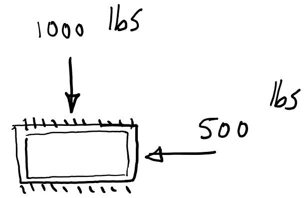
2-21-22

EXERCISE PROBLEMS FOR THE EXAM

Prob. #3



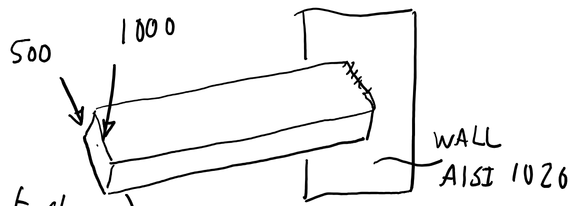
Given : 4X2 beam

E60

$h = 0.25$

AISI 1020 wall

$L = 10''$



(ignore direct shear)

Find

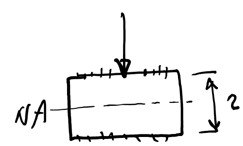
n_s , n_f

Stress due to 1000 lb load

$$\tau_1 = \frac{Mc}{I_t} = \frac{10\,000\,(1)}{1.414} = 7072.1 \text{ psi} \quad (\text{OOP})$$

Where

$M = 1000(10) = 10{,}000$ in-lb



$C = 1$

$I_t = I_u t = 8\ (0.17675) = 1.414\ in^4$

where

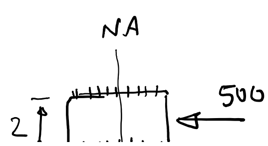
$$\overline{\,2} = \frac{4(2)^2}{2} = 8 \ in^3$$

Stresses due to 500 lb load

$$\tau_2 = \frac{MC}{I_t} = \frac{5000\,(2)}{1.885} = 5305 \text{ psi} \quad (\text{OOP})$$

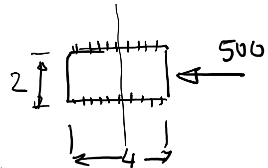
Where

$M = 500(10) = 5000$ in-lb



where

$$M = 500(10) = 5000 \quad \text{in-lb}$$



$$c = 2$$

$$I_t = I_u t = (10.667)(0.17675)$$

where

$$I_u = \frac{d^3}{6} = \frac{(4)^3}{6} = 10.667 \quad \text{in}^3$$

$$I_t = 1.885 \quad \text{in}^3$$

Total stress

$$\tau = \tau_1 + \tau_2 = 12377.1 \quad \text{psi}$$

Strength

$$S_{ys} = 0.3 \ S_{ut}$$

$$S_{ut} \ (\text{weld metal}) = 62 \quad \text{ksi}$$

$$S_{ut} \ (\text{AISI 1020 in Hot Rolled}) = 55 \quad \text{ksi}$$

$$S_{ys} = 0.3(55) = 16.5 \quad \text{ksi}$$

Factor of safety (static)

$$n_s = \frac{S_{ys}}{\tau} = \frac{16.5}{12.38} = 1.3 \leftarrow$$

Fatigue Analysis (fully alternating)

$$n_f = \frac{S_{es}}{\tau_{a,act}} = \frac{12}{33.4} = 0.36 \leftarrow$$

where

$$S_{es} = K_a K_c S_e' = (0.74)(0.59)(27.5) = 12 \quad \text{ksi}$$

where $\qquad -0.995$

$$K_a = a\,S_{ut}^{\,b} = 39.9\,(55)$$

$$K_a = 0.74$$

$$K_c = 0.59$$

$$S_e' = \frac{1}{2} S_{ut} = \frac{1}{2}(55) = 27.5 \text{ ksi}$$

Also

$$\tau_{a,act} = K_f\,\tau_{a,nom}$$

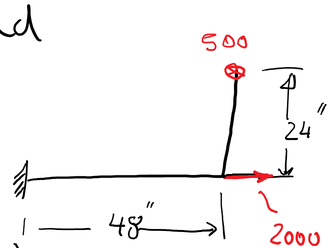
$$= 2.7\,(12.377) = 33.4 \text{ psi}$$

Prob #4

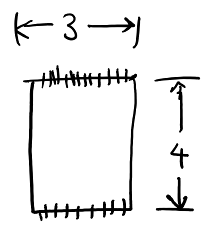
Given: geometry and load

$h = 3/8''$

E70



Find n_s (static loading)



