3-2-2022

Gear geometry Review

$P \equiv$ Pitch (diametral pitch) $= \frac{N}{d}$

$d \equiv$ diameter of pitch Circle

$N \equiv$ Number of teeth

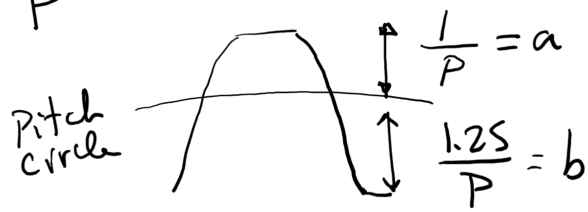
$p \equiv$ Circular pitch $\quad pP = \pi$

$\phi \equiv$ Pressure angle ($\phi = 20^\circ$)

$d_b \equiv$ diameter of the base circle

$$d_b = d \cos\phi$$

$h =$ tooth height $\quad \frac{2.25}{P}$




$a \equiv$ addendum

$b \equiv$ dedendum

$$m \equiv \text{gear ratio} \equiv \frac{n_{driver}}{n_{driven}} = \frac{N_{driven}}{N_{driver}} = \frac{d_{driven}}{d_{driver}}$$

Checking for interference (undercutting)

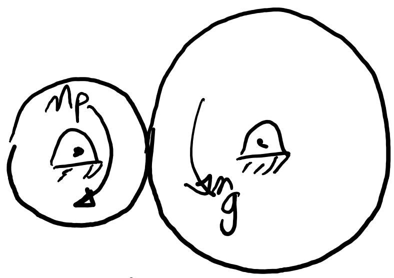
$N_{min,P} = \checkmark$

checking contact ratio

$CR \equiv \checkmark \quad > 1.2$

Gear Trains (Gear Kinematics)

Regular gear Trains




$n_P = +100$ rpm

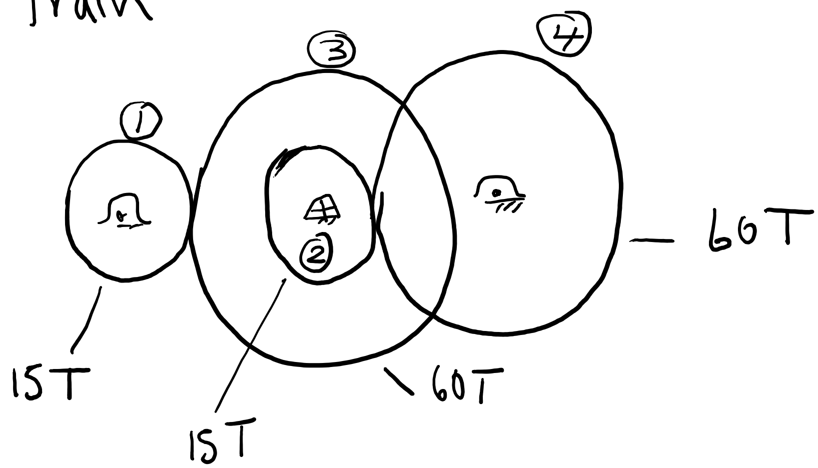
$N_P = 15 \qquad N_G = 60$

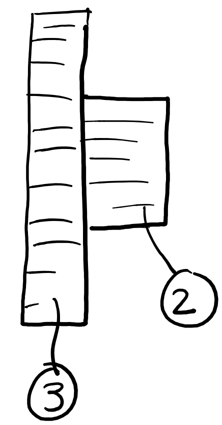
$m = +\frac{n_P}{n_G}$ [Two external gears]

$m = +\frac{100}{n_G} \Rightarrow -4 = \frac{100}{n_G} \Rightarrow n_G = -25$ rpm

$m = -\frac{N_G}{N_P} = -\frac{60}{15} = -4$

Gear Train




Find $m = \frac{n_1}{n_4}$

$$m = \frac{n_1}{n_3} \cdot \frac{n_3}{n_2} \cdot \frac{n_2}{n_4}$$

$$= \left(-\frac{N_3}{N_1}\right) \cdot (1) \cdot \left(-\frac{N_4}{N_2}\right)$$

