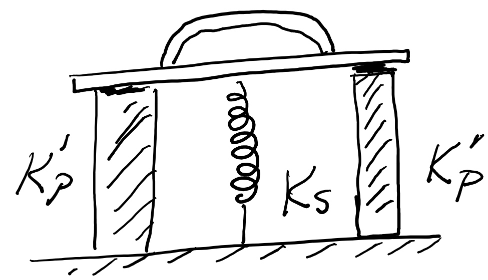
Test HW

$F_i = 100$ lbs



$K_p = K_p' + K_p''$

$K_p = 50$ lbs/in

$K_s = 100$ lbs/in

For Spring

$\Delta F = K \Delta X$

state-I : free length

state 2 : $F_s = 100$ lbs

$F_2 - F_1 = K(X_2 - X_1)$

length-1 Length-2

$\Delta F = F_2 - F_1 = F_i - 0 = F_i = 100$

$\Delta X = ?$

$100 = 100\,\Delta X \Rightarrow \Delta X = 1$ inch spring stretch

For Pads

$\Delta F = K_p \Delta X$

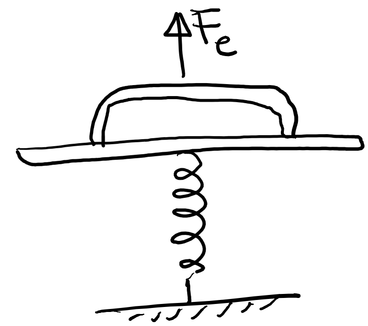
$F_i - 0 = K_p \underbrace{\Delta X}_{\text{Compression of Pads}}$

$$100 = 50\,\Delta X_p \Rightarrow \Delta X_p = 2''$$

Solving Problem using reasoning

To lift the pads, the external force must stretch the spring an additional $2''$

$\Rightarrow$ Total spring stretch $\equiv 3$ inches



$F_e = F_s$ (after lifting)

