Experiment 6

Sudden Expansion

Purpose

The objective of this experiment is to investigate the relationship between pressure drop, velocity profile, and area change for a sudden expansion in a duct. The experiments are designed to answer the following questions:

- What is the velocity profile downstream of the sudden expansion?
- What is the relationship between the velocity profile and the flow rate?
- What is the relationship between the pressure change across the sudden expansion and the magnitude of the centerline velocity immediately downstream of the sudden expansion?

Apparatus

Figure 6.1 depicts the main components of the laboratory apparatus. A blower draws air through a duct constructed from acrylic tubing of two diameters. The inlet end of the duct has diameter d_1 and length L_1. A longer section of tubing with diameter $d_2 > d_1$ connects the inlet section to the blower. The transition from d_1 to d_2 is abrupt.

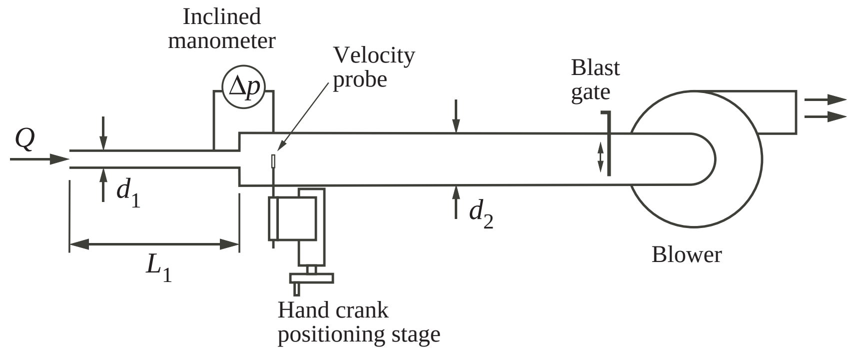

Figure 6.1: Apparatus for measuring flow downstream of a sudden expansion.

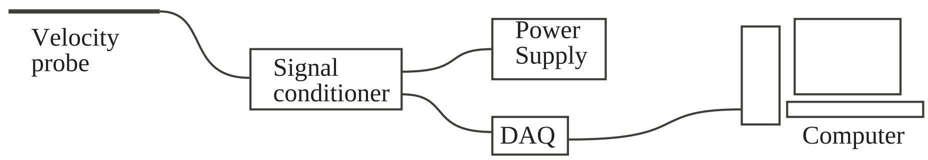

Figure 6.2: Data acquisition system for velocity measurements.

The flow rate through the duct is controlled by adjusting the blower speed switch (either high or low) and a sliding damper, also called a *blast gate*. Velocity downstream of the sudden expansion is measured by a velocity sensor. The pressure change across the sudden expansion is measured with an inclined manometer.

Velocity Sensor

The air velocity is measured with a TSI Model 8455 Air Velocity Transducer[1]. The velocity transducer is a thermal anemometer. A small electrical current is passed through the probe tip and the rate of cooling is calibrated to the oncoming velocity. A velocity sensor is mounted on a manual positioning stage that allows the sensor to be moved radially across the larger duct. As shown in the schematic in Figure 6.2, the velocity sensor is connected to a signal conditioner, a power supply, and a data acquisition DAQ unit. The DAQ is connected to a computer via a USB cable.

Figure 6.3 shows the location of the sensing element in the probe tip. The sensing element looks like a small cylinder suspended in the center of an opening in the otherwise solid probe assembly. For the purpose of this laboratory exercise the velocity is assumed to be measured at a *virtual sensor* location at the center of the opening in the probe tip. In other words the velocity measurement is assumed to occur at a distance $w_t/2$ from the physical end of the probe.

The signal conditioning unit controls the power to the sensing tip and provides an output voltage. The signal conditioner linearizes the signal so that the output voltage is linearly related to the air velocity past the sensor. The DAQ converts the analog voltage to digital values that are transmitted

[1] TSI Incorporated, Environmental Measurements and Controls Division, 500 Cardigan Road, Shoreview, Minnesota, `www.tsi.com`.