$$m = + \frac{N_3 N_4}{N_1 N_2}$$

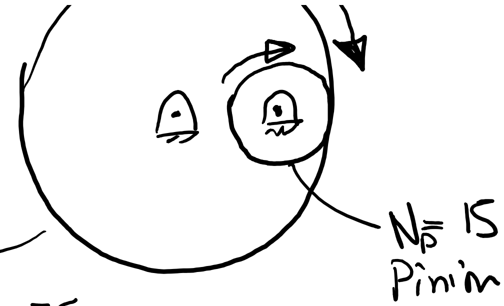
$N_1 = 15$ $N_2 = 15$ $N_3 = 60$ $N_4 = 60$

$$m = \frac{60(60)}{15(15)} = +16$$

Internal gears in mesh

$n_p$

$$m = \frac{n_P}{n_G}$$




$$m = + \frac{N_G}{N_P}$$

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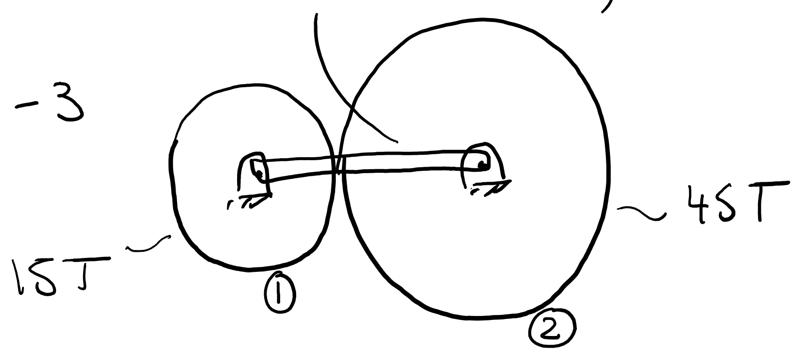
# Plantary Gear Trains (Epicyclic gears)

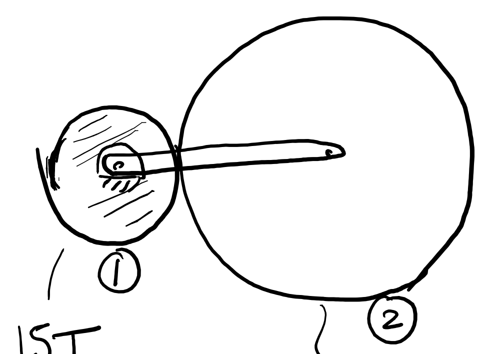
$$m = -\frac{N_2}{N_1} = -\frac{45}{15} = -3$$




Releasing the Arm

2-dof System

Need 2 inputs



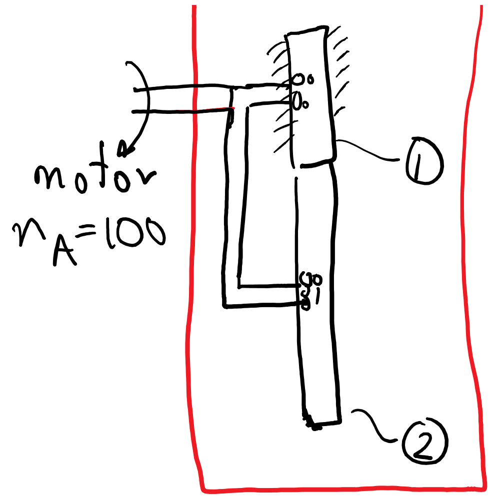

Example

$n_1 = 0 \qquad n_{1/ground} = 0$

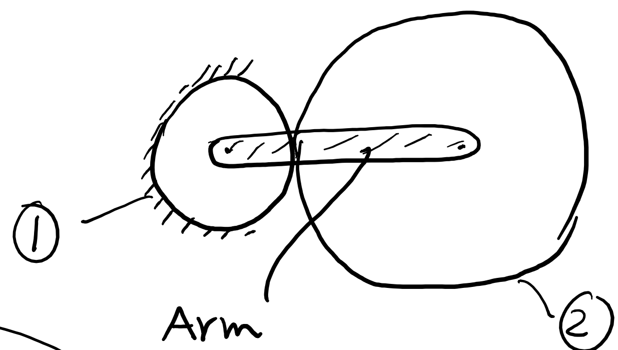
$n_A = +100$ rpm

$n_A = +100$

Find $n_2$




How do we solve all Plantary gear train Problems




$$\frac{n_{1/A}}{n_{2/A}} = -\frac{N_2}{N_1} = -\frac{45}{15} = -3$$

$n_{1/A} \equiv$ rpm of gear 1 w/r to Arm

$n_{1/g} \equiv n_1 \equiv$ rpm of gear 1 w/r to ground

$$n_{1/A} = n_{1/G} - n_{A/G} = n_1 - n_A$$

$$n_{2/A} = n_2 - n_A$$

$$\frac{n_{1/A}}{n_{2/A}} = \frac{n_1 - n_A}{n_2 - n_A} = \frac{0 - (100)}{n_2 - (100)}$$

