

Experiment 5

Bernoulli Equation

Purpose

The objective of this experiment is to measure the variation in air velocity along the axis of a duct with variable cross sectional area. The measurements enable experimental verification of the Bernoulli Equation.

Apparatus

Figure 5.1 depicts the apparatus used in the experiment. It consists of a blower connected to a duct with a Venturi. At the exit of the duct is a support for a Pitot probe. The support allows the probe to be positioned at different axial locations in the duct.

The Pitot tube (see Munson et al., §3.5) is a device that enables simultaneous measurement of the stagnation and static pressure of a moving fluid. Figure 5.2 is a schematic of the measurement tip of the Pitot tube used in the experiment.

A Pitot tube has an opening that faces upstream. Fluid approaching this opening is assumed to be brought to rest isentropically. Thus, the upstream-facing opening measures the stagnation pressure. The stagnation pressure port is the open end of a continuous tube that extends to a pressure tap outside of the flow stream. The stagnation tap is attached to a pressure measuring device.

Downstream a short distance from the stagnation port are a series of openings around the circumference of the probe. These ports sense the static pressure. Inside the probe body, the static pressure ports are connected to a continuous tube that extends to a pressure tap outside of the flow stream. By connecting multiple static pressure ports to a single pressure transmitting duct, the effects of misalignment of the probe with the local flow direction are minimized. The static pressure tap is attached to a pressure measuring device.

Although the static pressure ports are downstream from the stagnation port, one normally assumes that the stagnation port and the static ports measure their respective pressures at the same axial location. More precisely, the assumption is that the static pressure does not change significantly as the fluid moves the short distance from the stagnation port to the static port(s). For the Pitot probe used in this laboratory exercise, the distance between the stagnation and static pressure ports is 11/16 inch.

In the laboratory exercise, a Pitot tube is used to measure air velocity in a duct with variable cross sectional area in the flow direction. Because the duct area is not constant, the average fluid velocity varies in the flow direction. The Bernoulli equation shows that changes in velocity along

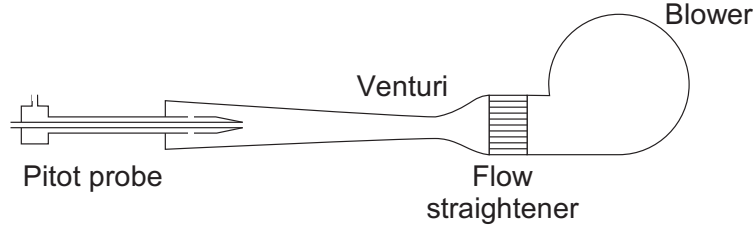


Figure 5.1: Apparatus for verifying the Bernoulli Equation.

a horizontal streamline cause a change in static pressure. Thus, in regions of the duct where the area is rapidly changing in the flow direction, it is not safe to assume that the static pressure is the same at the stagnation and static pressure ports. The measurement procedure and data reduction steps in the laboratory exercise must compensate for the variation in static pressure due to changes in duct area.

On the opposite end of the Pitot probe from the measuring tip are taps that allow the stagnation and total pressure ports to be connected to a pressure measuring device. Not shown in Figure 5.1 are three U-tube manometers that are used to measure the static pressure, dynamic pressure and stagnation pressure from the Pitot probe. The manometer measuring dynamic pressure measures the difference between the stagnation and static pressure ports. This measurement will indicate the true dynamic pressure only in sections of the duct where the area does not change in the flow direction.

Theory

The Bernoulli equation applies to steady, incompressible flow along a streamline with no heat or work interaction. One form of the Bernoulli equation is

$$p_0 = p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

where p is the pressure, ρ is the density, V is the fluid velocity, γ is the specific weight of the fluid, and z is the elevation measured from an arbitrary datum. The subscripts 1 and 2 denote two positions along the streamline. The total or stagnation pressure, p_0 , is a measure of total energy of the flowing stream. When the Bernoulli equation applies the stagnation pressure is constant along a streamline.

Figure 5.2 is a sketch of the flow field near the tip of the Pitot probe. The streamline that terminates at point A is called the stagnation streamline because as the fluid approaches A it decelerates until it has zero velocity. The fluid velocity is V on the stagnation streamline far upstream of A . If we assume that the deceleration is reversible, the pressure tap at point A measures the total pressure in the vicinity of the tip. Note that there is no flow through the Pitot tube.