Stress from tensile force

$$\tau_d = \frac{F}{A_t} = \frac{2000}{1.59} = 1257.86 \text{ psi}$$

(OOP)

where

$$A_t = (2)(3)\,t = 6(0.265) = 1.59$$

where

$$t = 0.707\,h = 0.707\,(3/8) = 0.265''$$

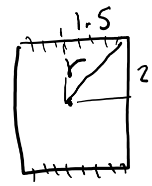
Stress from torsion

$$\tau_t = \frac{Tr}{J_t} = \frac{12000\,(2.5)}{7.55} = 3973.5 \quad \text{(IP)}$$

where

$$T = 500(24) = 12000$$

$$r = \sqrt{2^2 + 1.5^2} = 2.5$$



where

$$J_t = J_u t = (28.5)(0.265) = 7.55 \quad in^4$$

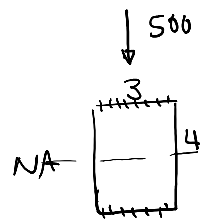
where

$$J_u = \frac{d(3b^2 + d^2)}{6}$$

$$= \frac{3(3 * 4^2 + 3^2)}{6} = 28.5 \quad in^3$$

Stresses from bending

$$\tau_b = \frac{Mc}{I_t} = \frac{500(48)(2)}{6.36}$$



where

$$I_u = \frac{bd^2}{2} = \frac{3(4)^2}{2} = 24 \quad in^3$$

$$I_t = 24(0.265) = 6.36$$

$$\tau_b = 7547.1 \text{ psi} \quad (OOP)$$

Total IP stress

$$\tau_{IP} = 3973.5 \text{ psi}$$

Total OP Stress

$$\tau_{OP} = 1257.8 + 7547.1 = 8805 \text{ psi}$$

Resultant stress

______ psi

$$\tau = \sqrt{\tau_{1p}^2 + \tau_{op}^2} = 9660$$

Factor of Safety

$$n = \frac{S_{ys}}{\tau} = \frac{21000}{9660} = 2.17$$

where $S_{ys} = 0.3\ S_{ut} = 0.3\ (70) = 21^{ksi}$

Prob. #1 Spring Design

Given: $D_o = 1.8$

$d = 0.2$

A227

$F_{min} = 40^{lbs}$ $\quad L_{F_{min}} = 3''$

$F_{max} = 120^{lbs}$ $\quad L_{F_{max}} = 2''$

Square ends

Set-removed

Find: L_o, ξ, τ_s, n_s, n_f

Calculate K from forces

$$\Delta F = K\, \Delta x$$

$$(120-40) = K\ (3-2)$$

$$\Rightarrow K = 80 \text{ lbs/in}$$

Calculating free length

$$\Delta F = K\, \Delta x$$

$$(40-0) = 80\ (L_o - 3)$$

$$\Rightarrow L_o = 3.5 \text{ in}$$

fractional overrun to closure

$$\xi = \frac{F_s - F_{max}}{F_{max}} = \frac{120-120}{120} = 0$$

where

$$\Delta F = K \Delta x$$

$$(F_s - 0) = 80\,(3.5 - L_s) = 80(3.5-2)$$

$$\Rightarrow F_s = 120 \text{ lbs}$$

where

$$L_s = d\,(N_t + 1) = 0.2\,(9+1) = 2''$$

where

$$N_t = N + 2 = 7 + 2 = 9$$

where

$$K = \frac{d^4 G}{8 D^3 N}$$

$$80 = \frac{(0.2)^4 * (11.5 * 10^6)}{8\,(1.6)^3 N}$$

where

$$D = D_o - d$$

$$= 1.8 - 0.2$$

$$D = 1.6$$

$$\Rightarrow N \cong 7 \text{ turns}$$

Shear stress at solid length

$$\tau = K_B \frac{8 F_s D}{\pi d^3} = 1.17 \frac{8\,(120)(1.6)}{\pi\,(0.2)^3} = 71627.3 \text{ Psi}$$

where
$$K_B = \frac{4C+2}{4C-3} = \frac{4(8)+2}{4(8)-3} = 1.17$$

where
$$C = \frac{D}{d} = \frac{1.6}{0.2} = 8$$

Factor of Safety
$$n_n = \frac{S_{ys}}{\tau} = \frac{123.5}{71.62} = 1.72 \leftarrow$$

where
$$S_{ys} = 0.65\ S_{ut} \quad \text{(set-removed)}$$

where
$$S_{ut} = \frac{A}{d^m} = \frac{140}{(0.2)^{0.19}} = 190^{ksi}$$

where
$$\left.\begin{aligned} A &= 140^{ksi} \\ m &= 0.19 \end{aligned}\right\} \text{Table } 10\text{-}4$$

