3}$$

where D is the internal diameter of the pipe. Substituting Equation (3) into Equation (2) gives

$$\frac{p_4}{\gamma} + \frac{8Q^2}{\pi^2 g D_4^4} + h_L(Q) + \Delta z_{41} = h_p(Q) \tag{4}$$

where the dependence of h_p and h_L on Q has been made explicit, and the overall elevation change has been written Δz_{41}. Equation (4) can be used in a root-finding procedure to find the system balance point where the head loss, pump head, nozzle exit pressure and elevation gain are in equilibrium.

Viscous and Minor Losses

The head loss due to friction and minor losses is

$$h_L = \left[f\frac{L}{D} + K_L \right] \frac{V_4^2}{2g} = \left[f\frac{L}{D} + K_L \right] \frac{8Q^2}{\pi^2 g D_4^4} \tag{5}$$

where f is the friction factor and K_L is the sum of minor loss coefficients. The friction factor is obtained from a user-written m-file that evaluates the Colebrook equation for turbulent flow and uses the analytical solution for laminar flow

$$f_{\text{lam}} = \frac{64}{\text{Re}} \qquad \text{Re} \leq 3000 \tag{6}$$

When evaluating the Colebrook equation, use a roughness for PVC pipe of $\varepsilon = 0.003\,\text{mm}$.

Comparing Equation (4) and Equation (5) we see that the kinetic energy term in Equation (4) has the same form as a minor loss. Therefore, with no approximation necessary, the kinetic energy term can be absorbed as a minor loss, and Equation (4) can be rewritten

$$\frac{p_4}{\gamma} + h_L(Q) + \Delta z_{41} = h_p(Q) \tag{7}$$

This is the final form of the equation used in the root-finding procedure for finding the balance point for the system and pump.

Minor losses in the piping system are due to elbows, valves, and area changes. All four pumps considered in this project have 1.5 inch (nominal) inlet and outlet flanges. We assume that there are minor losses at the pump where the inlet and outlet flanges are joined to the rest of the pipe system. We further assume that the minor losses at the pump flange change with the size of the pipe connected to the pump. We will use a simplified model of minor loss coefficients that is a constant for each of the different pipe diameters as shown in Table 1.

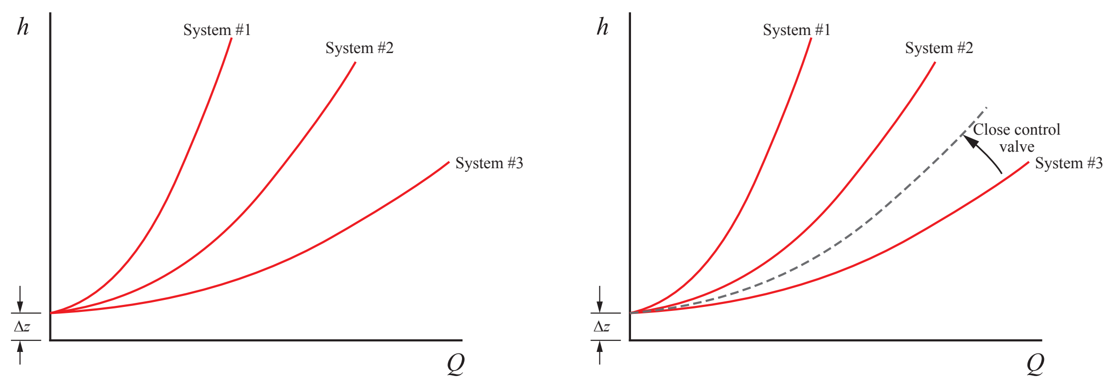

Figure 2: Three system curves with elevation offset, i.e., $h_{\mathrm{tot}}(Q)$. The plot on the right shows the effect of closing a control valve on system 3.

System Curve

The system curve is the $h_L(Q)$ function for a given combination of pipe diameters, pipe lengths, pipe roughness and minor losses. Equation (5) shows that the h_L function is approximately a quadratic of Q. Since h_L also depends on f, the $h_L(Q)$ function is not purely dependent on Q^2.

For a given application, the pump needs to overcome the total head requirement, which is the sum of the head loss and elevation gain.

$$h_{\mathrm{tot}} = h_L(Q) + \Delta z \tag{8}$$

Note that the left hand sides of Equation (4) and Equation (7) have the $h_L(Q) + \Delta z$ terms.