$F_e^* \equiv$ ext. force leading to pad separation

$F_s^* \equiv$ spring force corresponding to pad separation

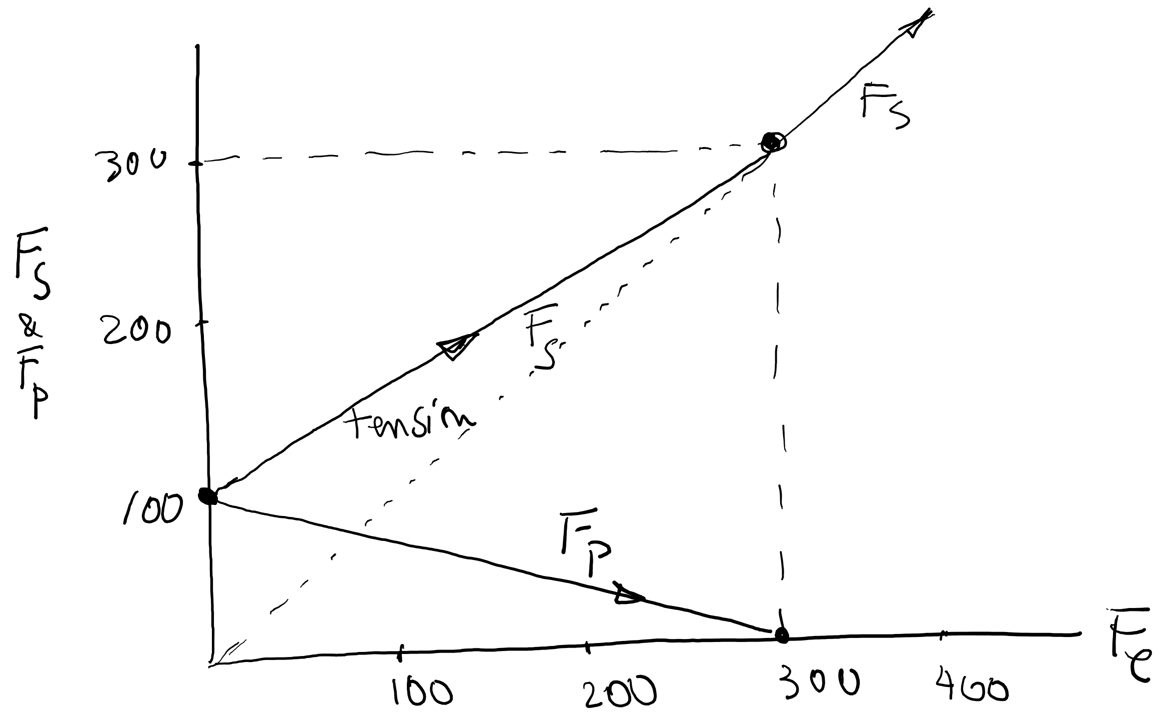
$$\Delta F = K_s\,\Delta X_s$$

$$F_s^* = 100\,(3) = 300 \text{ lbs}$$

$$\Rightarrow F_e^* = 300 \text{ lbs} \leftarrow$$

Draw F_s w/r F_e

Draw F_s ...



From Graph

$$F_s = 100 + \frac{2}{3} F_e \qquad \text{tension}$$

$$F_p = 100 - \frac{1}{3} F_e \qquad \text{Compression}$$

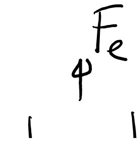
Solve w/o reasoning

After hook up $F_i = 100$ lbs

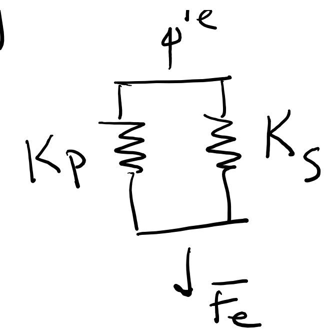
$$F_s = F_i = 100 \qquad \text{extesion}$$

$$F_p = F_i = 100 \qquad \text{compression}$$

when the external force comes, the load portion that goes into spring



Portion that goes into spring



Portion for spring

$$C = \frac{K_s}{K_p + K_s} = \frac{2}{3}$$

Portion for Pads

$$1 - C = \frac{K_p}{K_p + K_s} = \frac{1}{3}$$

Therefore

$$F_s = F_i + C\,F_e$$
$$F_p = F_i - (1-C)\,F_e$$

or

$$\boxed{\begin{aligned} F_s &= 100 + \tfrac{2}{3}\,F_e \\ F_p &= 100 - \tfrac{1}{3}\,F_e \end{aligned}}$$

Similar to bolt & clamped materials

$$F_b = F_i + C\,F_e$$
$$F_m = F_i - (1-C)\,F_e$$

where $\quad K_b$

where $$C = \frac{K_b}{K_b + K_m}$$

F_i = initial tension (designer)

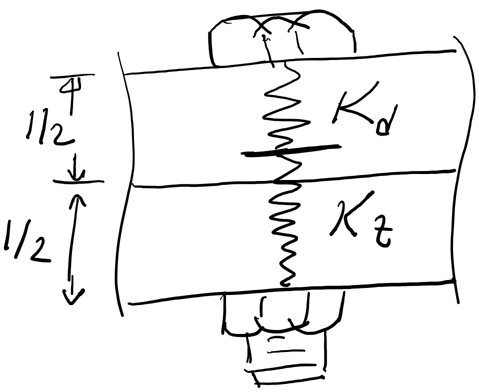
K_b = bolt spring constant

K_m = clamped material spring c.

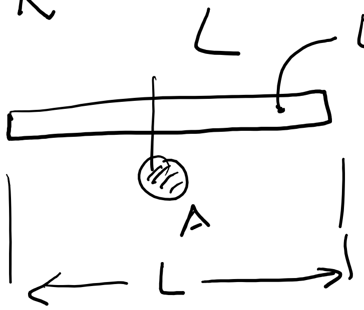
Calculating bolt (screw) spring constant

