ME 350 Lab Exercise 8 Solving Linear Systems
Winter 2017

1. A formula in a statistics calculation is written $p^T Ap$, where A is a $n \times n$ matrix, and p is a $n \times 1$ column vector of ones.

   (a) What type of quantity is $p^T Ap$? (e.g., a matrix, vector, scalar, or undefined?)

   (b) Given the following matrix A and vector p, compute $p^T Ap$.

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}, \qquad p = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

2. Given

$$A = \begin{bmatrix} 2 & 4 & -2 & -2 \\ 1 & 2 & 4 & -3 \\ -3 & -3 & 8 & -2 \\ -1 & 1 & 6 & -3 \end{bmatrix}, \qquad b = \begin{bmatrix} -4 \\ 5 \\ 7 \\ 7 \end{bmatrix}.$$

   create the MATLAB variables `A` and `b` that store matrix A and vector b. Make sure `b` is a column vector.

   (a) Write the MATLAB expression that extracts row 3 from `A`.

   (b) Write the MATLAB expression that extracts column 2 from `A`.

   (c) Create a new MATLAB matrix `B` from

$$\begin{bmatrix} a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

   (d) Solve the system $Ax = b$ for x.

   (e) Once you have the solution, x, compute the residual, $r = b - Ax$. Is r acceptable?

3. Use MATLAB's built-in \ operator to solve the following system of equations:

$$\begin{aligned} x_1 + 2x_2 - x_4 &= 9, \\ 2x_1 + 3x_2 - x_3 &= 9, \\ 4x_2 + 2x_3 - 5x_4 &= 26, \\ 5x_1 + 5x_2 + 2x_3 - 4x_4 &= 32. \end{aligned}$$

   Hints:

   - What are the unknowns? What are the known constants?
   - Use the row view of matrix-vector multiplication to identify each row of the coefficient matrix.
   - Define the coefficient matrix as the MATLAB variable `A` and the right hand side vector as the MATLAB variable `b`. Make sure `b` is a *column* vector.

- Solve the system with `x = A\b`.
- Verify the result by computing the residual, `r = b - A*x`. How do you know if `r` is small?

4. Centrifugal pumps are common devices used to move liquid through piping systems. Figure 1 shows a typical *pump curve*, which relates the flow rate, q to the pressure head h. Different pump designs have different pump curves.

   To automate design calculations, it is advantageous to have a formula for the pump curve. The $h(q)$ curve shown in Figure 1 appears to be roughly parabolic, so it is reasonable to seek a relationship of the form

$$h = c_1 q^2 + c_2 q + c_3.$$

   Three convenient (q, h) data points are listed in the following table

q (m^3/s)	1×10^{-4}	8×10^{-4}	1.4×10^{-3}
h (m)	115	110	92.5

   These points are shown as open circles in Figure 1. Substituting the (q, h) pairs into the preceding equation results in three equations for the unknown c_i:

$$\begin{aligned} 115 &= 1 \times 10^{-8}\ c_1 + 1 \times 10^{-4}\ c_2 + c_3, \\ 110 &= 64 \times 10^{-8}\ c_1 + 8 \times 10^{-4}\ c_2 + c_3, \\ 92.5 &= 196 \times 10^{-8}\ c_1 + 14 \times 10^{-4}\ c_2 + c_3, \end{aligned}$$

   which can be written in matrix form as

$$\begin{bmatrix} q_1^2 & q_1 & 1 \\ q_2^2 & q_2 & 1 \\ q_3^2 & q_3 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}.$$

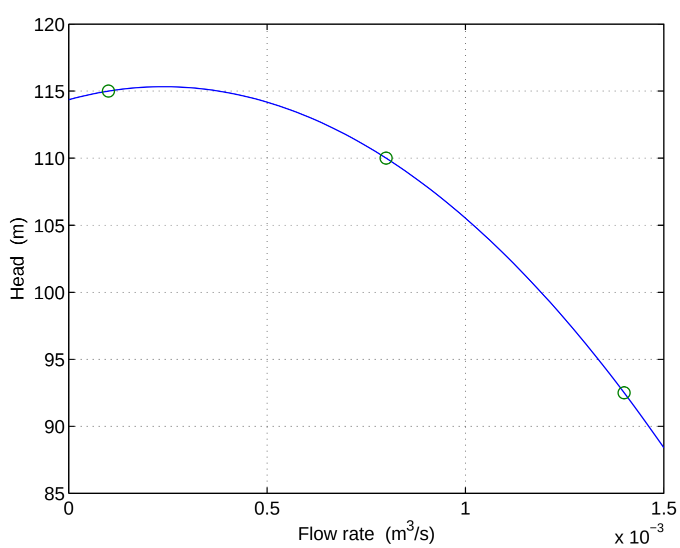

Figure 1: Typical performance curve for a centrifugal pump.

Using more compact symbolic notation the matrix equation is $Ax = b$, where

$$A = \begin{bmatrix} q_1^2 & q_1 & 1 \\ q_2^2 & q_2 & 1 \\ q_3^2 & q_3 & 1 \end{bmatrix}, \qquad x = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}, \qquad b = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}.$$

(a) Write an m-file that solves the system of equations for determining the equation of the pump curve, where the pump data are defined by the two vectors

```
h = [115, 110, 92.5];
q = [1.0e-4, 8.0e-4, 1.4e-3];
```

Use the `h` and `q` vectors to define the coefficients of matrix A and column vector b.

(b) Extend your m-file to plot the interpolant and the original data points on the same axis. In other words, recreate Figure 1. This step requires you to write a one-line formula for computing h from a vector of q values.

Hint: `plot(q,h,'o',qplot,hpplot,'-')` where `qplot` and `hplot` provide the data for the curve over the range $0 \leq q \leq 1.5 \times 10^{-3}$ m^3/s.

5. The `pumpcurve` function has data for Goulds pump model GT10, which is shown in the picture to the right. The data is from the Grainger's catalog at `www.grainger.com`.

The direct URL for the pump is `https://www.grainger.com/product/GOULDS-WATER-TECHNOLOGY-1-HP-Centrifugal-Pump-1N440`.

Run the `pumpcurve` function and answers the following questions.

(a) What is the meaning of the warning message?

(b) Does the interpolating polynomial match the data?

(c) Modify the `pumpcurve` function to include a third order polynomial curve fit.

```
nfit = 3;
cp = polyfit(q,h,nfit);
qfit = linspace( min(q), max(q) );
hfit = polyval(cp,qfit);    %  Evaluate curve fit at qfit points
```

Add the curve fit to the plot.

(d) Modify the `pumpcurve` function so that it creates a second figure window with just the raw data and the curve fit. Is the curve fit more reasonable than the polynomial interpolant?