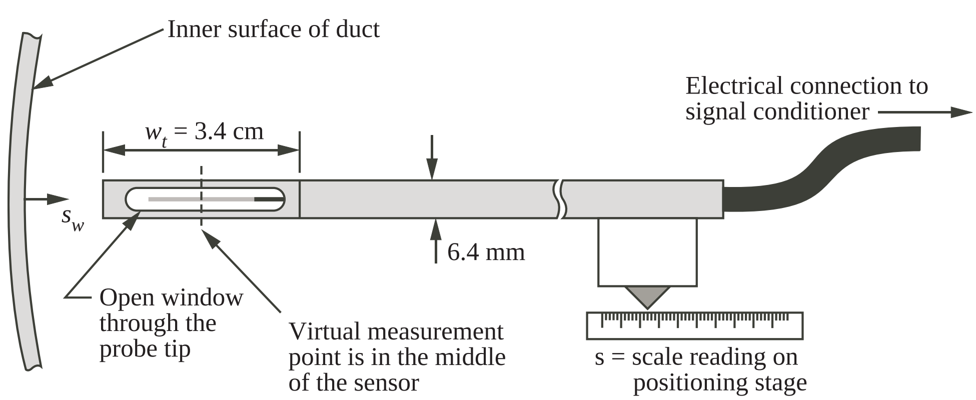

Figure 6.3: Detail of the velocity probe tip.

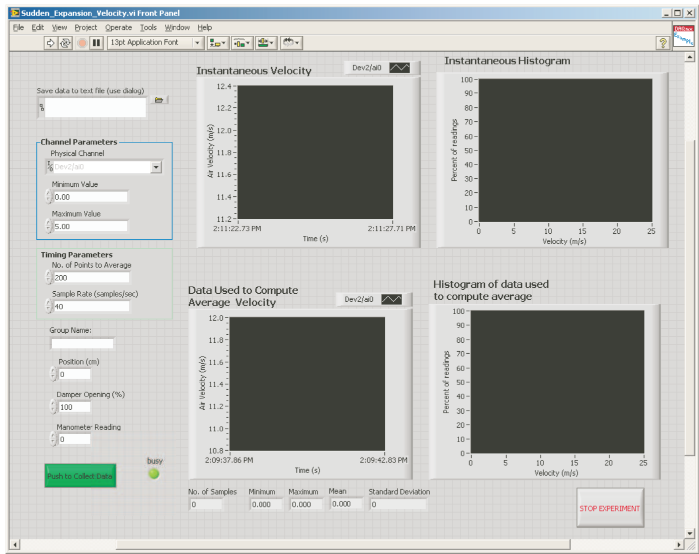

Figure 6.4: Screen image of the LabVIEW virtual instrument (VI) used to collect data.

to and stored on the computer.

The location of the velocity sensor in the duct is controlled by a manual positioning stage. The user cranks the stage to move the probe. The position is indicated by a linear scale on the side of the positioning stage. Figure 6.3 shows the schematic relationship between the probe position in the duct and the position indicated on the positioning stage. The indicator on the positioning stage defines the value of s. When the probe tip is touching the inner surface of the duct wall the indicator on the positioning stage has a value of s called s_0.

Pressure Measurement

The pressure change across the sudden expansion is measured by a Dwyer Model 102.5 Durablock portable inclined manometer[2]. The two pressure taps on the duct wall are located 3 inches upstream and 6 inches downstream of the sudden expansion, respectively.

Data Acquisition

Output of the velocity sensor is digitized with a USB-6008 data acquisition device from National Instruments[3]. The LabVIEW software system (also from National Instruments) is used to control and obtain readings from the DAQ device. A LabVIEW *virtual instrument* (VI) provides a user interface for the data collection. Figure 6.4 is a screen image of the VI. Along the left boundary of the VI are input areas for configuring the DAQ device, and for entering values for the probe position and the pressure difference measured by the manometer.

Figure 6.5 is an enlarged view of the upper left corner of Figure 6.4. The start and stop buttons in the toolbar are used to run (and stop) the VI. While the VI is running, samples of the velocity signal are displayed in the upper two plot frames in Figure 6.4. Figure 6.6 is an enlarged view of the lower left corner of Figure 6.4.