Figure 2 shows a representative plot of h_{tot} for three piping systems with different flow resistances and the same elevation gain. For the term project, consider these three flow resistances to correspond to different pipe diameters. For example, System 1 would have a smaller pipe diameter, and hence higher flow resistance, than either system 2 or system 3. System 3 has the least flow resistance, which would correspond to the largest pipe diameter. If the piping application resulted in no elevation gain between the inlet and the outlet, $\Delta z = 0$ and the pump only has to overcome the flow resistance. Note that the p_4/γ term could be absorbed into a modified Δz that offsets the system curves from $h = 0$.

The plot on the right side of Figure 2 shows what happens when a control valve on Systems #3 is partially closed. Of course, a similar effect would occur when a valve is closed in any of the systems. As the valve closes, the minor loss for the valve (and the entire system) increases. An

Table 1: Pipe dimensions and associated minor loss coefficients. For viscous losses, assume a roughness of $\varepsilon = 0.003\,\mathrm{mm}$ for PVC pipe.

Nominal D (inch)	I.D. (inch)	K_L
1.5	1.754	2.0
2.0	2.193	3.5
2.5	2.655	5.0

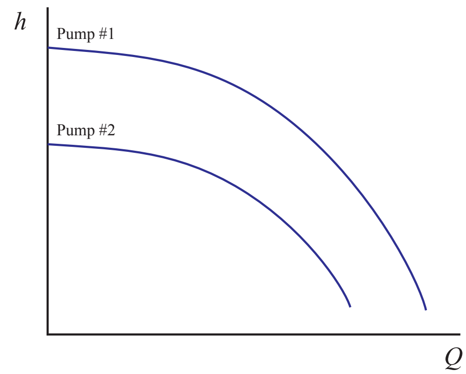

Figure 3: Two pump curves. At a given Q, pump 1 has a higher head than pump 2.

increase in flow resistance results in a steeper $h_L(Q)$ curve. Later we will show how this increase in flow resistance caused the balance point to shift to a lower flow rate, which is what we expect when we close a valve.

When designing a piping system, we size the pump based on a h_{tot} curve corresponding to all control valves being fully open. All valves have a minimum minor loss at the full open position. Assume that the values of K_L in Table 1 are for control valves that are fully open.

Pump Model

For our irrigation project, we will choose a pump from the IRRI-GATOR GT line from Gould[1]. There are other manufacturers, but Gould is a popular brand with a range of pumps suitable for this application.

Figure 3 is a conceptual example of two pump curves that we will use in the rest of the problem description. Neither of these pumps is from the IRRI-GATOR line. The pump curves from the IRRI-GATOR datasheet were digitized and stored as data in the `pumpData` m-file. You can plot the pump curve for the GT07 model with the following code snippet

```
[q07,h07] = pumpData('GT07');
plot(q07,h07,'o')
```

To use the pump data in the piping analysis, you will either need to create a curve fit or use interpolation. Linear interpolation is easily achieved with the built-in `interp1` command

```
q = 3e-3;                %  find head at this flow rate, m^3/s
h = interp1(q07,h07,q);  %  interp1 does linear interpolation by default
```

Creating a curve fit will probably make your code marginally more efficient. However, modern computers are sufficiently fast that interpolation is also acceptable, and you will probably not notice any difference in time to run the code.

Matching the Pump to the System

A given pump connected to a given piping system will operate at the natural balance point where the system head and flow rate match the pump head and flow rate. In engineering design, the operating condition of a pump in a piping system can be predicted by finding the point where the (q, h) curve

[1] `http://goulds.com/centrifugal-pumps-boosters/self-priming-end-suction/irri-gator-model-gt/`

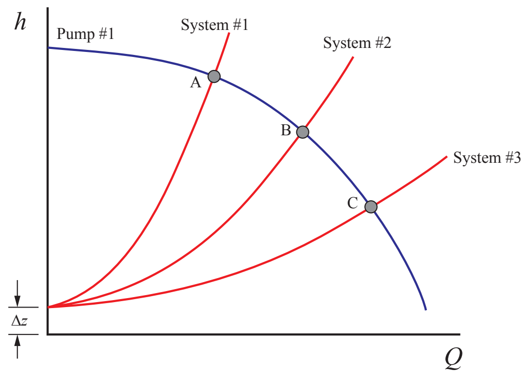

Figure 4: Matching conditions for a pump with three different system curves.

of the pump intersects the (q, h) curve of the system. Graphically, the matching condition can be obtained by overlaying the system curve(s) on a plot of the pump curves.

Figure 4 shows the intersection of our model "pump 1" with the three hypothetical systems. In Figure 4, system 1 has a higher resistance than system 2 so the matching condition A has a higher head, but lower flow rate than matching condition B. System 3 has the least flow resistance, and for a given pump, the balance point for system 3 will be at a lower head and higher flow rate than either system 1 or system 2.

