

# Computational Photography

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**Prof. Feng Liu**

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<http://www.cs.pdx.edu/~fliu/courses/cs510/>

**04/28/2022**

# Last Time

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- Panorama
  - Feature detection
  - Feature matching

# Today

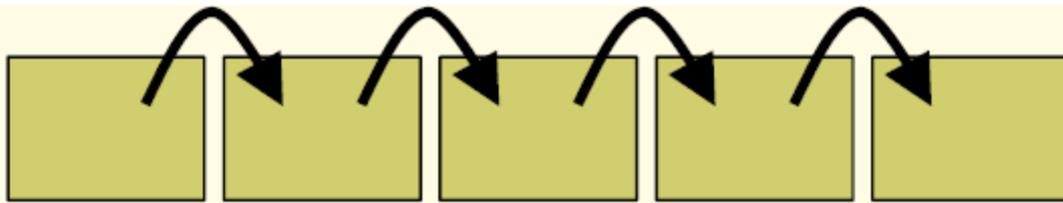
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- Panorama
  - Homography estimation
  - Cylindrical panorama
  - Blending

# Stitching Recipe

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- Align pairs of images
  - Feature Detection
  - Feature Matching
  - Homography Estimation



- Align all to a common surface
- Adjust (Global) & Blend



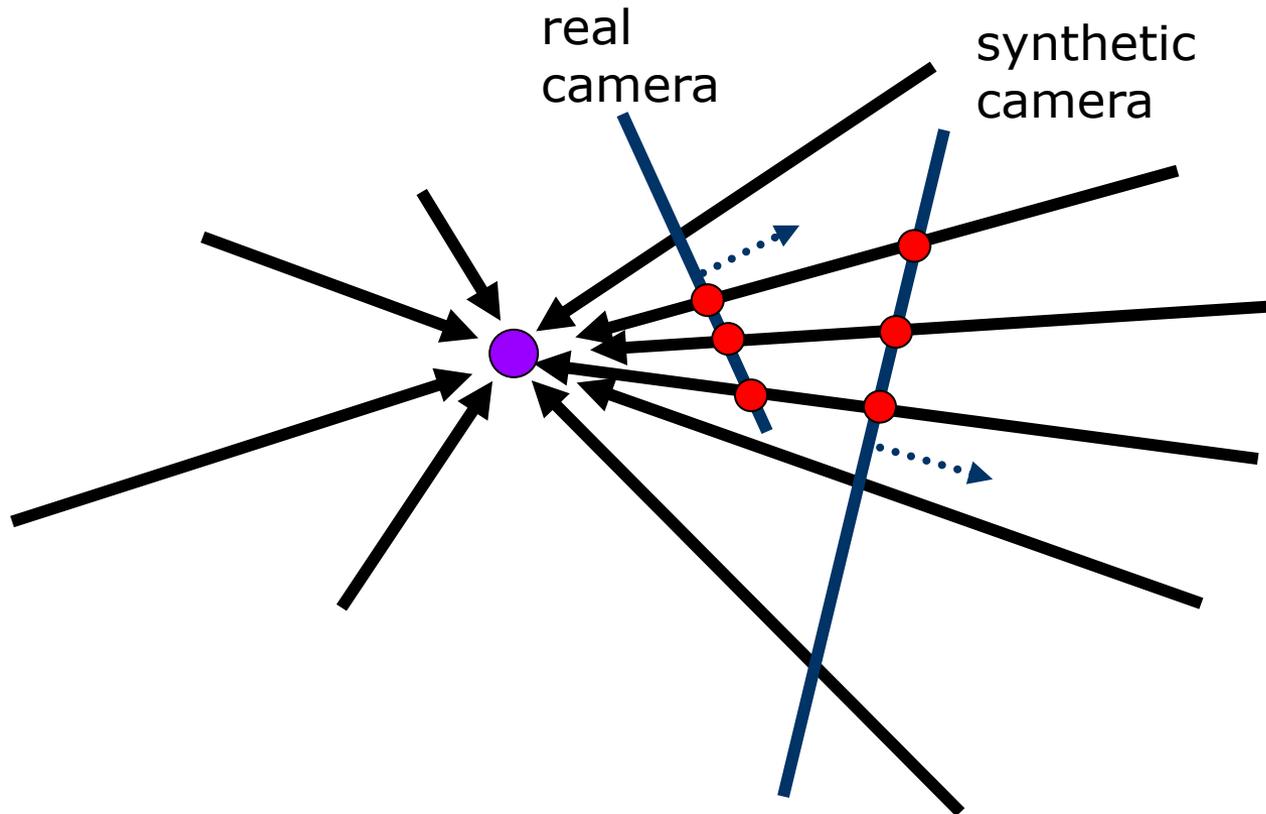
# What can be globally aligned?

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- In image stitching, we seek for a model to globally warp one image into another. Are any two images of the same scene can be aligned this way?
  - Images captured with the same center of projection
  - A planar scene or far-away scene

# A pencil of rays contains all views

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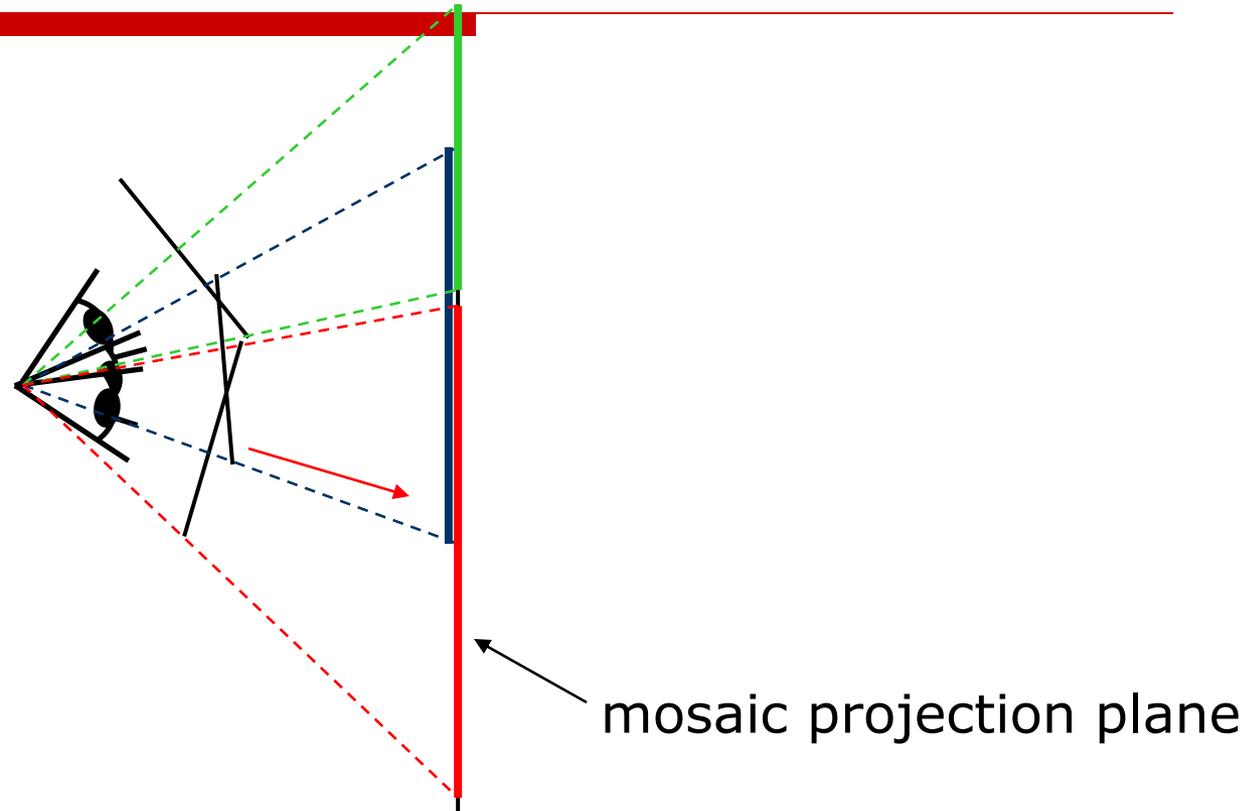


Can generate any synthetic camera view  
as long as it has the same center of projection!

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# Mosaic as an image reprojection

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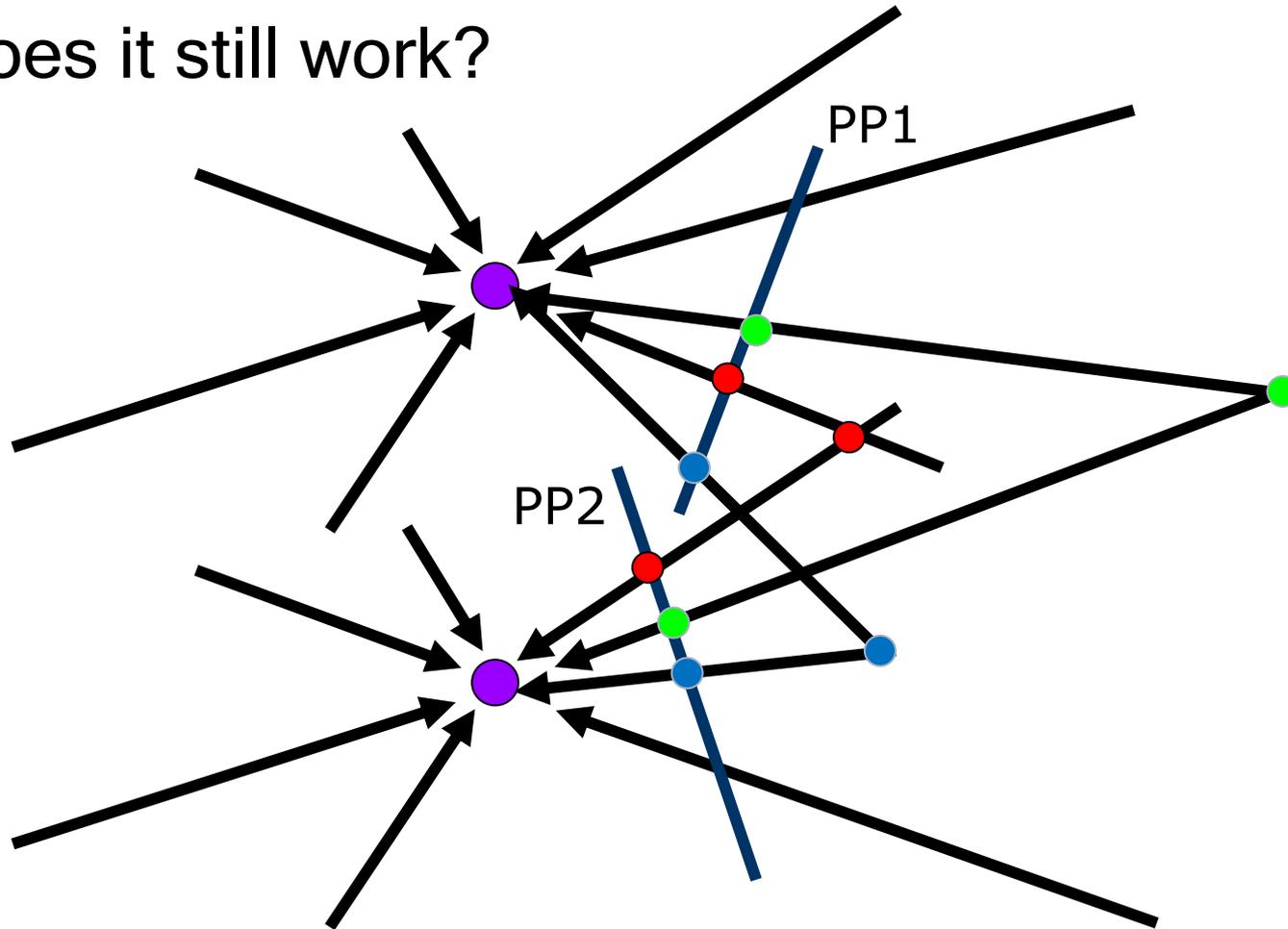


