Review of Planetary gear Trains

1) Identify the Two inputs
   a) Two rpms of two gears (n_1, n_2)
   b) one gear rpm & arm (n_1, n_A)

[if n_1 & n_2 then one would be a motor n_1
and one would be fixed n_2

[if n_1 & n_A, then n_A would be motor]
and n_1 would be fixed

Find
[if n_1 & n_2 are given] Solve for n_A
or any other gear n_3

[if n_1, n_A are given] Solve for
any other gear n_3

Case-I (n_1, n_2 are given), Solve for n_A first

$$\frac{n_{1/A}}{n_{2/A}} = \frac{\overset{\checkmark}{n_1} - n_A}{\overset{\checkmark}{n_2} - n_A}$$

with respect to the Arm (Arm is held stationary) the gear train becomes a regular gear train

$$\frac{n_{1/A}}{n_{2/A}} = m \checkmark$$ (regular gear train speed ratio between n_1 and n_2

we set

$$\frac{n_1 - n_A}{n_2 - n_A} = m \Rightarrow \text{Solve for } n_A$$

To Solve for any other gear (n_3), we do the Same using n_3 and n_A and one of the known gear rpms (n_1)

$$\frac{n_{1/A}}{n_{3/A}} = \frac{\overset{\checkmark}{n_1} - \overset{\checkmark}{n_A}}{n_3 - n_A} = m_1 \Rightarrow n_3$$

Regular gear train (Arm stationary)

$$\frac{n_{1/A}}{n_{3/A}} = m_1 \checkmark$$

Case-II) if n_1 and n_A are given and n_3 is to be found

$$\frac{n_{1/A}}{} = \frac{\overset{\checkmark}{n_1} - \overset{\checkmark}{n_A}}{} \quad m$$

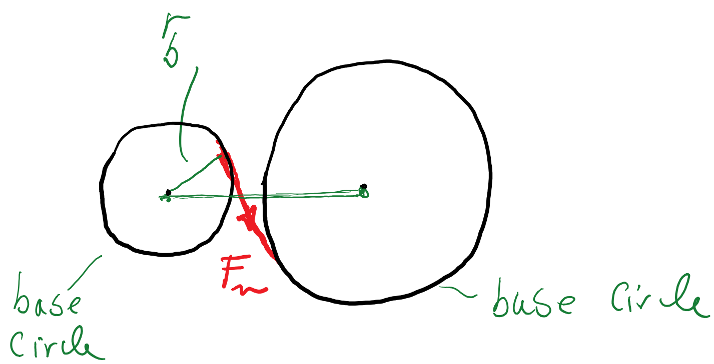
$n_{3/A}$ $\qquad \dfrac{\quad}{n_3 - n_A} = m_2$

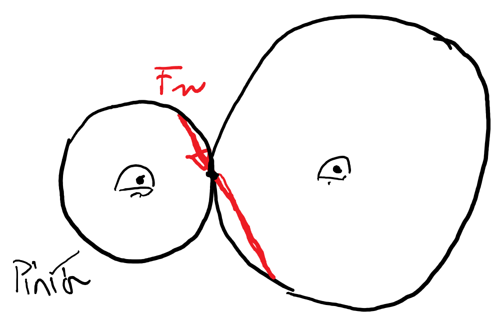
where $\dfrac{n_{1/A}}{n_{3/A}} = m_2$ (regular gear train)

Gear Stress Analysis for tooth Breakage

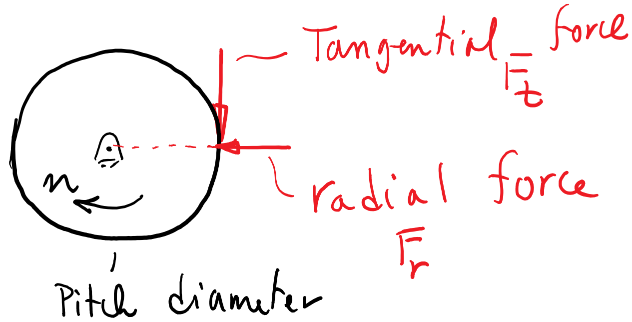
First we find the average force acting on a gear tooth

$$T = F_n \cdot r_b$$





$$F_n = \sqrt{F_t^2 + F_r^2}$$



Example

$n = 1000$ rpm

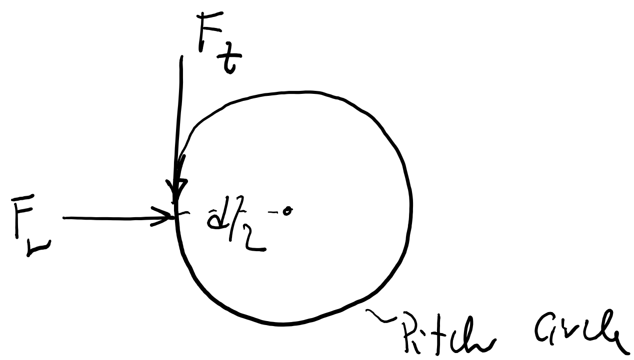
ft-lbs

$n = 1000$ rpm

$h_p = 5$ hp

Finding torque $\quad h_p = \dfrac{\overset{\text{ft-lbs}}{T}\,\overset{\text{rpm}}{n}}{5252}$