The Pitot probe is a small aerodynamic body that does not significantly disturb the flow field except for the stagnation streamline. Although the streamlines curve as the fluid passes around the tip there is negligible change in velocity for fluid that follow streamlines near the probe. Along the streamline through point C , for example, the fluid velocity is assumed to be constant. In addition, over distances on the order of the Pitot tube diameter, elevation changes are negligible. All the streamlines in Figure 5.2, therefore, have the same stagnation pressure

$$p_{0,A} = p_{0,C}$$

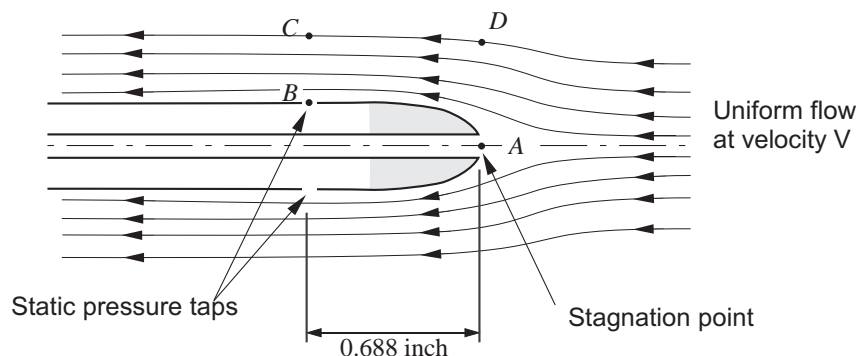


Figure 5.2: Flow field near the tip of a Pitot probe.

There is a different relationship between points B and C . Since there is no velocity in the direction between B and C the static pressure is the same at these points,

$$p_B = p_C$$

In other words the pressure tap at point B measures the static pressure in the vicinity of the probe tip. Combining the foregoing equations and assumptions, the pressure difference between point A and point C is the dynamic pressure

$$p_A - p_B = \frac{1}{2}\rho V^2 \quad (5.1)$$

Equation (5.1) applies if the fluid velocity does not change between points D and C .

Procedure

The Pitot tube is used to measure the variation of stagnation, static, and dynamic pressure along the length of the variable area duct. The basic measurement procedure is to record the differential heights of the fluid columns for the three manometers attached to the Pitot tube. The manometer readings are recorded at a series of probe positions along the duct. Care must be taken in selecting the probe locations so that the correct dynamic pressure can be obtained from measurements of stagnation and static pressure at two separate probe locations.

Figure 5.3 shows the location of the Pitot probe at two measurement stations located $\Delta x = 11/16 = 0.688$ inch apart. The term *station* refers to the position of the probe in the apparatus. At each station, the static ports are 0.688 inch downstream from the stagnation port. If the probe is moved so that subsequent stations are 0.688 inches apart, then the stagnation port at station $i + 1$ is at the same location as the static pressure port for station i . Therefore, by moving the probe in increments of 0.688 inch, the correct dynamic pressure can be obtained by subtracting the static pressure at station i from the stagnation pressure at station $i + 1$.

Table 5.1 is a suggested layout of a table to record raw data during the experiment. The first column is a station number. The second column is the axial position of the probe measured from an arbitrary reference point. It makes sense to define x as the position of the probe tip. The location of $x = 0$ is up to you. The last three columns in the table are for recording the height differential of the three U-tube manometers on the apparatus. Record all three manometer readings for each position of the probe.