- ❑ The images are reprojected onto a common plane
  - ❑ The mosaic is formed on this plane
  - ❑ Mosaic is a *synthetic wide-angle camera*
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# Changing camera center

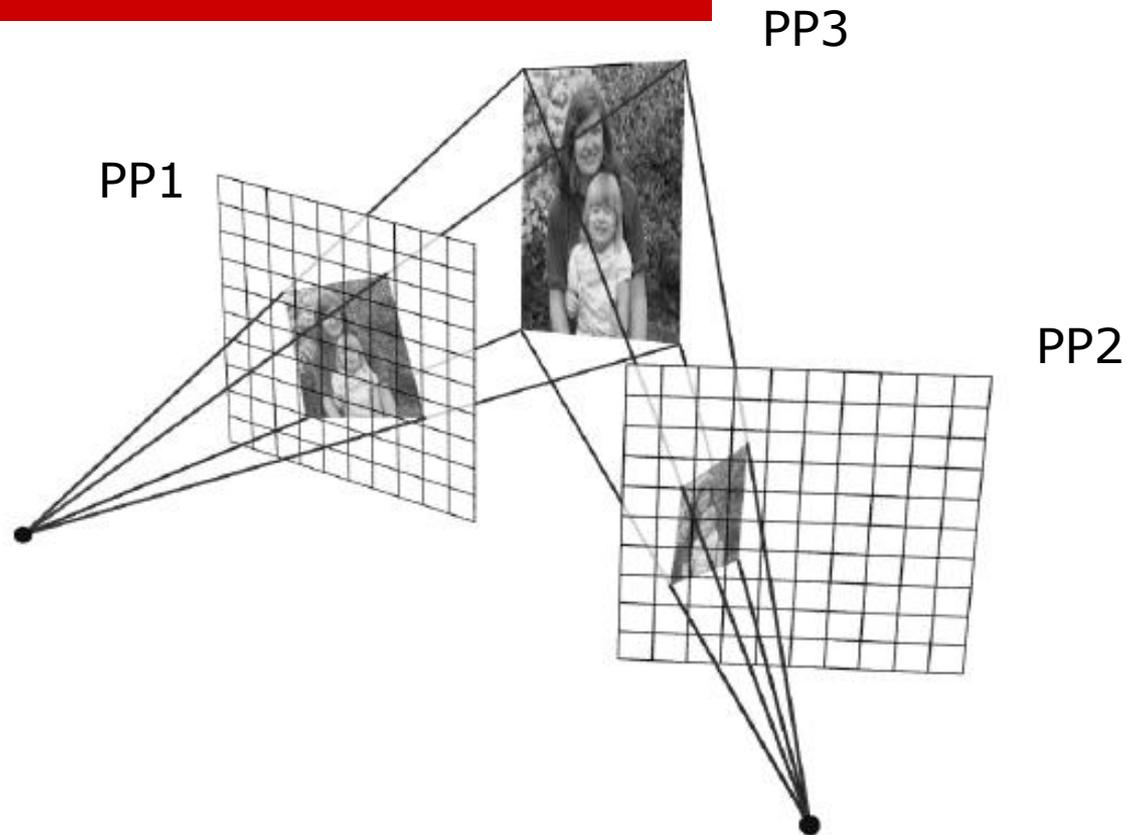
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Does it still work?



# Planar scene (or a faraway one)

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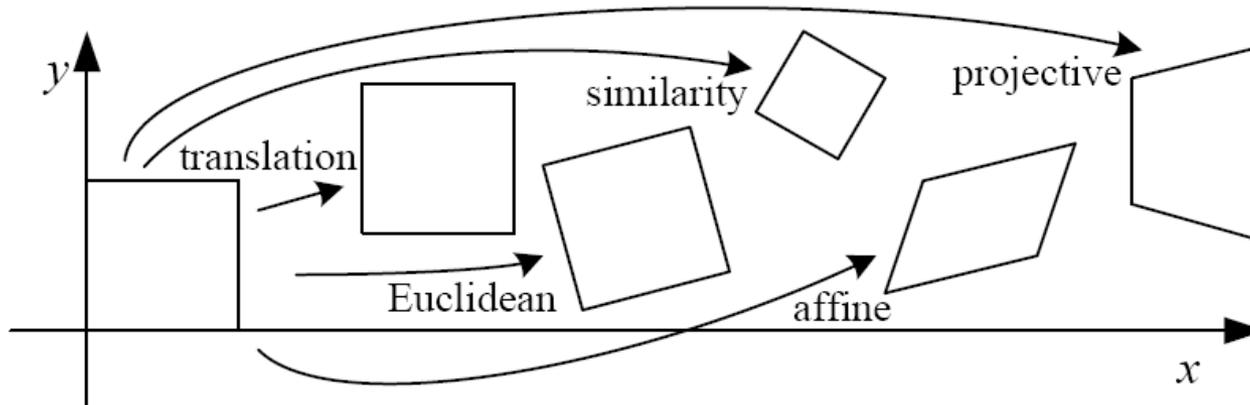
- ❑ PP3 is a projection plane of both centers of projection, so we are OK!
- ❑ This is how big aerial photographs are made

# Motion models

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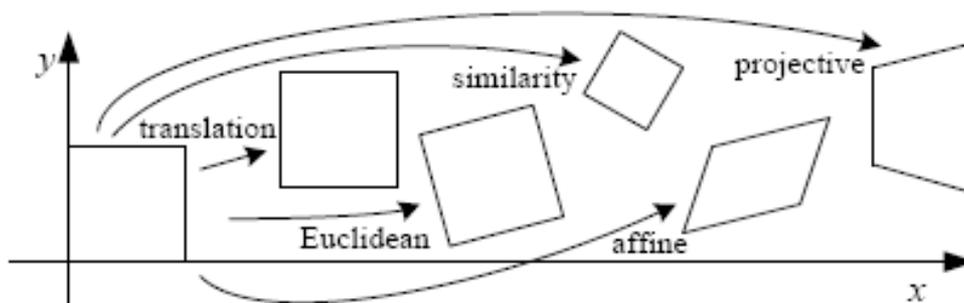
- Parametric models as the assumptions on the relation between two images.

# 2D Motion models



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	2	orientation + ...	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	3	lengths + ...	
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	4	angles + ...	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$	6	parallelism + ...	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$	8	straight lines	

# Motion models

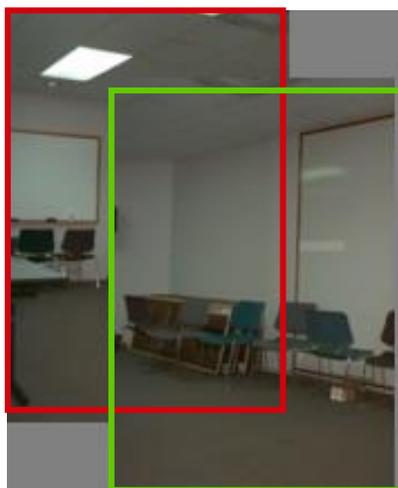


Translation

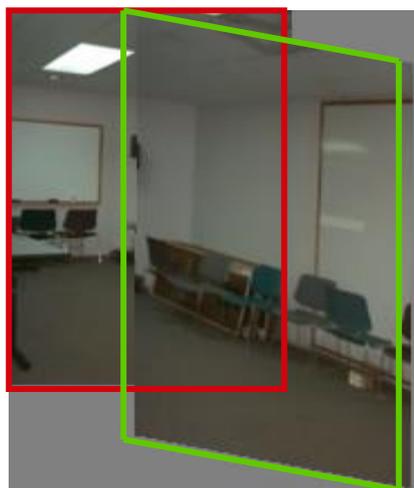
Affine

Perspective

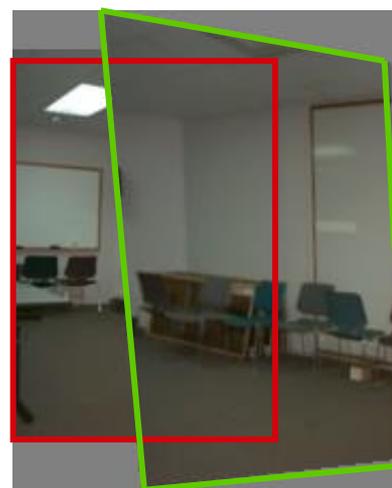
3D rotation



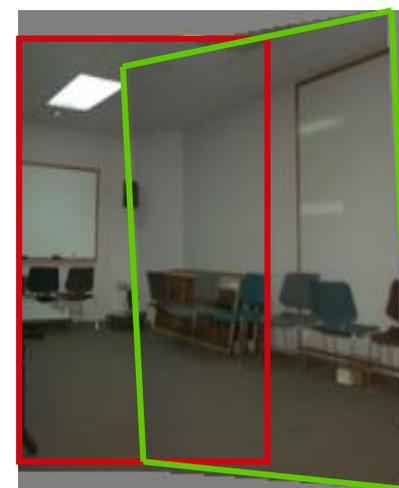
2 unknowns



6 unknowns



8 unknowns



3 unknowns

# Determine pairwise alignment?

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- ❑ Feature-based methods: only use feature points to estimate parameters
- ❑ We will study the “Recognising panorama” paper published in ICCV 2003
- ❑ Run SIFT (or other feature algorithms) for each image, find feature matches.