$$5 = \frac{T(1000)}{5252} \Rightarrow T = 26.26 \text{ ft-lbs}$$



$$F_t\left(\frac{d}{2}\right) = T$$

Example $\quad d = 3$ inches $\quad$ (for Pinion)

$$F_t\left(\frac{\overset{\text{inch}}{3}}{2}\right) = \underbrace{26.26\ (12)}_{\text{in-lb}}$$

$$\Rightarrow F_t = 210 \text{ lbs}$$

Finding F_t directly

$$\underbrace{F_t}_{\text{lbs}} = \frac{33000\ hp}{\underbrace{V}_{\text{Pitch line velocity in ft/min}}}$$

τ lbs v Pitch-line velocity in ft/min

$$\underbrace{V}_{\text{ft/min}} = \frac{\pi \overbrace{n}^{\text{rpm}} \underbrace{d}_{\text{inches}}}{12}$$

$$V = \frac{\pi(1000)(3)}{12} \Rightarrow V = 785.4 \text{ ft/min}$$

$$\Rightarrow F_t = \frac{33000(5)}{785.4} = 210 \text{ lbs}$$

Calculating F_r (bearing support)

$$F_r = F_t \tan\phi$$

$$F_r = 210 \tan(20)$$

$$F_r = 76.46 \text{ lbs}$$

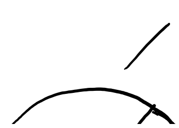
If F_n is needed (resultant force)

$$F_n = \sqrt{F_t^2 + F_r^2} = \sqrt{(210)^2 + (76.46)^2}$$

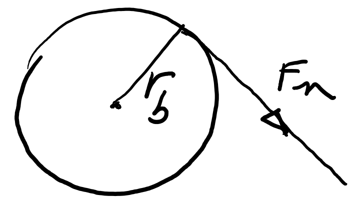
$$F_n = 223.5 \text{ lbs}$$

Alternatively

base circle



Alternatively



$$T = F_n\, r_b$$

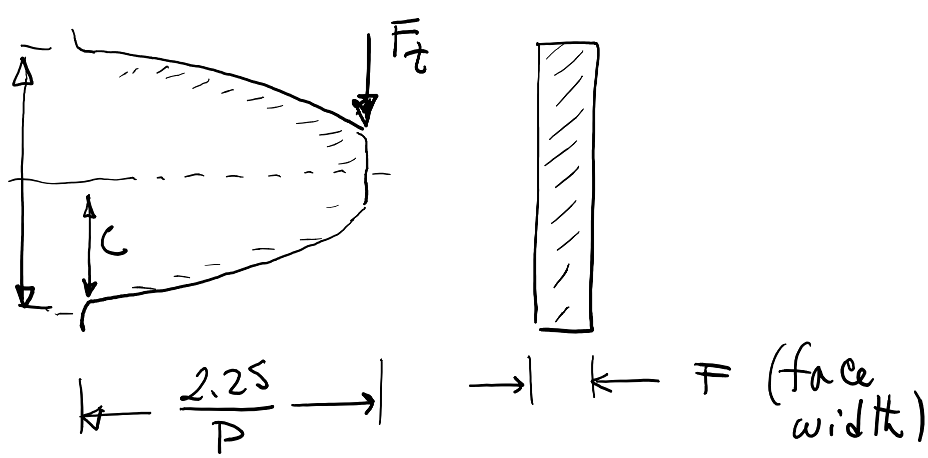
$$= F_n \left(d/2\right) \cos\phi$$

$$\underbrace{26.26\,(12)}_{\text{in-lb}} = F_n \left(\frac{3}{2}\right) \cos 20$$

$$\Rightarrow F_n = 223.5 \text{ lbs}$$

Calculate stresses (bulk) - responsible for tooth breakage (fatigue mechanism)

Historical perspective (1893 - Lewis)



load

$$\sigma = \frac{Me}{I}$$

geometry

geometry

load

$$\sigma = \frac{F_t \, P}{F \, J}$$ geometry, geometry, geometry

(Lewis's Formula)

AGMA formula

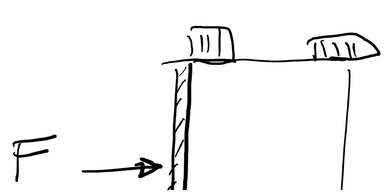
includes
- Load sharing
- location of F_t
- stress concentration @ the root