[2]Dwyer Instruments Inc., Michigan City, Indiana, `www.dwyer-inst.com`.
[3]National Instruments, Austin, Texas, `www.ni.com`.

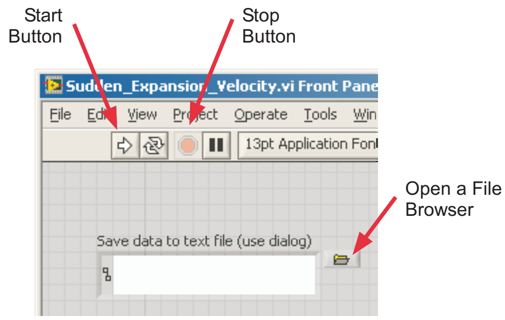

Figure 6.5: Partial view of the virtual instrument used to collect data.

For each measurement configuration, the user manually enters the manometer reading and probe position into the input controls shown in Figure 6.6. The user then clicks the "Push to Collect Data" button on the screen to initiate the recording of several velocity sensor values.

The central region on the left side of the VI contains a box labeled "Channel Parameters". Do not adjust these values unless told to do so by the teaching assistant in the lab. Typically 200 velocity readings are made at 40 readings per second.

Procedure

The laboratory exercise involves two experimental procedures: (1) measurement of the velocity profile for a fixed flow rate; and (2) measurement of the pressure drop as a function of flow rate (or centerline velocity).

Measurement of the Velocity Profile

The purpose of these measurements is to obtain data for the axial velocity as a function of position across the duct.

1. Record the inside diameter, d_1, and wall thickness of the small tubing at the inlet. Measure

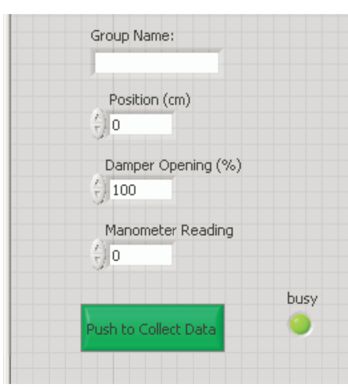

Figure 6.6: Partial view of the virtual instrument used to collect data. The input controls in this section are used to record the probe position, and the pressure drop.

the outside diameter of the large tubing. From the outside diameter of the large tubing and the wall thickness, compute the inside diameter d_2 of the large tubing.

2. Use the Bernoulli equation and the duct diameters to estimate the pressure difference that will be read by the manometer when the average velocity in the larger diameter pipe is 15 m/s.
3. Turn on the blower and set the air speed by adjusting the motor speed (high/low) and blast gate. Keep those fixed for remainder of data collection. Turn on the power supply to the signal conditioner for the velocity sensor.
4. Move the probe tip fully into the duct so that it *gently* touches the far wall. Record the indicator on the positioning stage. Later you will use this value, called s_0, to determine the position of the probe relative to the centerline of the duct.
5. Choose one member of your group to operate the VI. That person logs uses the lab computer to log on to their MCECS Windows account. Locate the VI on the hard drive of the lab computer. Ask the lab TA for help, if necessary. Double click the file icon for the VI to start LabVIEW. You should eventually see a screen like that in Figure 6.4. Start the VI by clicking the "Run" arrow shown in Figure 6.5. Do not stop the VI until all the data for the velocity profile are collected.
6. Click the File Browser button shown in Figure 6.5. Navigate to a convenient directory for your account on the file server. Do not choose a directory on the `C:` drive of the lab computer. Enter a file name with either a `.txt` or `.dat` extension.
7. Move the probe to a new location and read the probe location from the scale on the manual positioning stage. Enter the value of position in the "Position" box on the virtual instrument. (See Figure 6.6.) Wait for the fluctuations in the velocity signal to settle. It is especially important to wait 15 to 30 seconds after moving the probe.
8. After the velocity signal settles, manually read the pressure indicated by the inclined manometer. Enter the pressure drop in the "Manometer Reading" box on the virtual instrument. See Figure 6.6. Press the "Push to Collect Data" button to start recording the output of the velocity probe. Wait for the "Busy" indicator on the VI to change from red to green.
9. Repeat step 7 for several (between 12 and 25) locations of the probe tip across the duct. Randomize the order.
10. Press the "Stop Experiment" button to close the data file for the current measurements.