# Determine pairwise alignment

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- $p' = Mp$ , where  $M$  is a transformation matrix,  $p$  and  $p'$  are feature matches
- It is possible to use more complicated models such as affine or perspective
- For example, assume  $M$  is a 2x2 matrix

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- Find  $M$  with the least square error

$$\sum_{i=1}^n (Mp - p')^2$$

# Determine pairwise alignment

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$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x_1 m_{11} + y_1 m_{12} = x'_1$$

$$x_1 m_{21} + y_1 m_{22} = y'_1$$

$$\begin{pmatrix} x_1 & y_1 & 0 & 0 \\ 0 & 0 & x_1 & y_1 \\ x_2 & y_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 0 & 0 \\ 0 & 0 & x_n & y_n \end{pmatrix} \begin{pmatrix} m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \end{pmatrix} = \begin{pmatrix} x'_1 \\ y'_1 \\ x'_2 \\ \vdots \\ x'_n \\ y'_n \end{pmatrix}$$

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Over-determined system

# Normal equation

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Given an over-determined system

$$\mathbf{Ax} = \mathbf{b}$$

the normal equation is that which minimizes the sum of the square differences between left and right sides

$$\mathbf{A}^T \mathbf{Ax} = \mathbf{A}^T \mathbf{b}$$

Why?

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# Normal equation

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$$\begin{aligned} E &= (\mathbf{Ax} - \mathbf{b})^2 \\ &= (\mathbf{Ax} - \mathbf{b})^T (\mathbf{Ax} - \mathbf{b}) \\ &= \left( (\mathbf{Ax})^T - \mathbf{b}^T \right) (\mathbf{Ax} - \mathbf{b}) \\ &= \left( \mathbf{x}^T \mathbf{A}^T - \mathbf{b}^T \right) (\mathbf{Ax} - \mathbf{b}) \\ &= \mathbf{x}^T \mathbf{A}^T \mathbf{Ax} - \mathbf{b}^T \mathbf{Ax} - \mathbf{x}^T \mathbf{A}^T \mathbf{b} + \mathbf{b}^T \mathbf{b} \\ &= \mathbf{x}^T \mathbf{A}^T \mathbf{Ax} - \left( \mathbf{A}^T \mathbf{b} \right)^T \mathbf{x} - \left( \mathbf{A}^T \mathbf{b} \right)^T \mathbf{x} + \mathbf{b}^T \mathbf{b} \\ \frac{\partial E}{\partial \mathbf{x}} &= 2\mathbf{A}^T \mathbf{Ax} - 2\mathbf{A}^T \mathbf{b} \end{aligned}$$

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# Determine pairwise alignment

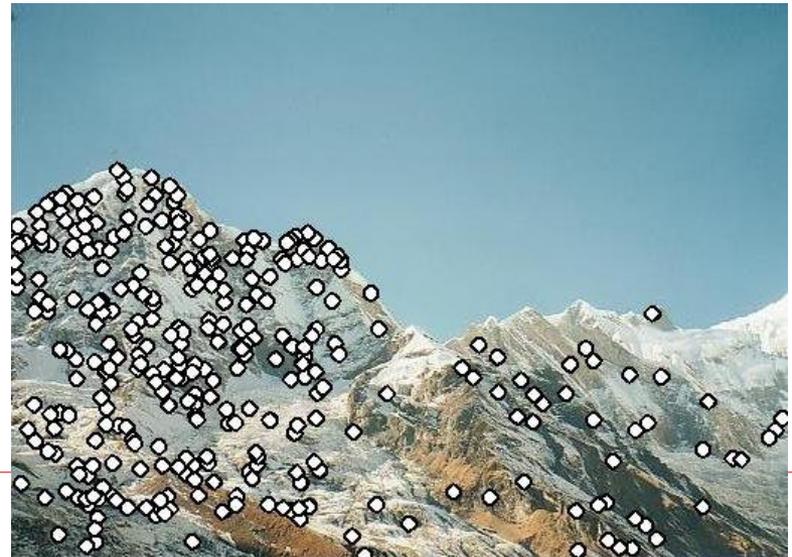
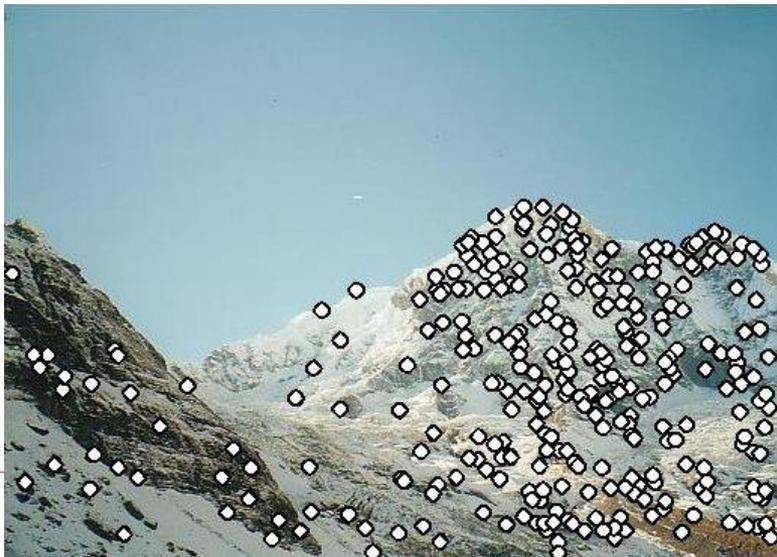
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- $p' = Mp$ , where  $M$  is a transformation matrix,  $p$  and  $p'$  are feature matches
- For translation model, it is easier.

$$E = \sum_{i=1}^n \left[ (m_1 + x_i - x'_i)^2 + (m_2 + y_i - y'_i)^2 \right]$$

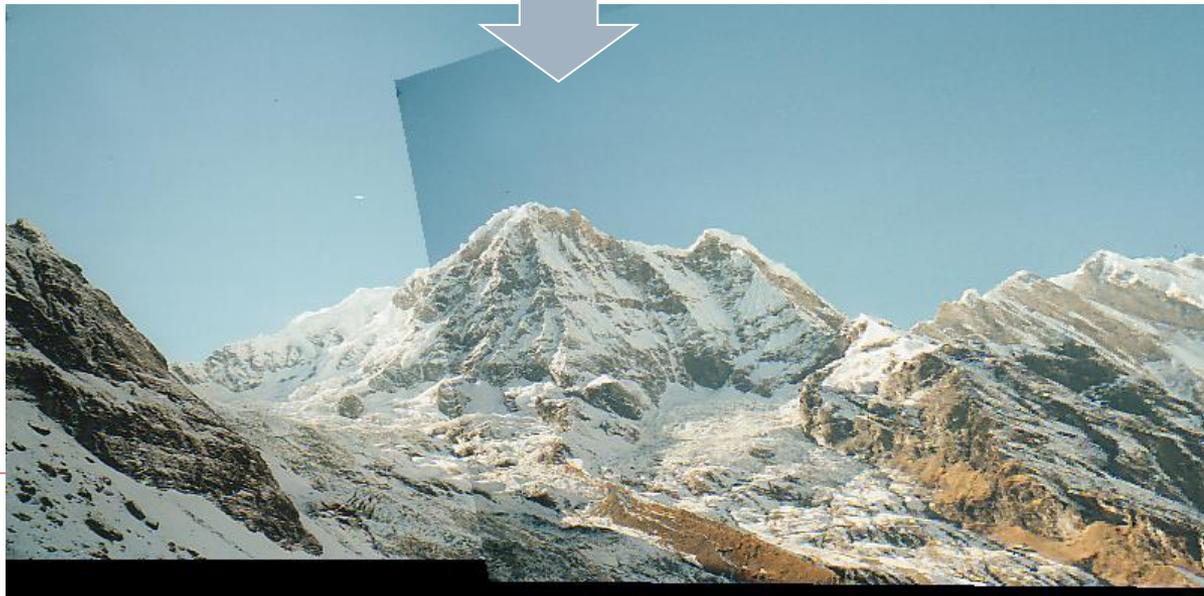
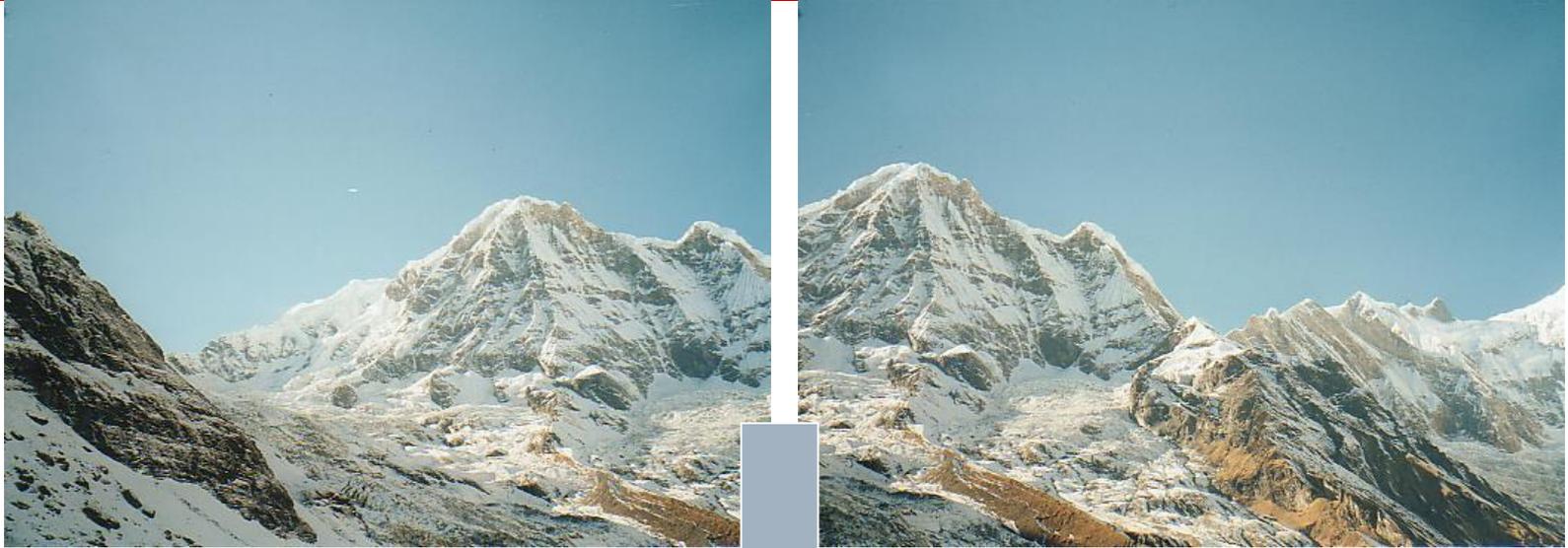
$$0 = \frac{\partial E}{\partial m_1}$$