In a piping design problem, there is a *minimum* head and a *minimum* flow rate to meet the requirements of the application. Figure 5 shows how Q_{min} and h_{min} thresholds would determine which pump would meet the requirements for a hypothetical pump and three different piping systems. Pump 1 and system 1 do not meet the design conditions because the match point is at a flow rate below Q_{min}. Pump 1 and system 2 satisfy the design conditions because the matching condition exceeds the Q_{min} and h_{min} requirements.

Pump 1 and system 3 do not initially appear to meet the h_{min} requirement. However, by closing

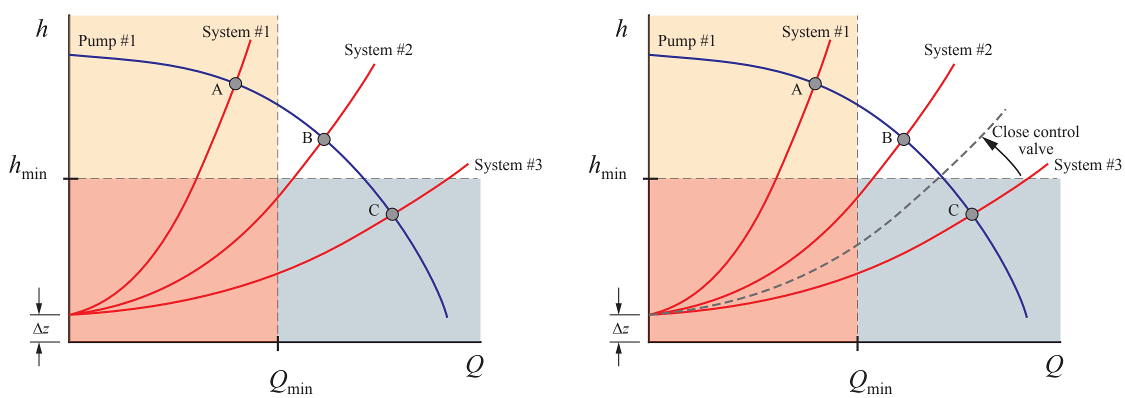

Figure 5: Pump and system matching conditions relative to design requirements for minimum flow rate, Q_{min} and minimum heat, h_{min}. Matching condition B satisfies both the Q_{min} and h_{min} requirements, so the combination of pump 1 and system 2 meet the design requirements.

a control valve, the flow rate for the pump 1 and system 3 combination can meet both the $Q_{\min}$ and $h_{\min}$ requirements.

Assignment

The ultimate goal of this assignment is to find a pump that will supply $1.2 \times 10^{-3}\,\mathrm{m^3/s}$ of water with a lift of 10 m and a nozzle backpressure of $p_4 = 200\,\mathrm{kPa}$. You will choose one of the four IRRI-GATOR pumps with PVC pipe.

1. Modify the m-file (from HW 5) for calculating the friction factor from the Moody chart so that it correctly handles low Re i.e., laminar flow. Refer to Equation (6). The modification is a simple extension to the m-file so that either the laminar or turbulent flow friction factor is calculated based on the Reynolds number.

2. Write an m-file to evaluate and plot system curve h_L as a function of Q, L and d. Assume the fluid is water at 16°C. Create a plot with curves for PVC pipe with 1.5, 2.0 and 2.5 nominal diameter and $L = 100\,\mathrm{m}$.

3. On a single set of axes, plot the pump curves for the DT07, DT10, DT15, and DT20 models of the Gould family of IRR-GATOR pumps. The pump curves should be h in meters versus Q in $\mathrm{m^3/s}$.

4. Find a combination of pump and piping system that simultaneously meets these design requirements for water at 16 °C.

   - $L = 100\,\mathrm{m}$
   - $\Delta z_{14} = 10\,\mathrm{m}$
   - $Q_{\min} = 1.2 \times 10^{-3}, \mathrm{m^3/s}$
   - $p_4 = 200\,\mathrm{kPa}$

   This step amounts to finding the Q that satisfies Equation (7).

   Briefly describe the method you used (one paragraph maximum) and include the MATLAB code.

Note that this design problem is not complete without an economic analysis, which is beyond the scope of this project. The next step would be to analyze the cost of purchasing and installing the equipment, as well as the energy cost of running the system during its expected lifetime. The analysis presented here only determines whether a particular combination of pump and system meets the minimum flow rate requirements.