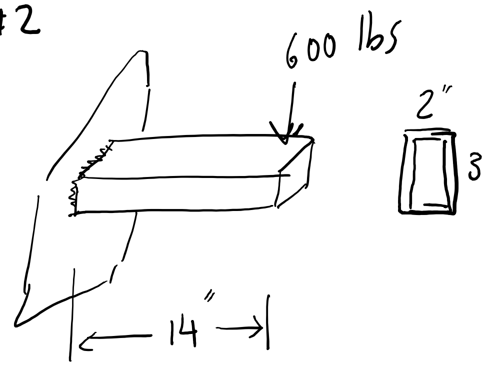
Solution of HW#3 Prob. #2



a)

Bending stress

$$\tau_b = \frac{Mc}{I_t} = \frac{8400(1.5)}{2.385} = 5283 \text{ Psi}$$ out of Plane

where

$$M = 600\,(14) = 8400 \text{ lb-in}$$

$$C = 1.5$$

$$I_t = I_u \cdot t = 13.5\,(0.1767) = 2.385 \text{ in}^4$$

where

$$I_u = \frac{d^2}{6}(3b+d) = 13.5 \text{ in}^3$$

and

$$t = 0.707\,(0.25) = 0.1767$$

Direct shear

$$\tau_d = \frac{F}{A_t} = \frac{600}{0.1767\,(2+3+2+3)}$$

$$\tau_d = 339.5 \text{ Psi}$$ (in plane)

Total shear stress

$$\tau = \sqrt{5283^2 + 339.5^2} = 5294 \text{ Psi}$$

Factor of Safety

$$n = \frac{0.3\,S_{ut}}{\tau} = \frac{0.3\,(70)}{5.294} = 3.97$$

$n \cong 4.00$

b) Nominal Shear stress 5294 Psi

$\tau_{act} = K_f\,\tau_{nom} = (1.5)(5294) = 7941$ Psi

K_f = toe of the weld

Endurance limit

$$S_{es} = K_a\,K_c\,S_e'$$

where $S_e' = \frac{1}{2} S_{ut} = 0.5(70) = 35$ ksi

and $K_c = 0.59$

and K_a = As forged

$K_a = 39.9\,(70)^{-0.995}$

$K_a = 0.582$

ksi

$$\Rightarrow \quad S_{es} = (0.582)(0.59)(35) \cong 12$$

The factor of safety

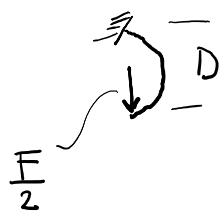
$$n = \frac{S_{es}}{\tau_{alt,act}} = \frac{12}{7.94} = 1.5$$

$$n = 1.5 \leftarrow$$

Continue with spring Design

check the shear stress (maximum) when the spring is under F_{max}

$$\tau = \frac{Tr}{J}$$

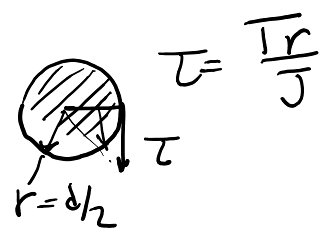


where $T = \frac{FD}{2}$

$D \equiv$ coil diameter

$r = \frac{d}{2}$ radius of the wire

$J = \frac{\pi d^4}{32}$



$$\tau_{max} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

There is also direct shear

$$= \frac{4F}{}$$

There is also

$$\tau_d = \frac{4F}{\pi d^2}$$

There is also the curvature effect

The Resultant shear stress is shown as

$$\tau_{max} = K \frac{8FD}{\pi d^3}$$

correct factor that takes into account both direct shear and curvature effect

Most conservative

$$K_B = \frac{4C+2}{4C-3}$$

where $C \equiv$ spring index

$$C = \frac{D}{d}$$

Recommendation $4 \leq C \leq 12$

example if $C = 3$

$$\Rightarrow K_B = \frac{4(3)+2}{4(3)-3} = 1.55$$

For the pencil spring

$$C = \frac{D}{d} = \frac{0.145}{} = 9.67$$

$$C = \frac{D}{d} = \frac{0.145}{0.015} = 9.67$$

$$K_B = \frac{4(9.67)+2}{4(9.67)-3} = 1.14$$

Calculating τ_{max}

$$\tau_{max} = 1.14 \frac{8(0.99)(0.145)}{\pi(0.015)^3}$$

$$\tau_{max} = 123473.7 \text{ Psi}$$

Strength in Shear

$S_{ys} = 0.45 S_{ut}$ (not set-removed)