Measurement of the Pressure Change versus Flow Rate

The purpose of these measurements is to obtain data for the pressure change across the sudden expansion as a function of average velocity in the duct.

1. Click the File Browser button shown in Figure 6.5. Navigate to a convenient directory for your account on the file server. Do not choose a directory on the `C:` drive of the lab computer. Enter a file name with either a `.txt` or `.dat` extension. Be sure to use *a different file name* from the one you used in the velocity profile measurements.
2. Move the velocity probe to the centerline. Read the probe position and enter the value into the "Position" box on the VI. Wait for the velocity signal to settle. Read the pressure indicated by the inclined manometer. Enter the pressure drop in the "Manometer Reading" box on the virtual instrument. Press the "Push to Collect Data" button to start recording the output of the velocity probe. Wait for the "Busy" indicator on the VI to change from red to green.

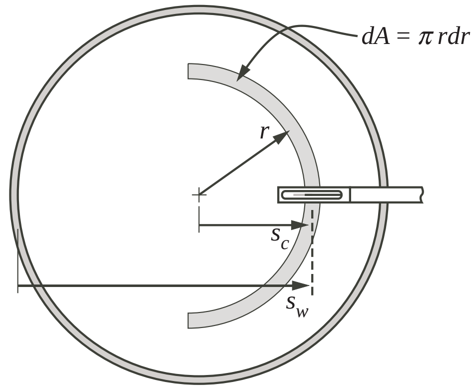

Figure 6.7: Coordinates used to indicate the probe position and integrate the velocity profile.

3. Without further adjustment of velocity probe position, adjust the blower speed to a new setting (change motor speed and/or damper position). Wait for the velocity signal and the manometer reading to settle. Enter the the pressure drop in the "Manometer Reading" box on the VI and press the "Push to Collect Data" button to record the output of the velocity probe. Wait for the "Busy" indicator on the VI to change from red to green.

4. Repeat step 3 for several (say 10 or more) flow rates.

Analysis

Conversion of Stage Position to Virtual Probe Position

Figure 6.7 identifies two ways to indicate the virtual location of the velocity measurement. The notion of a virtual location is necessary because the velocity sensor has an active length on the order of several millimeters. We define the virtual location of the velocity measurement as the midpoint of the open window at the tip of the velocity probe.

Let s be the indicated position on the scale for the positioning stage. Let s_w be the virtual position of the velocity measurement relative to the far wall of the duct. If s_0 is the reading on the scale of the positioning stage when the tip of the probe is touching the wall, then

$$s_w = s - s_0 + \frac{w_t}{2} \tag{6.1}$$

where w_t is the width of the active end of the probe as shown in Figure 6.3. If s_c is the the virtual position of the velocity measurement relative to the centerline of the duct, then

$$s_c = s_w - R_{i,2} = s - s_0 + \frac{w_t}{2} - R_{i,2}. \tag{6.2}$$

In a proper radial or axisymmetric coordinate system, the value of the radial coordinate, r, is never negative. As it is defined in Equation (6.2), the value of s_c is like a radial position that can be positive or negative.

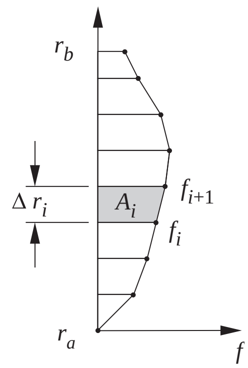

Figure 6.8: Trapezoid rule for numerical integration of the velocity profile. labelfig:trapRule

Numerical Integration of the Velocity Profile

The average velocity at any axial location in the duct is

$$V = \frac{1}{A}\int_A u dA \tag{6.3}$$

where u is the velocity component along the duct axis, and A is the cross-sectional area of the duct. Values of u at different radial locations are obtained when the velocity transducer is traversed across the duct. Because the $u(r)$ data consists of discrete measurements, Equation (6.3) must be evaluated numerically.