# Stitch with Homography



# Stitch with Homography

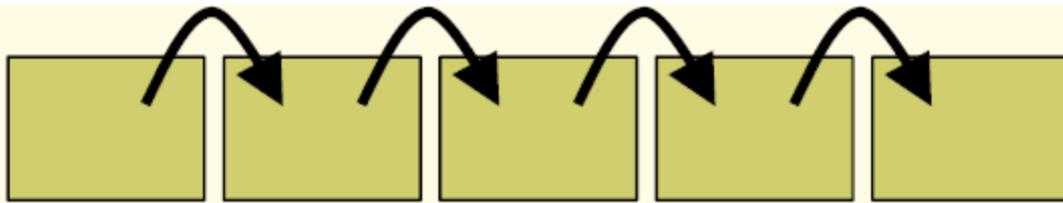
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# Stitching Recipe

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- Align pairs of images
  - Feature Detection
  - Feature Matching
  - Homography Estimation



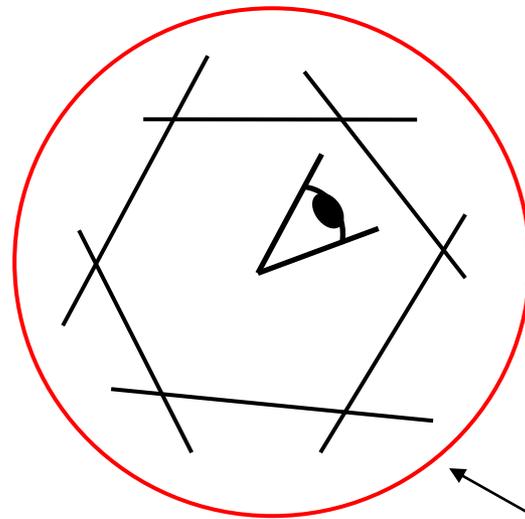
- Align all to a common surface
- Adjust (Global) & Blend



# A case study: cylindrical panorama

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- What if you want a 360° field of view?



mosaic projection cylinder

# Cylindrical panoramas

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## □ Steps

- Reproject each image onto a cylinder
- Blend
- Output the resulting mosaic

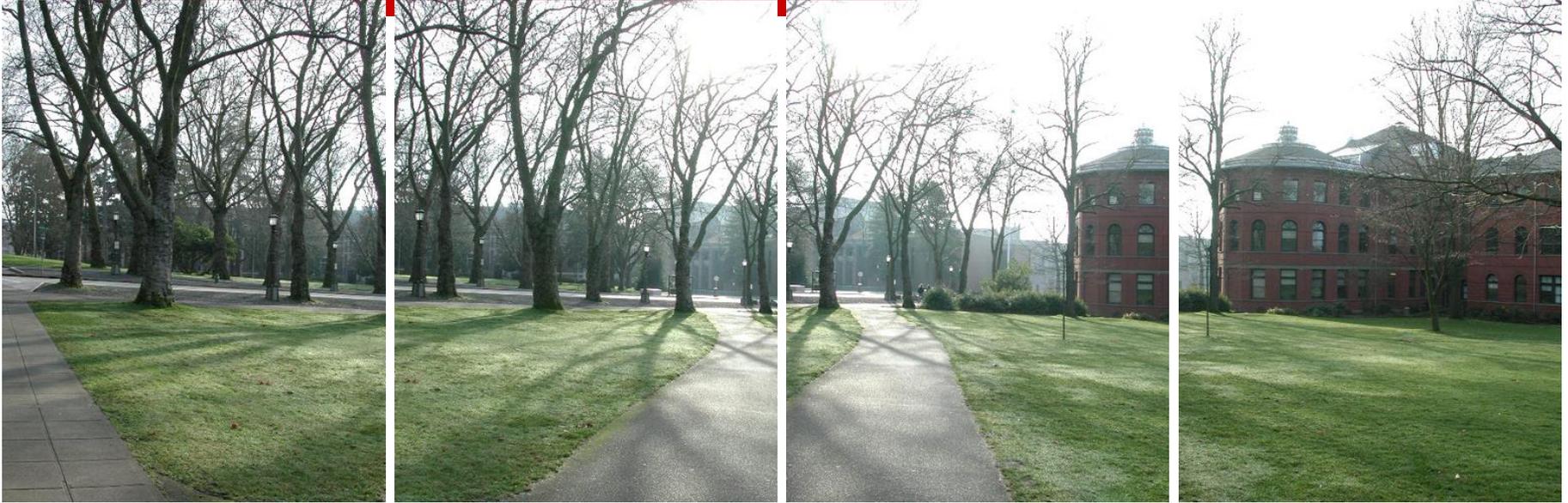
# Cylindrical panorama

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1. Take pictures on a tripod (or handheld)
2. Warp to cylindrical coordinate
3. Compute pair-wise alignments
4. Fix up the end-to-end alignment
5. Blending
6. Crop the result and import into a viewer

It is required to do radial distortion correction for better stitching results!