$S_{ys} = 0.65 S_{ut}$ (set-removed)

steel springs

weld $S_{ys} = 0.3 S_{ut}$

other components

$S_{ys} = 0.58 S_y$

Estimating S_{ut} for spring wires

$$S_{ut} = \frac{A}{d^m}$$

For A 227 wire $A = 140$ Ksi

$m = 0.19$

$$S_{ut} = \frac{140}{(0.015)^{0.19}} = 310.9 \text{ Ksi}$$ (Table 10-4)

$$S_{ys} = 0.45\ S_{ut} = 0.45\,(310.9)$$

$S_{ys} = 140$ ksi yield strength in Shear

Factor of Safety

$$n = \frac{S_{ys}}{\tau} = \frac{140\,000}{123473} = 1.13$$

Set-removal Process

Always loaded downward

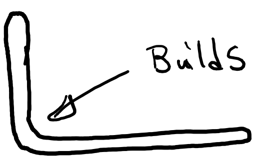
Desired Shape

1) 

stretched beyond yielding

2) 

Builds residual compressive stresses

3) 

it behaves as if its a stronger material

For spring ≡ 1) They make the free length longer
2) Compress it beyond the yielding
3) Release it to get to the desired free length

Design of a Mountain bike front suspension spring

Given: wire diameter $d = 0.15''$ inch

Coil outer diameter $D = 1.17$

Material: A 228

Squared ends

No move force (minimum force) Per spring = 20 lbs

Typical maximum force = 200 lbs Per spring

Set-removed

installation length $L_{F_{min}} = 10$ inches

Length @ maximum force $L_{F_{max}} = 5$ inches

Solution

Spring constant base on forces

$$\Delta F = K \, \Delta x$$

state-1 : F_{min} , $L_{F_{min}}$

State-2 : F_{max} , $L_{F_{max}}$

$$200 - 20 = K \, (10 - 5)$$

$$\Rightarrow K = 36 \ \text{lbs/in}$$

Free length

$$\Delta F = K \, \Delta x$$

(1) $F = 0$ L_o

(2) F_{min} $L_{F_{min}}$

$$(20 - 0) = 36 \, (L_o - 10)$$

$$\Rightarrow L_o = 10.55'' \leftarrow$$

Number of active Coils

$$K = \frac{d^4 G}{8 D^3 N} \Rightarrow 36 = \frac{(0.15)^4 (11.5 * 10^6)}{8 \, (1.02)^3 \, N}$$

where $D = D_o - d = 1.17 - 0.15 = 1.02$

Solving for N

$$N = 19 \text{ coils}$$

and

$$N_t = N + 2$$

$$N_t = 21 \text{ coils}$$

Checks

would the spring yield @ Solid force $n = 1.2$ or more

Solid length

$$L_s = (N_t + 1)\, d$$

$$L_s = (21 + 1)(0.15)$$

$$L_s = 3.3''$$

Solid Force

$$\Delta F = K\, \Delta X$$

(1) Free length $F = 0$ $\cancel{x} = L_o$

(2) Solid length $F = F_s$ $x = L_s$

$$(F_s - 0) = 36\,(10.55 - 3.3)$$

$$\Rightarrow F_s = 261 \text{ lbs}$$

$\Rightarrow F_S = 261$

✓ Fraction over-run to closure (Periouly clash allowance)

$$\xi = \frac{F_S - F_{max}}{F_{max}} = \frac{261 - 200}{200} = 0.305 \checkmark$$

Recommendation $\xi > 0.15$

Shear Stress @ Solid length

$$\tau = K_B \frac{8 F_S D}{\pi d^3} = 1.16 \frac{8(261)(1.02)}{\pi (0.15)^3} = 233005 \text{ Psi}$$

where $K_B = \frac{4C+2}{4C-3} = \frac{4(6.8)+2}{4(6.8)-3} = 1.16$

where

$$C = \frac{D}{d} = \frac{1.02}{0.15} = 6.8 \checkmark$$

$\tau = 233005$ Psi

Spring strength

$$S_{ut} = \frac{A}{d^m}$$

A228 — $m = 0.145$, $A = 201$ ksi

$$S_{ut} = \frac{201}{\ldots^{0.145}} = 264.6 \text{ ksi}$$

$$S_{ut} = \frac{}{(0.15)^{0.145}}$$

Spring strength in shear

$$S_{ys} = 0.65\ S_{ut} \qquad ksi$$

$$S_{ys} = 0.65\ (264.6) = 172$$

Factor of Safty guarding againt Set

$$n = \frac{S_{ys}}{\tau_{max}} = \frac{172}{233} = 0.74 \leftarrow$$

Fatigue Analysis

$$\tau_{min} = K_B\ \frac{8\ F_{min}\ D}{\pi d^3} = 1.16\ \frac{8\ (20)(1.02)}{\pi\ (0.15)^3}$$