Given: 1/2 in UNC 13

$L = 2''$



$$K = \frac{AE}{L}$$



$$\ell \equiv \text{grip} = 1''$$

$$\ell_d = L_d = 0.75$$

where $$L_d = L - L_T = 2 - 1.25 = 0.75''$$

where

$$L_T = 2d + 1/4 = 2(0.5) + 0.25$$
$$L_T = 1.25$$

$$\Rightarrow K_d = \frac{A_d E}{l_d} = \frac{0.196\,(30)(10^6)}{0.75}$$

where $A_d = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(0.5)^2 = 0.196 \text{ in}^2$

$$\Rightarrow K_d = 7.85 * 10^6 \text{ lbs/in}$$
or $K_d = 7.85$ Mlbs/in

For the threaded portion within grip

$$l_t = l - l_d$$
$$l_t = 1 - 0.75 = 0.25''$$

$$K_t = \frac{A_t E}{l_t} = \frac{0.1419 * (30 * 10^6)}{0.25} = 17.0 * 10^6 \text{ lbs/in}$$

or $K_t = 17$ Mlbs/in

Combining the two

$K_t K_d$

Combining the two

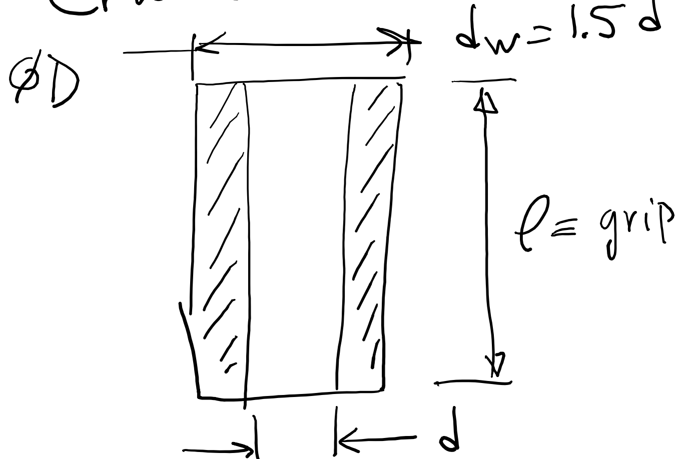
$$\frac{1}{K_b} = \frac{1}{K_t} + \frac{1}{K_d} \quad \text{or} \quad K_b = \frac{K_t K_d}{K_t + K_d}$$

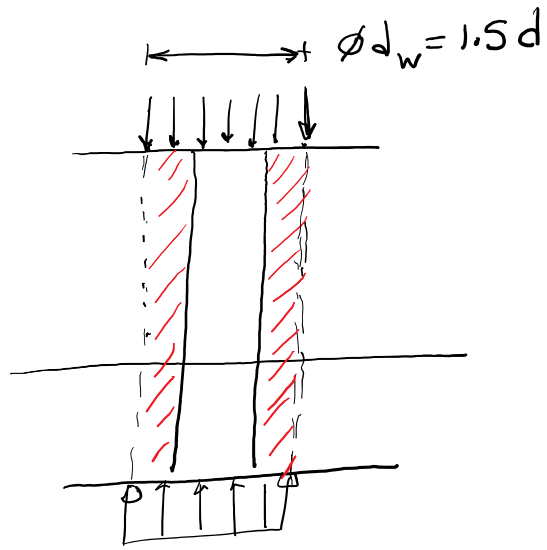
$$\frac{1}{K_b} = \frac{1}{17} + \frac{1}{7.85}$$

$$\Longrightarrow K_b = 5.37 \text{ Mlbs/in}$$

Calculating Spring Constant of clamped materials

Crude model





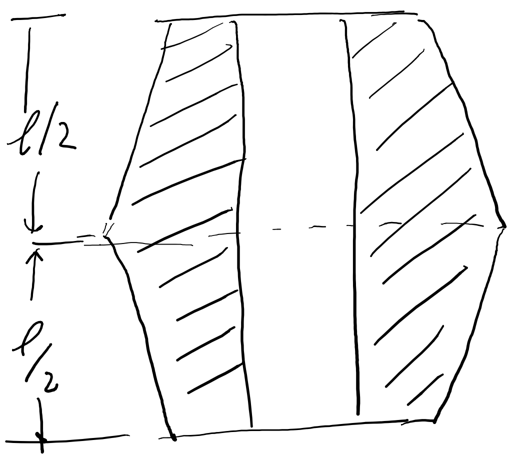
$$K_m = \frac{AE}{L} = \frac{(\pi (D^2 - d^2)/4)E}{\ell}$$