Approximate the velocity profile as a series of straight lines. The integral in Equation (6.3) is then a sum of trapezoidal areas as shown in the sketch to the right.

$$\int_{r_a}^{r_b} f(r)\,dr \approx \sum A_i$$

The area of a typical trapezoid is

$$\begin{aligned} A_i &= (r_{i+1} - r_i)f_i \ + \ \frac{1}{2}(r_{i+1} - r_i)(f_{i+1} - f_i) \\ &= \frac{1}{2}(f_{i+1} + f_i)(r_{i+1} - r_i) \\ &= \frac{1}{2}(f_{i+1} + f_i)\Delta r_i \end{aligned}$$

where $\Delta y_i = y_{i+1} - y_i$. The numerical approximation to the integral is known as the trapezoid rule

$$\int_{r_a}^{r_b} f(r)\,dr \approx \sum \frac{1}{2}(f_{i+1} + f_i)\Delta r_i \tag{6.4}$$

Evaluation of the mass flow rate integral can be done in any computing environment such as Matlab or Excel.

Data Reduction Steps

Analysis of the velocity profile data involves these steps:

1. Translate the $u(s)$ data to $u(s_c)$. This just means that a new value of s_c needs to be computed for each of the original values of s. There is no change to the u values, just the value of s_c that they are associated with.

2. Use interpolation to find the value of u for $s_c = 0$. Call this value u_{CL}, where "CL" indicates centerline.

3. Split the data in to two sets: one for measurements between the centerline and the far wall ($s_c < 0$), and another for measurements between the centerline and the near wall ($s_c > 0$). Call these data sets the "negative" and "positive". Include the value of u_{CL} in each data set.

4. Include the data point $u = 0$ at $r = R_{i,2}$, where $R_{i,2}$ is the inner radius of the larger duct.

5. Sort both the positive and negative data sets in order of increasing r. For both the negative and positive data sets all values of r will be zero or positive.

6. Apply the trapezoid rule to the computation of volumetric flow rate for each of the data sets. For each data set the integral for half of the duct is

$$Q_{\text{half}} = \int_0^{R_{i,2}} u(r)\pi r dr \tag{6.5}$$

   where $\pi r dr$ is the area element for half of an annular sector, as shown in Figure 6.7.

7. Evaluating Equation (6.5) for each half of the duct gives values of Q_{neg} and Q_{pos} for the negative and positive halves of the duct. Compute the total flow rate with

$$Q_{\text{tot}} = |Q_{\text{neg}}| + Q_{\text{pos}} \tag{6.6}$$

   where the absolute value sign accounts for the negative s_c values.

8. Compute the average velocity in the duct from

$$V = \frac{Q_{\text{tot}}}{A} = \frac{Q_{\text{tot}}}{\pi R_{i,2}^2} \tag{6.7}$$

Report

Perform the analysis described in the *Analysis* section.

1. Print a nicely formatted version of the raw data in the appendix of your report.

2. Estimate the volumetric flow rate by integrating the velocity profile data. Do this for each velocity profile data set.

3. Plot velocity profile(s): $u = u(s)$ and overlay the average velocity computed with Equation (6.7).

4. Create a table of dimensionless pressure drop ($\Delta p/(0.5\rho u^2(0))$) versus centerline velocity, $u(0)$, and versus the square of the centerline velocity. Add a column of the table for the dimensionless pressure change versus centerline velocity that is predicted by the Bernoulli equation.

5. Plot dimensionless pressure drop $(\Delta p/(0.5\rho u^2(0)))$ versus centerline velocity $u(0)$ and versus the square of the centerline velocity. On the same axis, plot the dimensionless pressure drop versus centerline velocity that is predicted by the Bernoulli equation.

Answer the following questions

1. Is the velocity profile uniform? Is it symmetric? Can you identify any causes for lack of symmetry?

2. How does the measured pressure change compare to the pressure change predicted by the Bernoulli equation? What could account for any discrepancies?

Reference

B.R. Munson, D.F. Young, and T.H. Okiishi, *Fundamentals of Fluid Mechanics*, 5th ed., 2006, Wiley and Sons, New York.