$$\Rightarrow \frac{-100}{n_2 - 100} = -3$$

$$\Rightarrow n_2 = \frac{400}{3} = +133.3 \text{ rpm}$$

Gear ratio $\left( \begin{array}{l} \text{input Arm} = 100 \\ \text{output } n_2 = 133.3 \end{array} \right)$

$$m = \frac{100}{133.3} \checkmark$$

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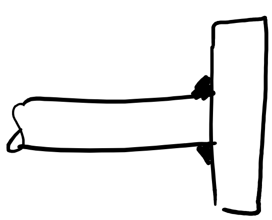
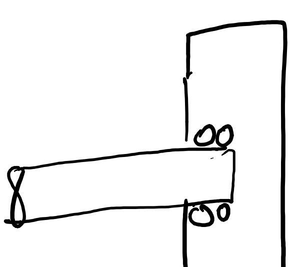
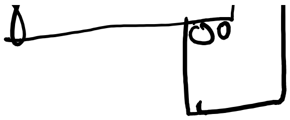
drawing Convention



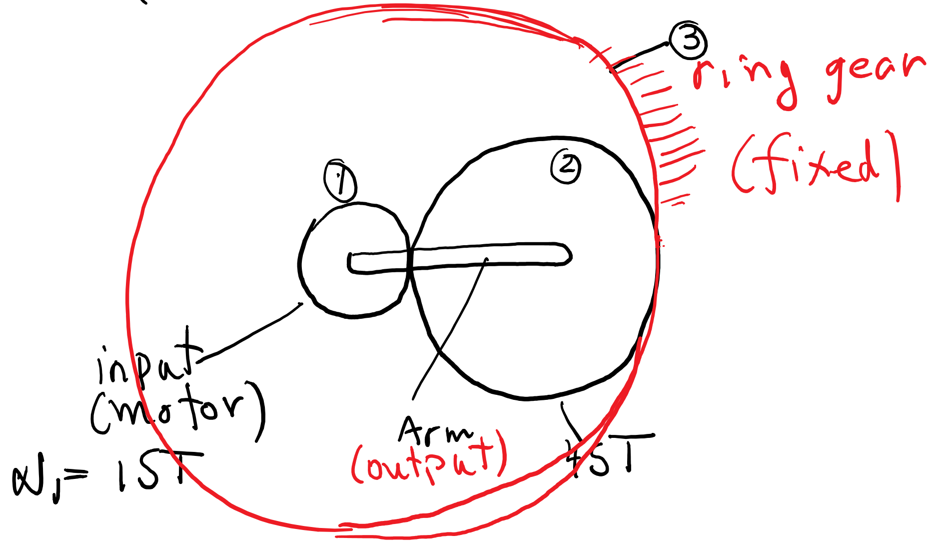
rigid Connection
(weld)



free to rotate



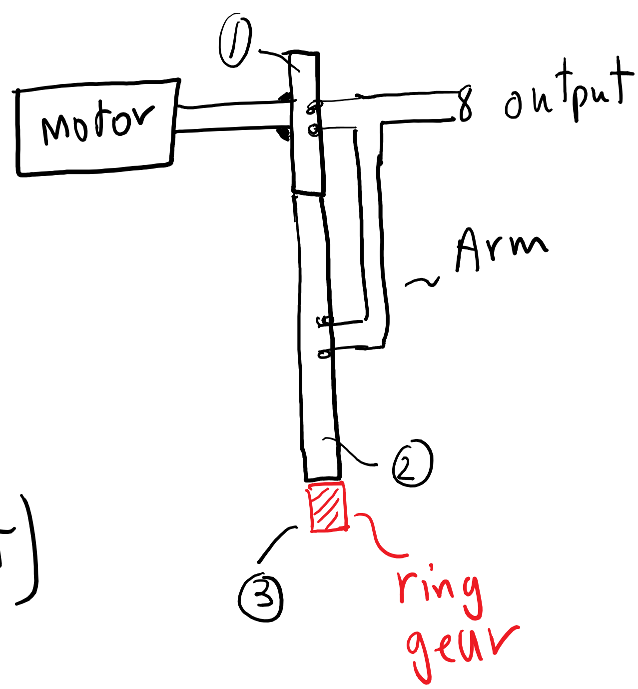
The Most Practical arrangement (Type-I)