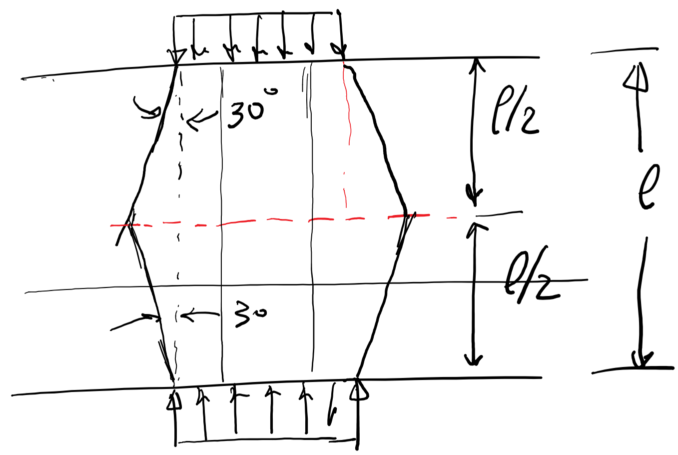
total grip

d = fastener major diameter

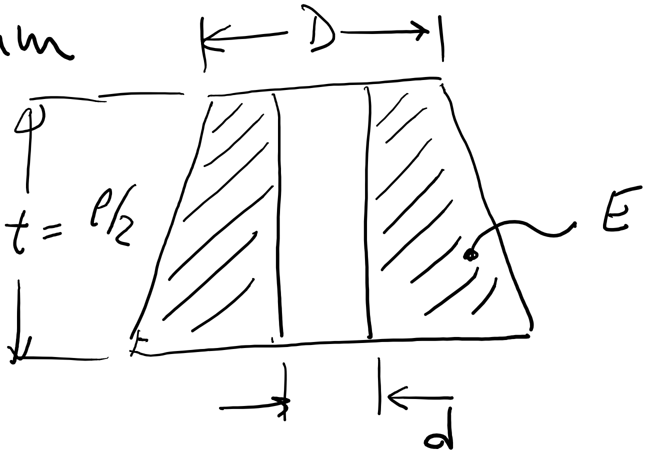
$D = 1.5\,d \equiv$ integrated washer diameter

Beter model :(Supported by experiments)





Frustum



$$K_F = f(D, d, t, E)$$

$$K_F = \frac{0.5774\,\pi E d}{\ln \dfrac{(1.155\,t + D - d)(D + d)}{(1.155\,t + D + d)(D - d)}} \qquad \text{(EQ 8-20)}$$

Q: Calculate the spring constant of a frustum for a 3/8 in UNC bolt where $t = 0.5''$

$d = 0.375''$

$D = 1.5d = 0.5625''$

$K_F = f(D, d, t, E)$

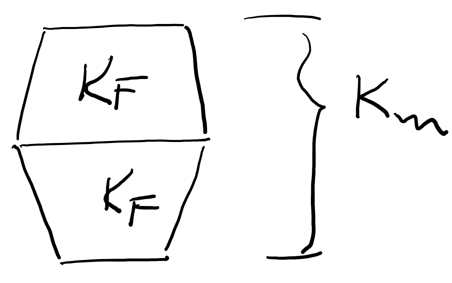
$\quad = f(0.5625, 0.375, 0.5, 30 \cdot 10^6)$

From EXCEL Sheet

$K_f = 22$ Mlbs/in

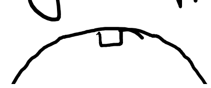
Q: what is the spring constant of the clamped materials

$$\frac{1}{K_m} = \frac{1}{K_F} + \frac{1}{K_F} \Rightarrow K_m = \frac{K_F}{2}$$