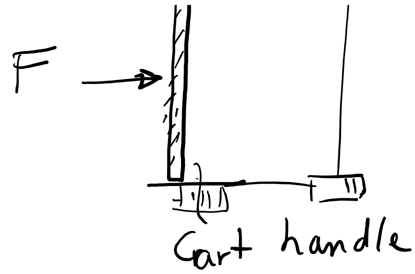
* Tooth contact misalignment
* Tooth geometry imperfections
* Non-uniform loads
* Radial load

$$\sigma = \underbrace{W^t}_{\text{Tangential force } F_t} \cdot K_o \cdot K_v \cdot K_s \cdot \frac{P}{FJ} \cdot K_B \cdot K_m$$

$W^t \equiv$ from Power & RPM

$K_o \equiv$ overload factor (Fig 14-17 P758)





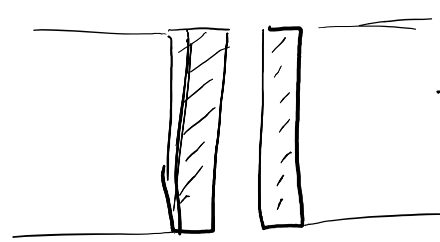
$K_v \equiv$ dynamic factor (Fig 14-9 P750)

$K_s = 1$ (size factor)

$K_B \equiv$ Rim Thickness factor (Fig 14-16 P756)

$J \equiv$ Geometry factor (Fig 14-6 P 745)

$K_m \equiv$ load distribution factor



C_{mc} — EQ 14-31

C_{PF} — EQ 14-32

C_{pm} — EQ 14-33

C_{ma} — EQ 14-34 & Fig 14-11

C_e — EQ 14-35

[We only need to analyze Pinion if both gear & Pinion have the same material]

[Because it sees more cycles]

Strength of gear teeth

$$\underbrace{S_b}_{\text{bending fatigue strength}} = \frac{Y_N}{K_T K_R} S_t$$

S_t — fatigue limit from direct test on gears

[stress number]

{Fatigue limit for 10,000,000 cycles @ 99% reliability}

$Y_N \equiv$ Life correction factor

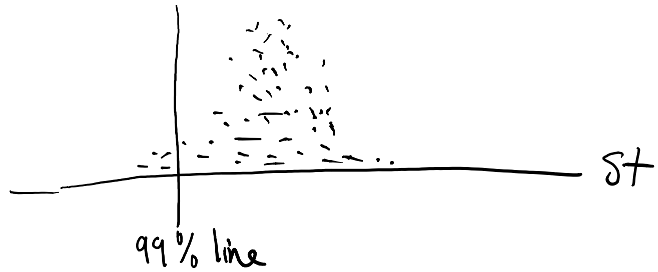
$Y_N = 1$ for 10 million

$K_T = 1$

$K_R \equiv$ reliability factor

$K_R = 1$ for 99%

Table 14-10 P 756 for other reliabilities



EXAMPLE

Given

$N_p = 16$

$N_g = 48$

$n_p = 300$ rpm

$hp = 5$

DC motor running a fan

$P = 6$

$F = 2$ inches

Material ≡ Through-hardened Grae-I Steel - hardened to 200 BHN

$Q_v = 6$ gear geometry precisim index

Not crowned

Accurate & rigid mounting

$L = 100$ millin cycles

Reliability (on strength data) = 90%

Find: Factor of Safety against tooth breakage

1) Calculate Pitch-line velocity

$$V = \frac{\pi n d}{12} = \frac{\pi (300)(2.667)}{12} = 209.47 \ \text{ft/min}$$

where $d = \frac{N_P}{P} = \frac{16}{6} = 2.667$

2) Calculate tangential force

$$W^t = \frac{33000\, hp}{V} = \frac{33000\,(5)}{209.47}$$

$$W^t = 787.7 \ \text{lbs}$$

3) overload factor $K_o = 1$ (Fig 14-17)

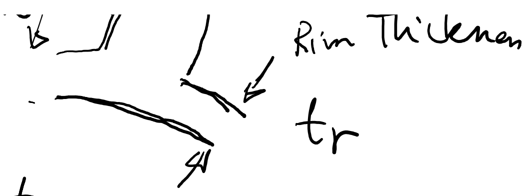
4) Dynamic facto $K_v = 1.2$ (Fig 14-9)

for $Q_v = 6$ and $V = 209$ ft/min

5) $K_S = 1$

5) K_S

6) Rim Thickness factor



Assume (find out) $m_B = \frac{t_R}{t_h} = 1$

$$K_B = 1.6 \ln \frac{2.242}{m_B}$$

$$= 1.6 \ln \frac{2.242}{1} = 1.3$$

$$K_B = 1.3$$

7) Load distribution factor K_m

$$K_m = 1 + C_{mc} \left(C_{PF} C_{pm} + C_{ma} C_e \right)$$