$$\tau_{min} = 17854.8 \quad Psi$$

$$\tau_{max} = K_B\ \frac{8\ F_{max}\ D}{\pi d^3}$$

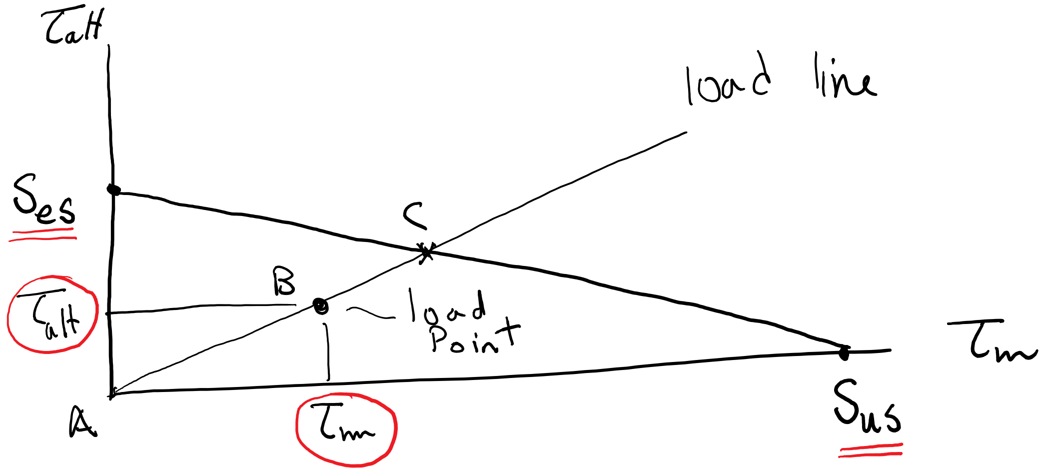
$$\tau_{max} = 178547.8 \quad Psi$$

Mean and alternating strenes

$$\tau_m = \frac{\tau_{min} + \tau_{max}}{2} = 98201.3 \quad Psi$$

$$\tau_{alt} = \frac{\tau_{max} - \tau_{min}}{2} = 80346.5 \quad Psi$$

Goodman diagram for shear stresses



$$n = \frac{AC}{AB}$$

$$\frac{1}{n} = \frac{\tau_a}{S_{es}} + \frac{\tau_m}{S_{us}}$$

Estimating S_{us} for steels

$$S_{us} = 0.67\, S_{ut} \quad (\text{EQ } 10\text{-}30)$$

$$S_{us} = 0.67\,(264.6)$$

$$S_{us} = 177.3 \text{ ksi}$$

Estimating S_{es} (non-steel or $d > 10^{mm}$)

$$S_{es} = K_a K_b K_c K_d K_e K_f S_e'$$

where $S_e' = \frac{1}{2} S_{ut}$ upto 200 ksi ... ksi

$S'_e = 100$ ksi when $S_{ut} > 200$

$\Rightarrow S'_e = 100$ ksi

$K_a \equiv$ Cold drawn wire

$\equiv a S_{ut}^{b} = 2.7\,(264.6)^{-0.265}$

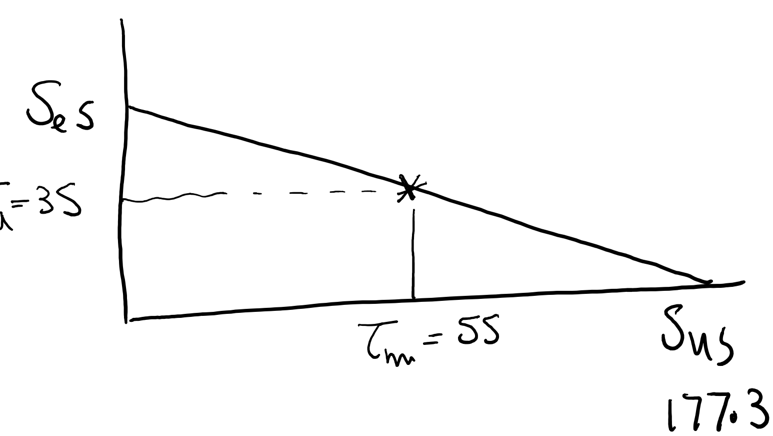
$= 0.615$

$K_e = 0.59$

$\Rightarrow S_{es} = (0.615)(0.59)(100) = 36.3$ ksi

Since $d < 10^{mm}$ and steel we can use the direct experimental results by Zimerelli

Zimerelli Data Method



$$\frac{S_{es}}{35} = \frac{177.3}{177.3 - 55}$$

$$\Rightarrow \quad S_{es} = 50.7 \text{ ksi}$$

Factor of Safety

$$\frac{1}{n} = \frac{\tau_a}{S_{es}} + \frac{\tau_m}{S_{us}}$$

$$\frac{1}{n} = \frac{80.3}{50.7} + \frac{98.2}{177.3}$$

$$\Rightarrow \quad n = 0.47 \leftarrow$$