inputs $n_1 = 100$

$n_3 = 0$

output $n_A$




Solving for $n_A$ (output)

$$\frac{n_{1/A}}{\ldots} = \frac{n_1 - n_A}{\ldots} = \frac{100 - n_A}{\ldots}$$

$$\bar{n}_{3/A} \qquad n_3 - n_A \qquad 0 - n_A$$

Also
$$\frac{n_{1/A}}{n_{3/A}} = \frac{n_{1/A}}{n_{2/A}} \cdot \frac{n_{2/A}}{n_{3/A}}$$

$$= \left(-\frac{\cancel{N_2}}{N_1}\right) \cdot \left(+\frac{N_3}{\cancel{N_2}}\right)$$

$$= -\frac{N_3}{N_1} = -\frac{105}{15} = -7$$

what is $N_3 = ?$

If the input & output shafts are in-line

$$r_1 + d_2 = r_3$$

$$2r_1 + 2d_2 = 2r_3$$

$$d_1 + 2d_2 = d_3$$

$$\Rightarrow N_1 + 2N_2 = N_3 \Rightarrow N_3 = 15 + 2(45)$$

$$\Rightarrow N_3 = 105 \text{ teeth}$$

Therefore
$$\frac{100 - n_A}{\phantom{x}} = -7$$

$$-n_A$$

$$\Rightarrow n_A = +12.5 \text{ rpm}$$

$$\text{Gear ratio} \quad m = \frac{n_1}{n_A} = \frac{100}{12.5} = 8 \quad (8 \text{ to } 1)$$

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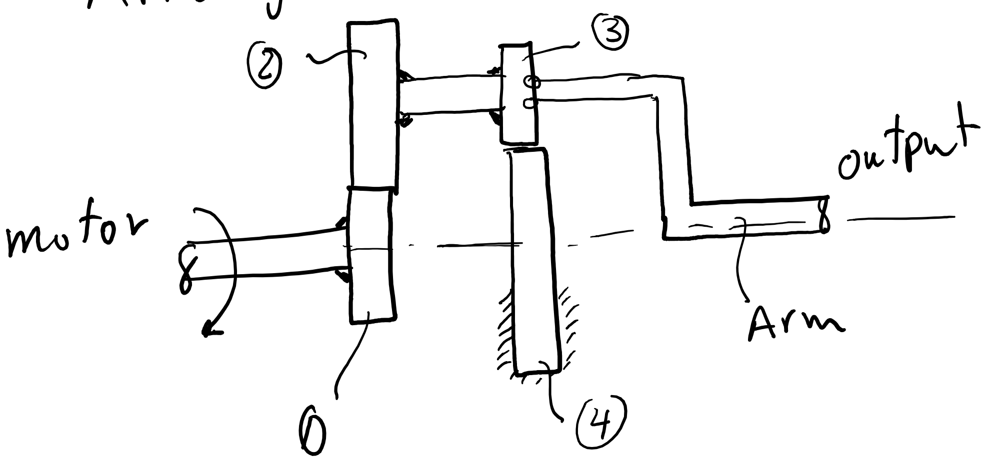
Arrangement #2 (Type - II)




$N_1 = 15$

$N_2 = 45$

$N_3 = 20$

for inline (input/output) shafts

$$r_1 + r_2 = r_3 + r_4$$

$$N_1 + N_2 = N_3 + N_4$$

$$15 + 45 = 20 + N_4 \Rightarrow N_4 = 40 \text{ teeth}$$

input $n_1 = +100$ rpm

output $n_A = ?$

$$\frac{n_{1/A}}{n_{4/A}} = \frac{n_1 - n_A}{n_4 - n_A} = \frac{100 - n_A}{0 - n_A}$$

Also

$$\frac{n_{1/A}}{n_{4/A}} = \frac{n_{1/A}}{n_{2/A}} \cdot \frac{n_{2/A}}{n_{3/A}} \cdot \frac{n_{3/A}}{n_{4/A}}$$

$$= \left(-\frac{N_2}{N_1}\right)\left(1\right)\left(-\frac{N_4}{N_3}\right)$$

$$= + \frac{N_2 N_4}{N_1 N_3} = \frac{(45)(40)}{(15)(20)} = +6$$

$$\Rightarrow \frac{100 - n_A}{0 - n_A} = 6 \Longrightarrow n_A = -20$$

gear ratio $m = \dfrac{100}{-20} = -5$

Toy gear box (one stage)