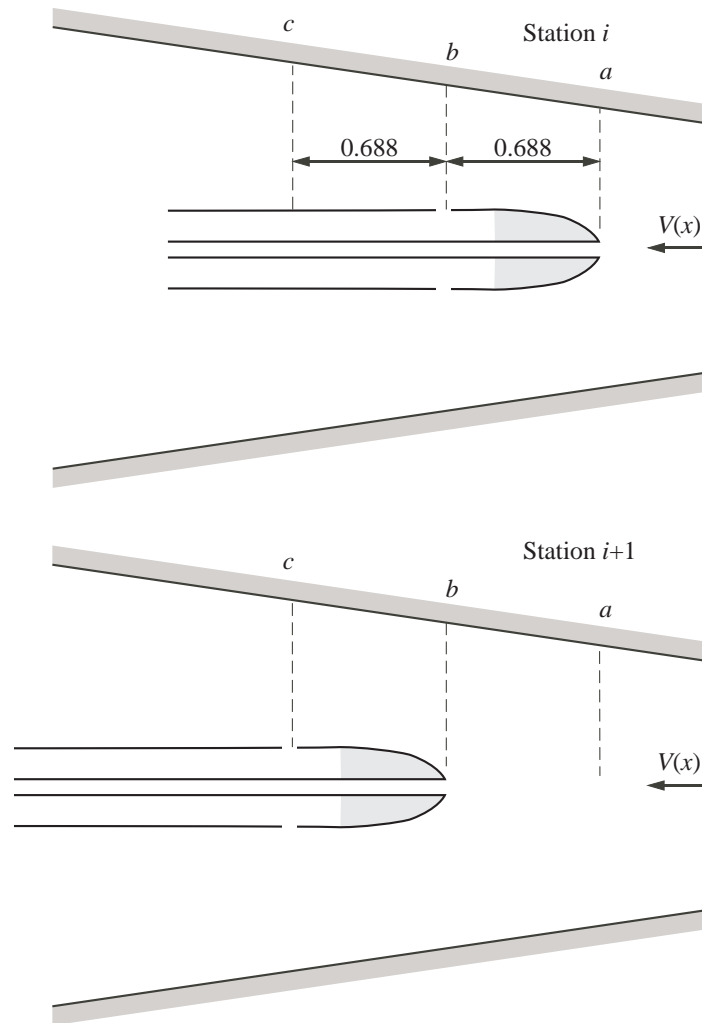


Figure 5.3: Using a Pitot probe to measure velocity in a duct with variable area. If the probe is moved a distance equal to the spacing between the stagnation port and the static port, the dynamic pressure can be measured without introducing an area due to the change in duct area.

Table 5.1: Layout of table to record pressure measurements

Station	x (in.)	Indicated Pressures (inch H ₂ O)		
		static	dynamic	stagnation
1				
2				
3				
⋮				

Analysis

The raw data must first be converted from manometer heights to pressures. The *indicated* dynamic pressure is the difference between the stagnation and static pressure taps at each probe position. In other words, the indicated dynamic pressure is obtained by converting the readings in the fourth column of Table 5.1 to pressure. The indicated dynamic pressure is then used to compute the indicated velocity.

The *corrected* dynamic pressure is obtained by subtracting the static pressure at location x (in the duct) from the stagnation pressure at location $x + \Delta x$. The corrected dynamic pressure can only be calculated if the probe was moved in increments of $\Delta x = 0.688$ inch. The corrected dynamic pressure is used to compute the corrected velocity.

Conversion of the raw data yields the variation of stagnation pressure $p_0(x)$, the static pressure $p(x)$, the indicated velocity $V_i(x)$, and corrected velocity $V_c(x)$ along the centerline of the duct.

1. Plot $p_0(x)$ and $p(x)$ on the same axes.
2. Plot $V_i(x)$ and $V_c(x)$ on the same axes.
3. Plot $V_i(x) - V_c(x)$.
4. Estimate the flow rate through the duct.

Report

Present and discuss the plots listed in the *Analysis* section. Answer the following questions.

1. When using a Pitot tube in a duct of constant cross-sectional area the total and static pressure are measured at two streamwise locations. What assumption is being made when the distance between these locations is neglected?
2. Suppose pressure gages with a range of 0 to 10 psi were used instead of the manometers. How would this affect the accuracy of the measurements?
3. Suppose mercury instead of water was used as the manometer fluid. How would this affect the accuracy of the measurements if no other changes in the apparatus or procedure were made?
4. Suppose the experiment was performed with an apparatus that was scaled up by a factor of two. Furthermore, suppose that the fan was adjusted so that the pressure readings for all of the manometers in the large apparatus was identical to the pressure readings for the manometers in the experiment you performed. Demonstrate mathematically how the velocity would be affected. What other flow properties would be affected by the change in scale?

Reference

B.R. Munson, D.F. Young, and T.H. Okiishi, *Fundamentals of Fluid Mechanics*, 4th ed., 2002, Wiley and Sons, New York.