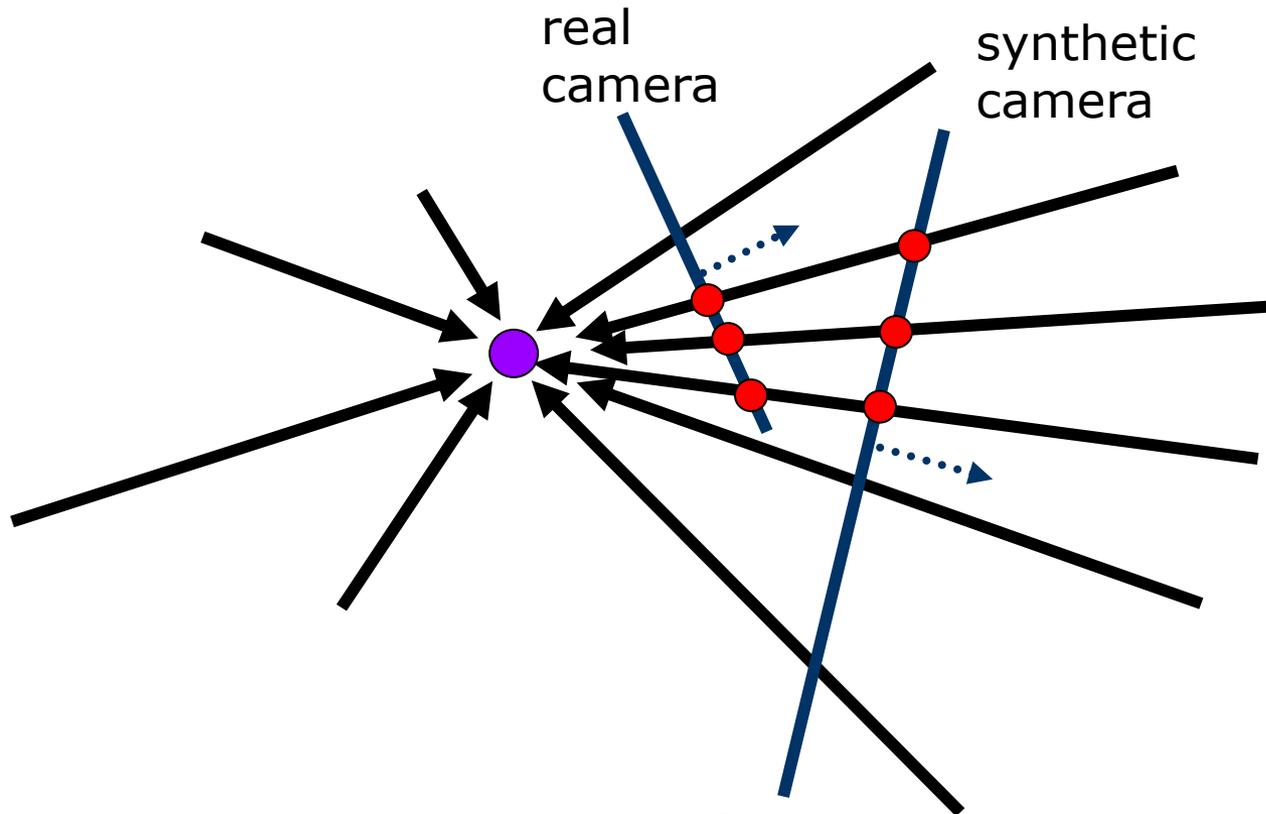
# Taking pictures



Kaidan panoramic tripod head

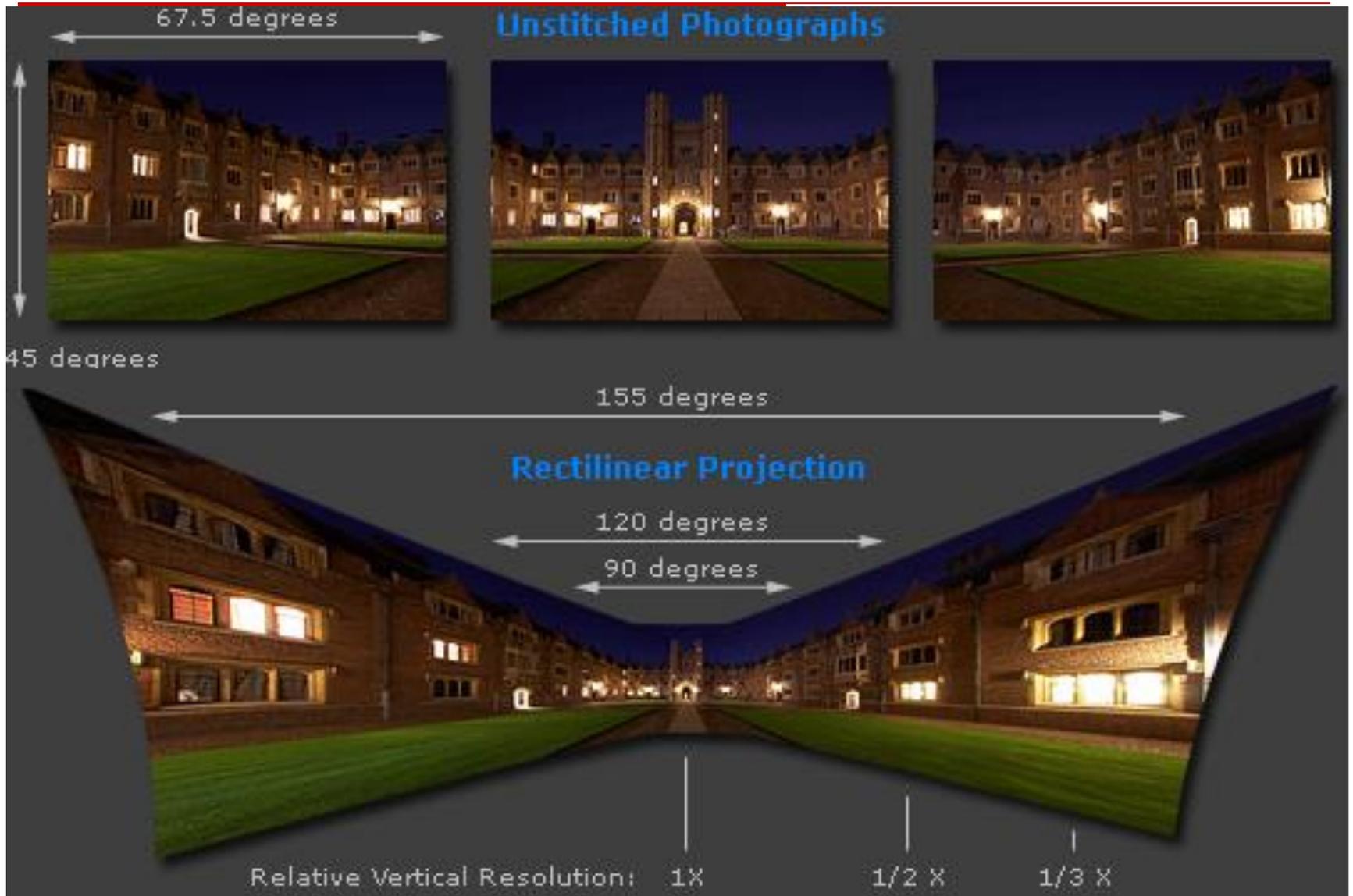
# Where should the synthetic camera be

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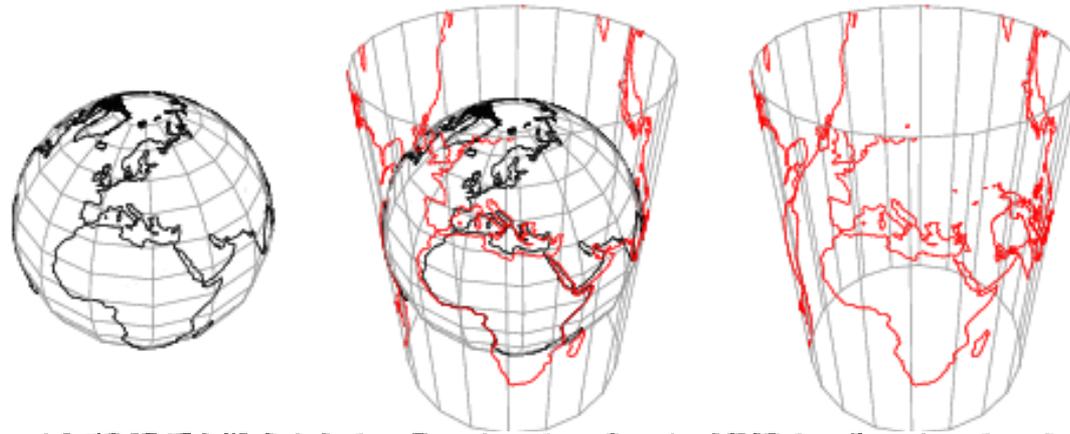
- The projection plan of some camera
- Onto a cylinder

# Rectilinear projection

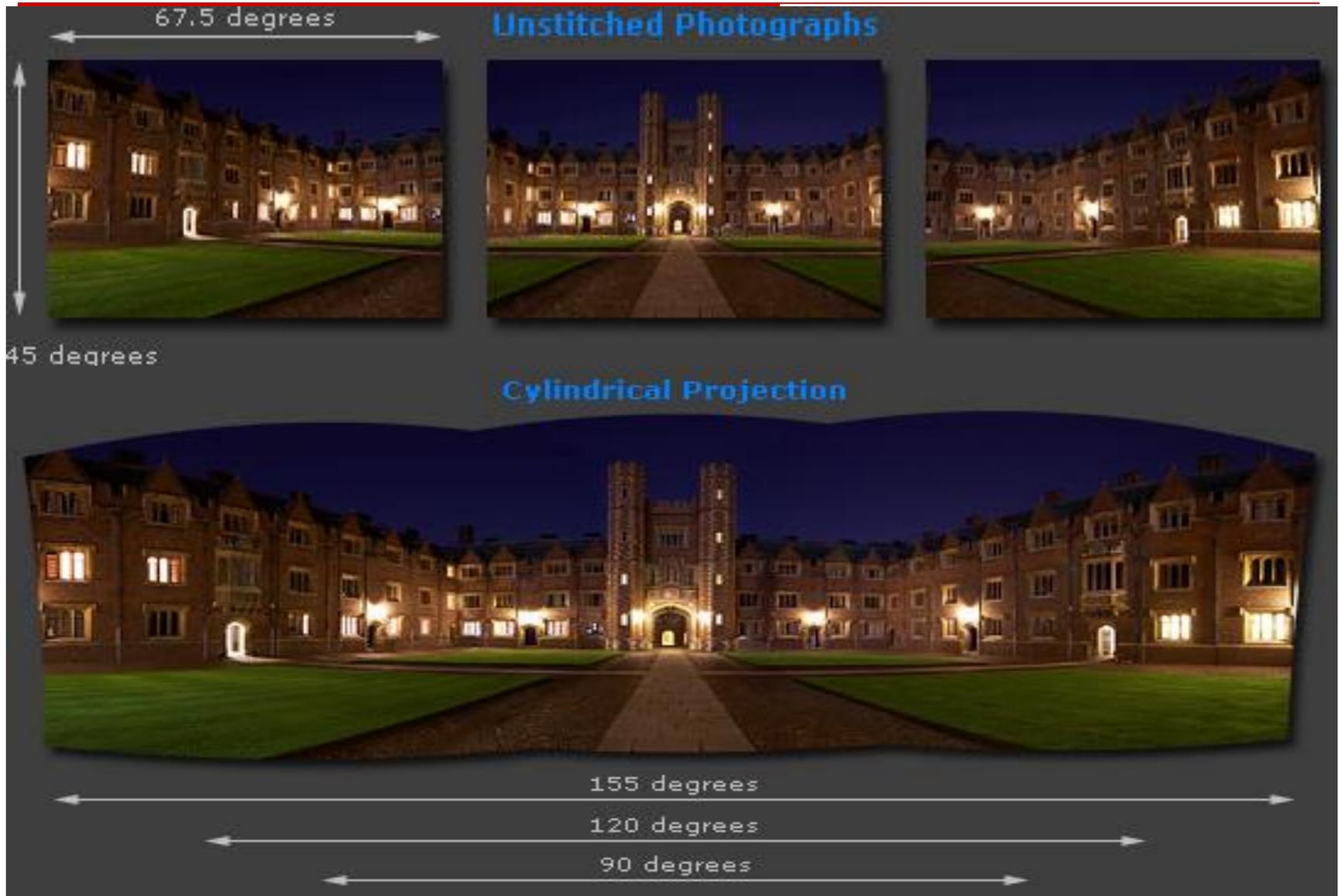


# Cylindrical projection

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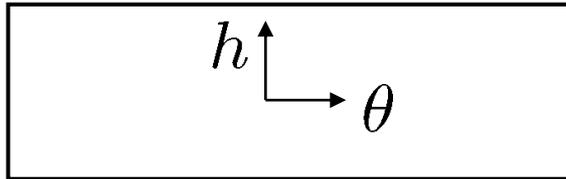


# Cylindrical projection

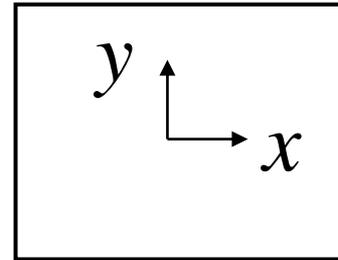


# Cylindrical projection

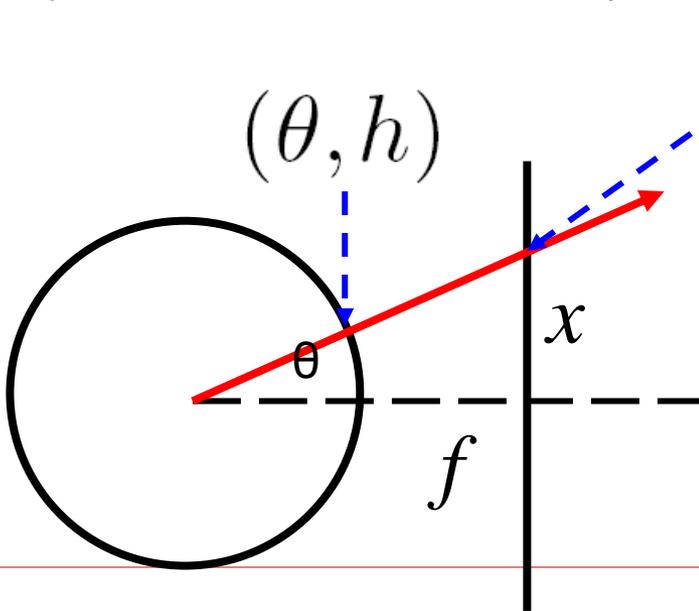
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unwrapped  
cylinder



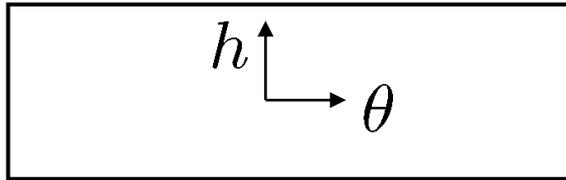
$$(\sin \theta, h, \cos \theta) \propto (x, y, f)$$