Toy gear box (one stage)

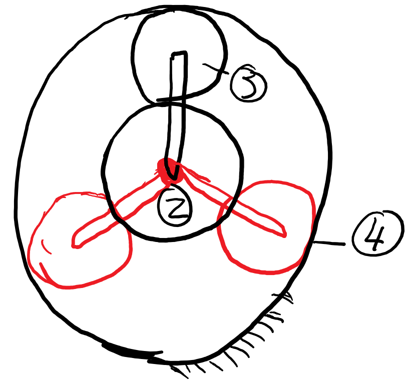
$N_2 = 24$

$N_3 = 18$

rpm

input $n_2 = +100$

Output $n_A$ (find)




Solution (answers) [calculate]

$N_4 \equiv 60$

$n_A = 28.57$

$$m = \frac{n_{input}}{n_{output}} \approx 3.5$$

2 stage $(3.5)^2$

⋮

8 stages $(3.5)^6 \equiv$ ✓ 1800

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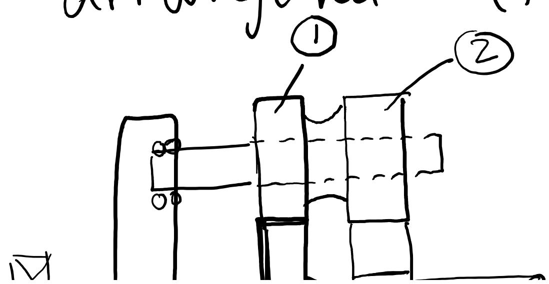
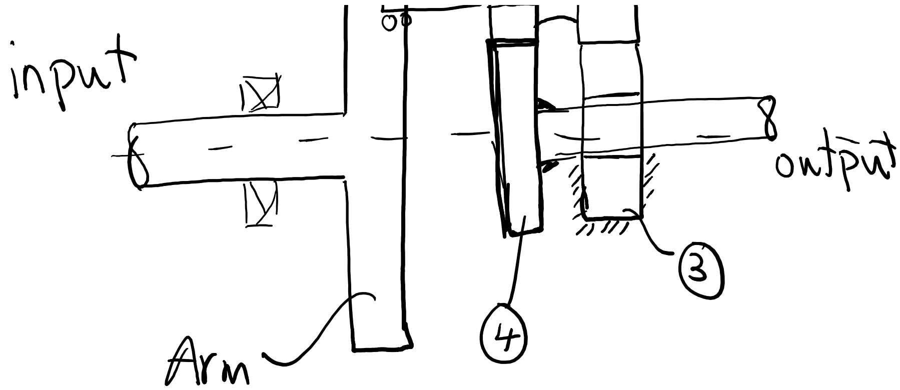
Special arrangement (for high gear ratio)






Given $N_1 = 101$ Teeth

$N_2 = 102$ "

$N_3 = 99$ "

Inputs Arm and gear 3

output $n_4$

Find gear ratio $\frac{n_A}{n_4}$

$n_A = +100$

Solution

$$r_1 + r_4 = r_2 + r_3$$

$$N_1 + N_4 = N_2 + N_3$$

$$\Rightarrow N_4 = 102 + 99 - 101 \Rightarrow N_4 = 100$$

Forming the speed ratio w/r to the arm

$$\frac{n_{4/A}}{n_{3/A}} = \frac{n_4 - n_A}{n_3 - n_A} = \frac{n_4 - 100}{0 - 100}$$

Also
$$\frac{n_{4/A}}{n_{3/A}} = \frac{n_{4/A}}{n_{1/A}} \cdot \frac{n_{1/A}}{n_{2/A}} \cdot \frac{n_{2/A}}{n_{3/A}}$$

$$= \left(-\frac{N_1}{N_4}\right)(1)\left(-\frac{N_3}{N_2}\right)$$

$$= +\frac{N_1 N_3}{N_2 N_4} = 0.98$$

$$\Rightarrow \frac{n_4 - 100}{0 - 100} = 0.98$$

$$\Rightarrow n_4 = 2$$

$$m = \frac{n_A}{n_4} = \frac{100}{2} = 50 \quad (50 \text{ to } 1)$$

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if not exactly in-line

$N_1 = 101$

$N_2 = 102$

$N_3 = 99$ $\Rightarrow$ $\underline{\underline{m = 100}}$

$N_4 = 99$