$$\Rightarrow S_{ys} = 0.65\,(190) = 123.5 \text{ ksi}$$

Fatigue Analysis

minimum & max stress
$$\tau_{min} = K_B \frac{8F_{min}D}{\pi d^3} = 1.172\ \frac{8(40)(1.6)}{\pi(0.2)^3}$$

$$\tau_{min} = 23875.8 \text{ Psi}$$

$$\tau_{max} = 71627.3 \text{ Psi}$$

mean & alternating stresses
$$\tau_m = \frac{\tau_{min} + \tau_{max}}{2} = 47751.5 \text{ Psi}$$

Psi

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = 23875.8$$

Using Goodman Criterim

$$\frac{1}{n} = \frac{\tau_a}{S_{es}} + \frac{\tau_m}{S_{us}}$$

where

$$S_{us} = 0.67\ S_{ut} = 0.67(190)$$

$$S_{us} = 127.3\ \text{ksi}$$

From Zimerelli method & Goodman

$$S_{es} = \frac{35}{1-\left(\frac{55}{S_{us}}\right)} = \frac{35}{1-\left(\frac{55}{127.3}\right)}$$

$$S_{es} = 61.6\ \text{ksi}$$

$$\frac{1}{n_f} = \frac{23.87}{61.6} + \frac{47.7}{127.3} = 0.76$$

$$\Rightarrow n_f = 1.3$$

Prob # 2 (motor cycle spring)

Given $L_o = 6''$

$F_{min} = 50$ lbs $\quad L_{F_{min}} = 5''$

$F_{max} = 150$ lbs $\quad L_{F_{max}}$

$d = 0.25$

$D_o = 2.75$

A 227

Set-removed

Squared & ground ends

Find

n_s

ξ

find K from forces

$$\Delta F = K \, \Delta X$$

$$(50 - 0) = K\,(6 - 5) \Rightarrow K = 50 \ \text{lb/in}$$

find N, number of active Coils

$$K = \frac{d^4 G}{8 D^3 N} \Rightarrow 50 = \frac{(0.25)(11.4 \times 10^6)}{8\,(2.5)^3 N}$$

where

$$D = D_{max} - d = 2.75 - 0.25 = 2.5$$

$$\Rightarrow N = 7.125 \ \text{turns}$$

Total # of Coils

$$N_t = N + 2 \Rightarrow N_t = 9.125 \ \text{Turns}$$

Finding $L_{F_{max}}$

$$\Delta F = K \, \Delta X$$

$$(F_{max} - 0) = 80\,(6 - L_{F_{max}})$$

$$\Rightarrow L_{F_{max}} = 3''$$

Solid Force calculation

$$\Delta F = K \, \Delta x$$

$$(F_s - 0) = 80\,(6 - 2.281) \Rightarrow F_s = 185.9 \text{ lbs}$$

where

$$L_s = d N_t$$

$$= 0.25\,(9.125)$$

$$L_s = 2.281$$

Fractional overrun to closure

$$\xi = \frac{F_s - F_{max}}{F_{max}} = \frac{185.9 - 150}{150} = 0.24$$

Shear stress at solid length

$$\tau_s = K_B \frac{8 F_s D}{\pi d^3} = 1.135 \frac{8(185.9)(2.5)}{\pi (0.25)^3}$$

where $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(10)+2}{4(10)-3} = 1.135$

where

$$C = \frac{D}{d} = \frac{2.5}{0.25} = 10$$

$$\Rightarrow \tau_s = 85968 \text{ Psi}$$

Strength in shear

$$S_{ys} = 0.65\, S_{ut} = 0.65\,(182.2) = 118.4 \text{ ksi}$$

where

$$S_{ut} = \frac{A}{d^m} = \frac{140}{(0.25)^{0.19}}$$

$$S_{ut} = 182.2 \text{ ksi}$$

Factor of Safety

$$n = \frac{S_{ys}}{\tau} = \frac{118.4}{85.97} = 1.38$$

Would the spring buckle?

For absolute stability

$$L_0 < 2.63 \frac{D}{\alpha}$$

where $\alpha = 0.5$ (fixed ends)

$$L_0 = 6''$$

$$2.63 \frac{D}{\alpha} = 2.63 \left(\frac{2.5}{0.5}\right) = 13.2$$

Since $L_0 \overset{\checkmark}{<} 2.63 \frac{D}{\alpha}$

$\Rightarrow$ The spring will not buckle