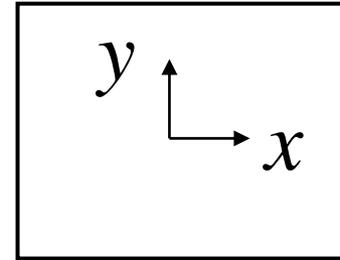
$$\theta = \tan^{-1} \frac{x}{f}$$



# Cylindrical projection



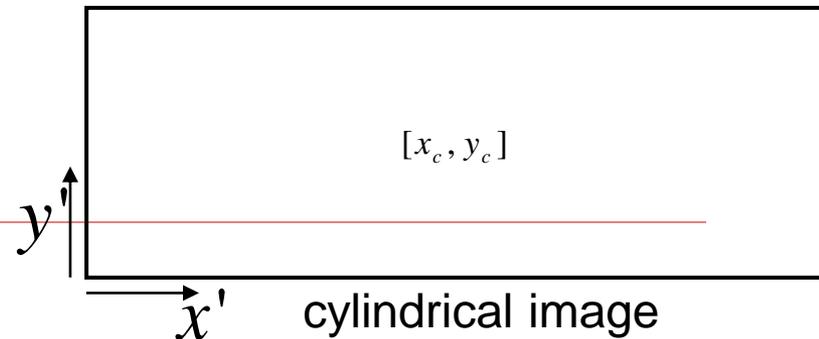
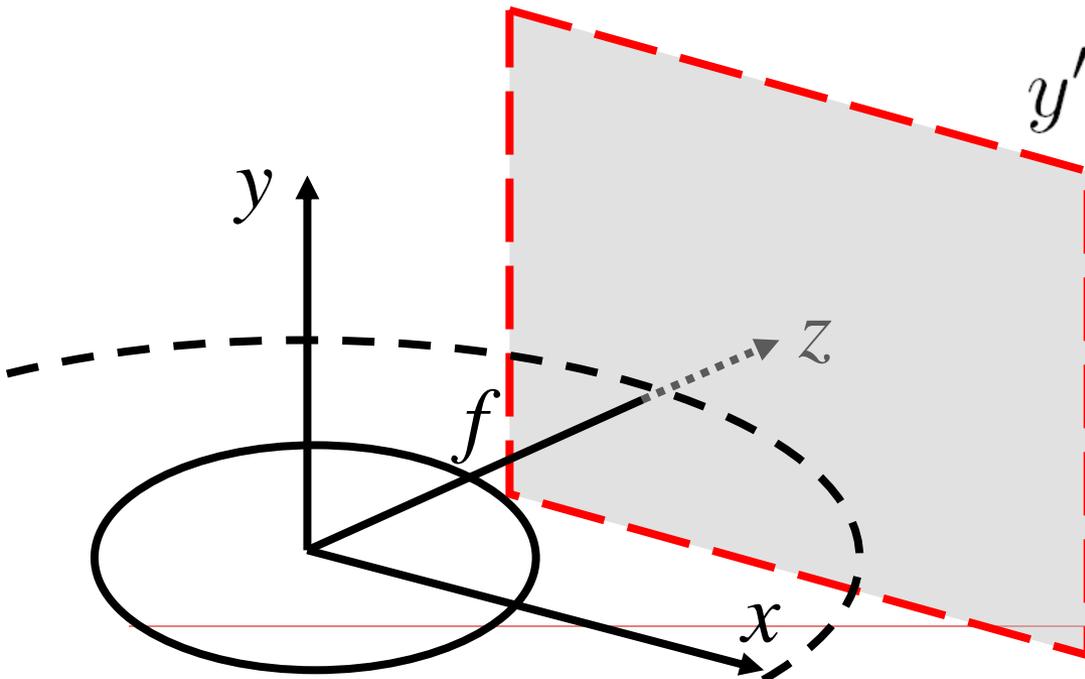
unwrapped  
cylinder



$$x' = s\theta = s \tan^{-1} \frac{x}{f}$$

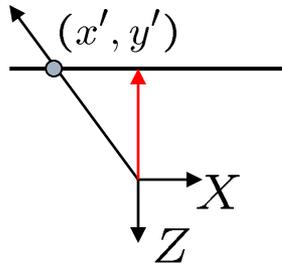
$$y' = sh = s \frac{y}{\sqrt{x^2 + f^2}}$$

$s$  defines size of the final image,  
often convenient to set  $s = f$

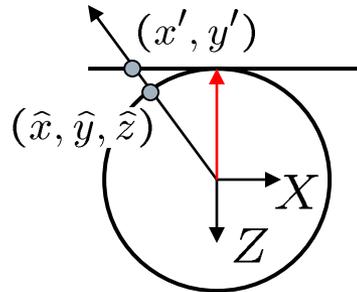


cylindrical image

# Cylindrical reprojection



top-down view



**Focal length** – the dirty secret...

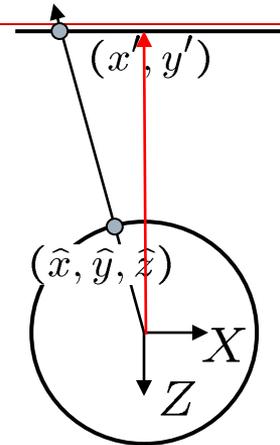
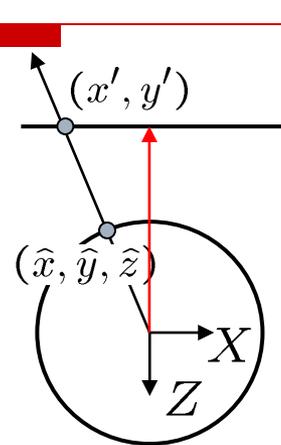


Image 384x300

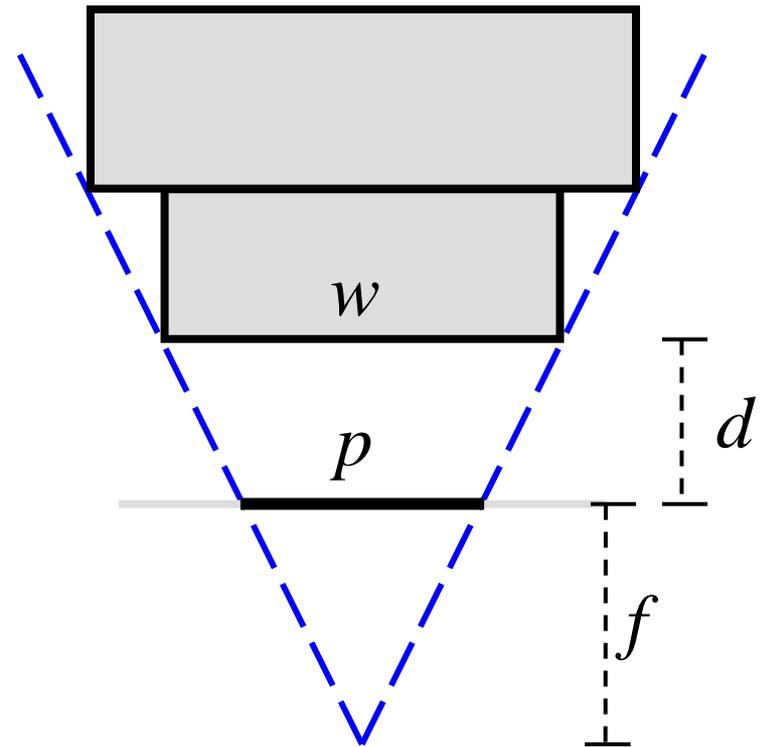
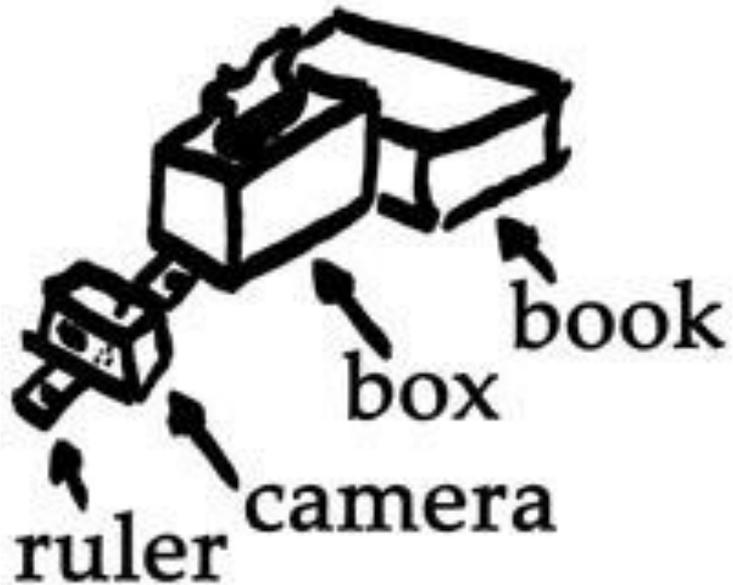
$f = 180$  (pixels)

$f = 280$

$f = 380$

# A simple method for estimating $f$

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Or, you can use other software, such as AutoStitch, to help.

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# Input images

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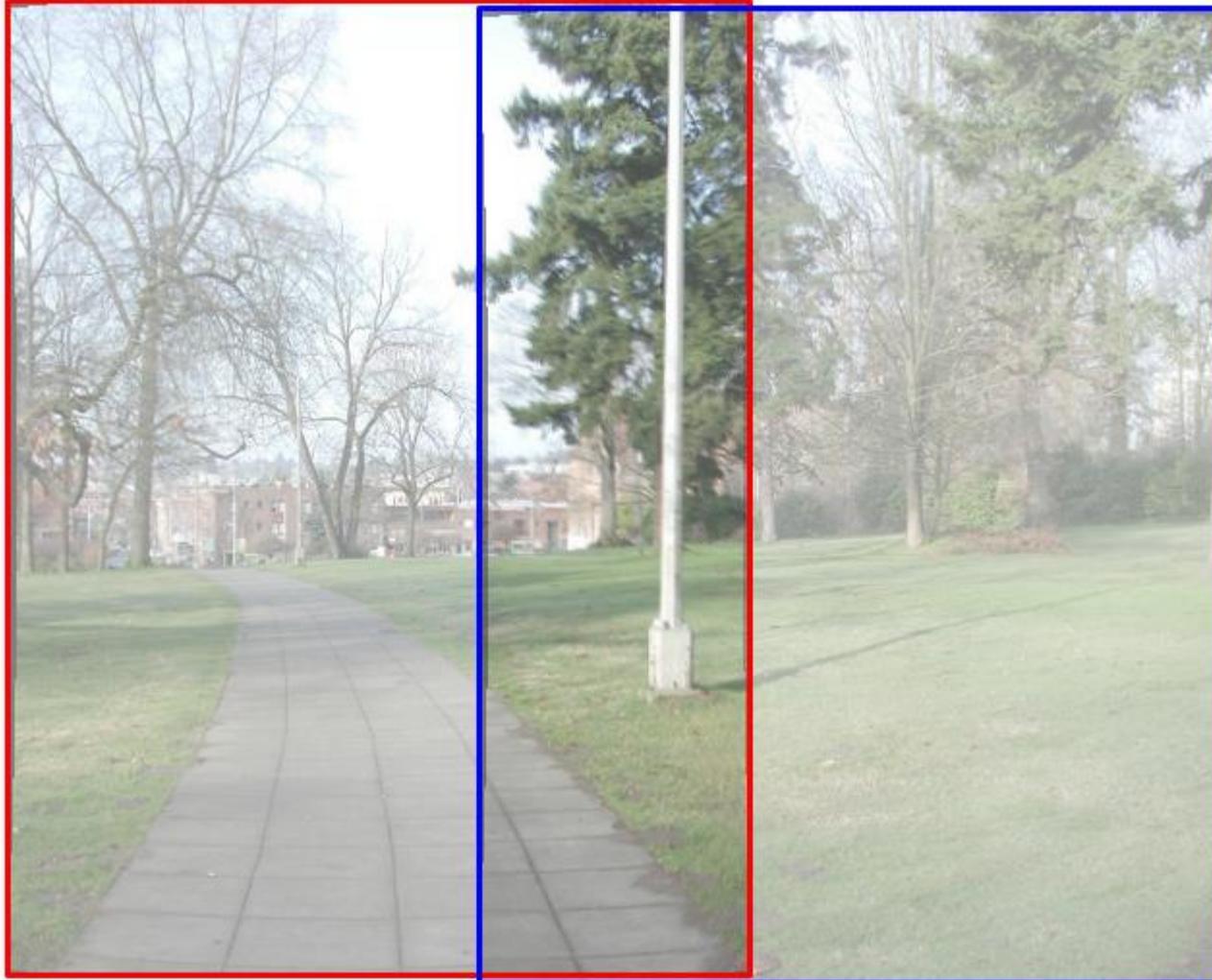
# Cylindrical warping

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# Alignment

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a rotation of the camera is a **translation** of the cylinder!