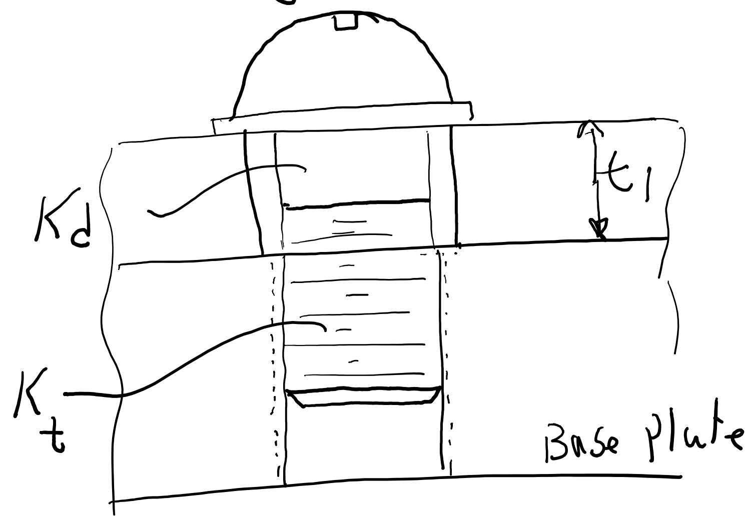


$K_m = 11$ Mlbs/in

Calculating Capscrew Spring Constant



Calculating Capscrew ...



$$K_d = \frac{A_d E}{l_d}$$

where $A_d = \frac{\pi}{4} d^2$

$l_d = L_d \equiv$ shank length

where $L_d = L - L_T$

and $L_T = 2d + 1/4$

Example

$t_1 = 1''$

$\frac{1}{2}$ in UNC 13

$l = 2''$ ... ^{2}

$$A_d = \pi/4\,(0.5)^2 = 0.1963\ in^2$$

$$E = 30 \times 10^6\ psi$$

$$\ell_d = L_d$$

$$L_T = 2d + 1/4 = 1.25''$$

$$L_d = L - L_T = 2 - 1.25 = 0.75''$$

$$\Rightarrow\quad K_d = \frac{0.1963 \times 30 \times 10^6}{0.75} = 7.85 \times 10^6\ lbs/in$$

or

$$K_d = 7.85\ Mlbs/in$$

For the threaded length

$$K_t = \frac{A_t E}{\ell_t} = \frac{0.1419\,(30 \times 10^6)}{0.5} = 8.51 \times 10^6\ lb/in$$

where

$$A_t = 0.1419$$

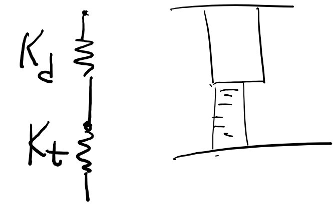
$$\ell_t = \ell - \ell_d = 1.25 - 0.75 = 0.5$$

$$\text{Total grip} \equiv \ell = t_1 + \frac{d}{2}$$

$$0.5 \qquad 1.25''$$

$$2 \qquad = 1.25$$

or $K_t = 8.51$ Mlbs/in



$$\frac{1}{K_b} = \frac{1}{K_d} + \frac{1}{K_t} = \frac{1}{7.85} + \frac{1}{8.51} \cong 0.25$$

$K_b = 4$ Mlbs/in

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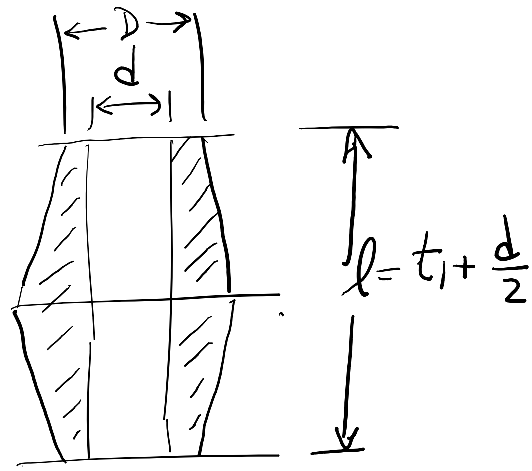
what about K_m

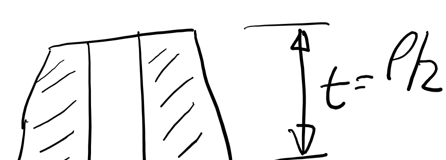


$$K_F = f(D, d, t_1, E)$$

where $l = t_1 + \frac{d}{2}$

$$l = 1 + \frac{0.5}{2} = 1.25$$

$$K_F = f\left(0.75,\ 0.5,\ \frac{1.25}{2},\ 30 \cdot 10^6\right)$$



$K_F = 30.2$ Mlbs/in

$$\Rightarrow \quad K_m = \frac{K_F}{2} = 15.1 \text{ Mlbs/in}$$

Scope

All materials are the same

(All Steel)

1 — steel

2 — Copper

3 — Steel