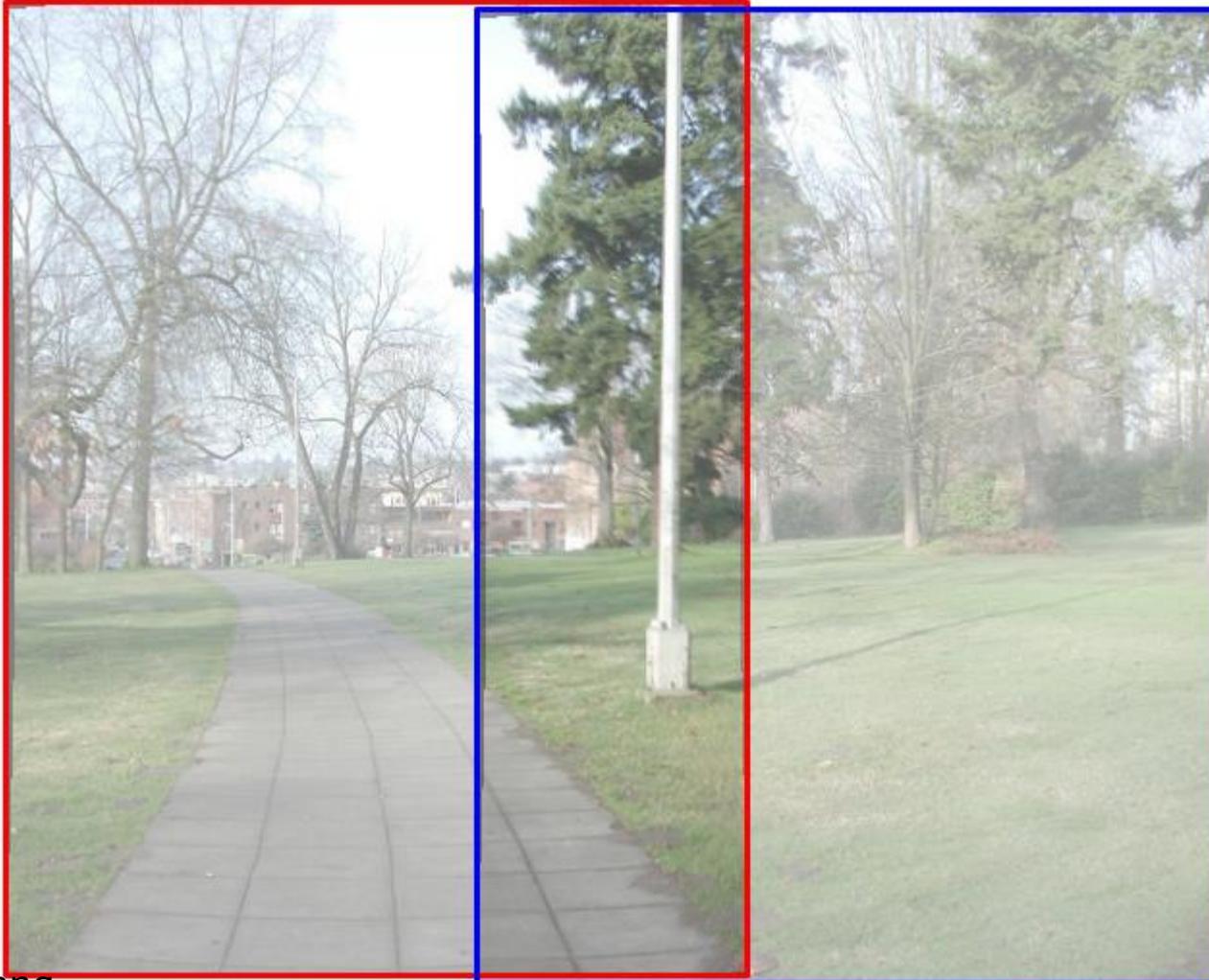
# Blending

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- ❑ Why blending: parallax, lens distortion, scene motion, exposure difference
  - ❑ Alpha-blending
  - ❑ Poisson blending
  - ❑ Adelson's pyramid blending
-

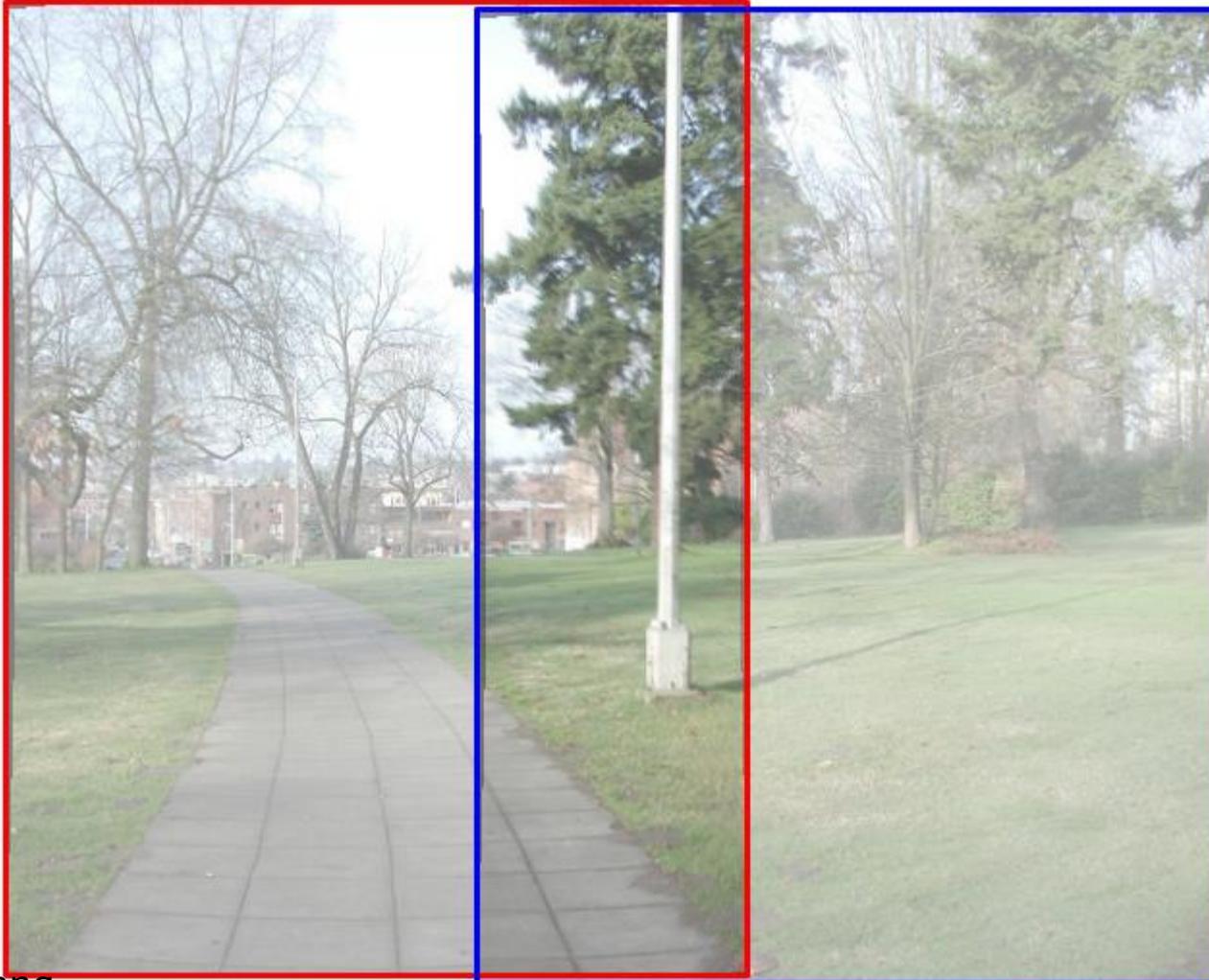
# Blending

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# Linear Blending

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# Linear Blending

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Credit: Y.Y. Chuang

# Linear Blending

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Image from <http://www.cs.ubc.ca/~lowe/425/slides/11-PanoramasAR.pdf>

# Multi-band Blending [BURT and ADELSON 83]

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Linear blending



Multi-band blending

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A multi-resolution spline with application to image mosaics.

Peter J. Burt and Edward Adelson. ACM Transactions on Graphics, 1983.

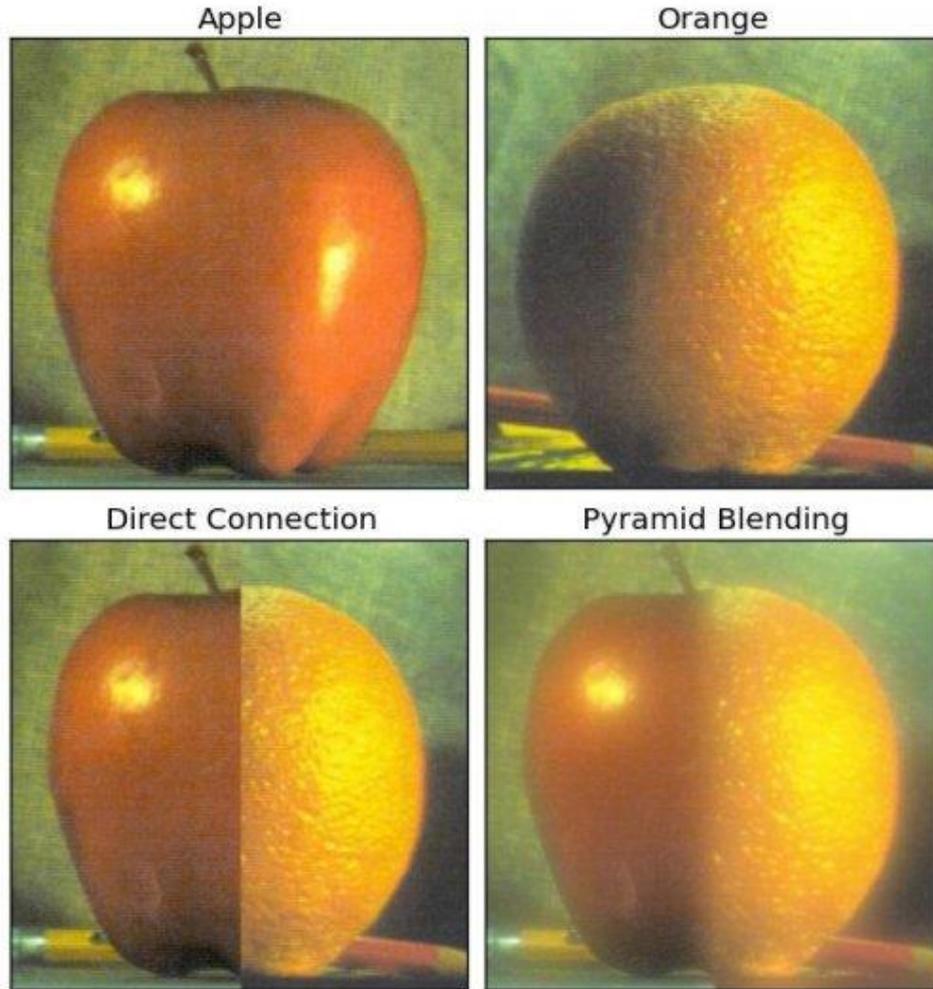
# Multi-band Blending

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1. Laplacian pyramids  $LA$  and  $LB$  are constructed for images  $A$  and  $B$  respectively.
2. A third Laplacian pyramid  $LS$  is constructed by copying nodes from the left half of  $LA$  to the corresponding nodes of  $LS$ , and nodes in the right half of  $LB$  to the right half of  $LS$ .
3. The final image  $S$  is obtained by expanding and summing the levels of  $LS$ .

# Multi-band Blending

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Source: [https://docs.opencv.org/3.1.0/dc/dff/tutorial\\_py\\_pyramids.html](https://docs.opencv.org/3.1.0/dc/dff/tutorial_py_pyramids.html)

# 2-band Blending

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Low frequency ( $l > 2$  pixels)



High frequency ( $l < 2$  pixels)

# Linear Blending

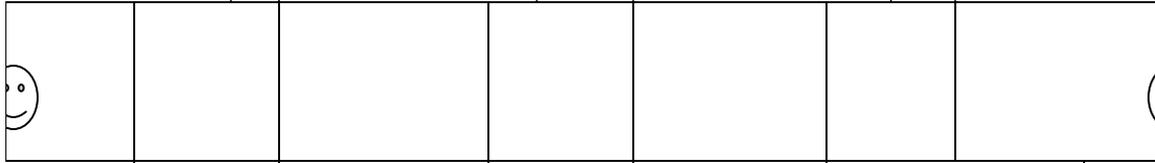


# 2-band Blending



# Assembling the panorama

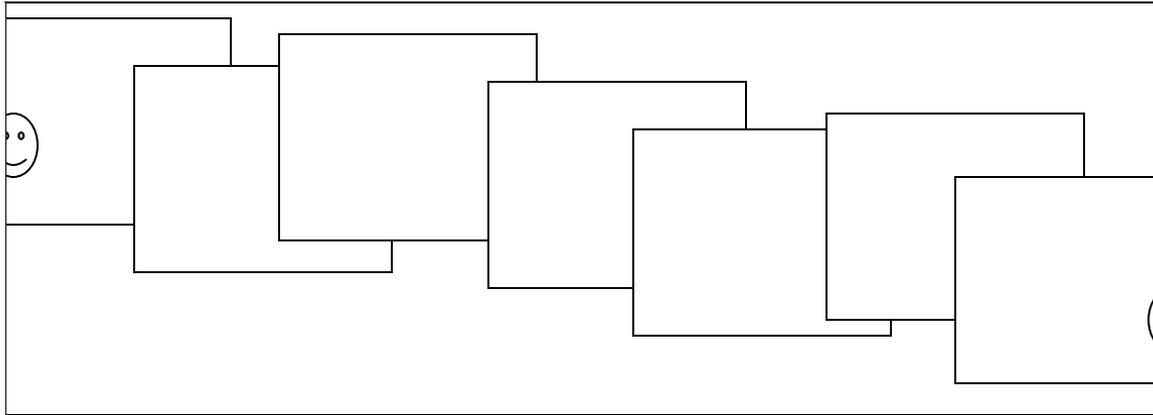
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- ❑ Stitch pairs together, blend, then crop

# Problem: Drift

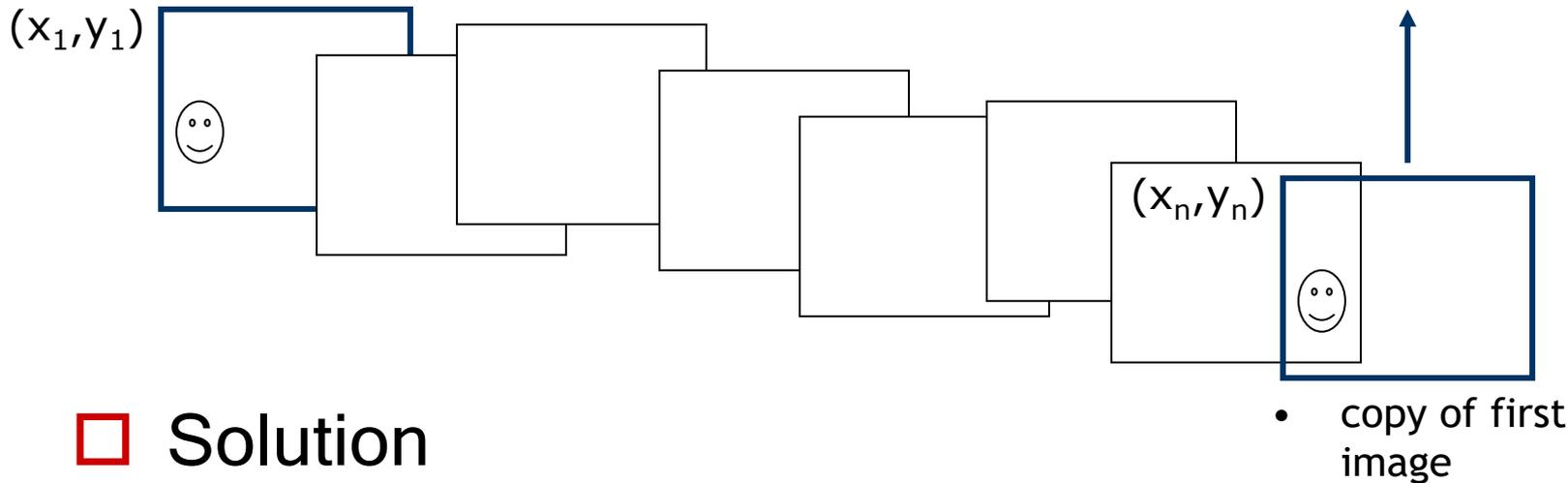
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- Error accumulation
  - small errors accumulate over time

# Problem: Drift

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## □ Solution

- add another copy of first image at the end
- there are a bunch of ways to solve this problem
  - add displacement of  $(y_1 - y_n)/(n - 1)$  to each image after the first
  - compute a global warp:  $y' = y + ax$
  - run a big optimization problem, incorporating this constraint
    - best solution, but more complicated
    - known as “bundle adjustment”

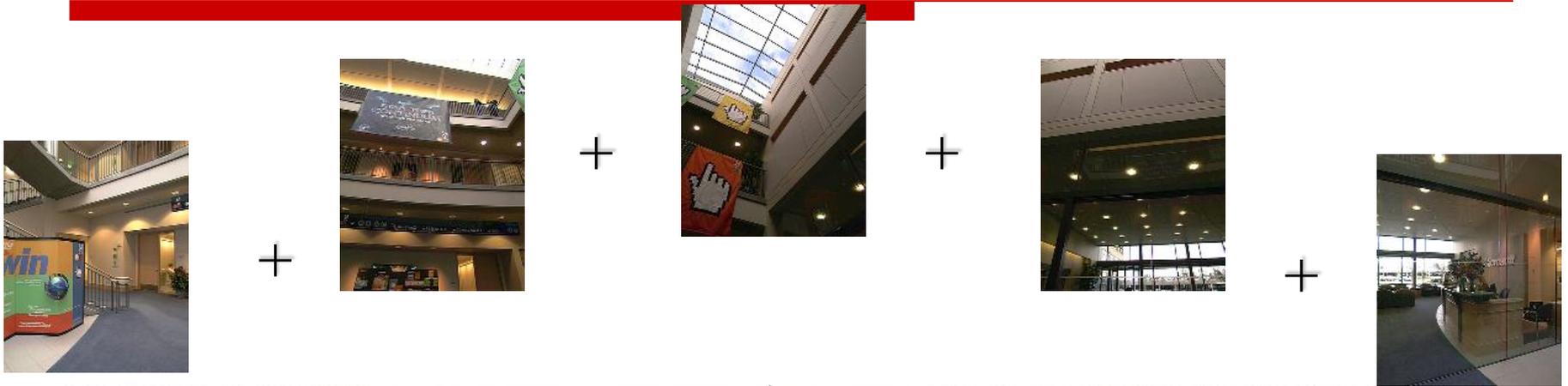
# End-to-end alignment and crop

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# Viewer: panorama

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example:

<http://www.cs.washington.edu/education/courses/cse590ss/01wi/projects/project1/students/dougz/index.html>

Credit: Y.Y. Chuang

# Student paper presentation

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## Color Image Colorization

R. Zhang, P. Isola, and A. Efros  
ECCV 2016

**Presenter: Seward, Garrett**

# Student paper presentation

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## Burst photography for high dynamic range and low-light imaging on mobile cameras

Samuel W. Hasinoff, Dillon Sharlet, Ryan Geiss, Andrew Adams,  
Jonathan T. Barron, Florian Kainz, Jiawen Chen, and Marc Levoy  
SIGGRAPH Asia 2016

**Presenter: Peters, Emerson**

# Next Time

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- Image segmentation
- Student paper presentations
  - 05/03: Little, Samuel
    - Deep High Dynamic Range Imaging of Dynamic Scenes.  
N. K. Kalantari and R. Ramamoorthi, SIGGRAPH 2017
  - 05/03: Joshi, Vijay
    - Night Sight: Seeing in the Dark on Pixel Phones
      - <https://ai.googleblog.com/2018/11/night-sight-seeing-in-dark-on-pixel.html>
      - <https://www.blog.google/products/pixel/see-light-night-sight/>