ME320 | Nov 9 2015

Announcements

1. Quiz #2 Friday 11/13

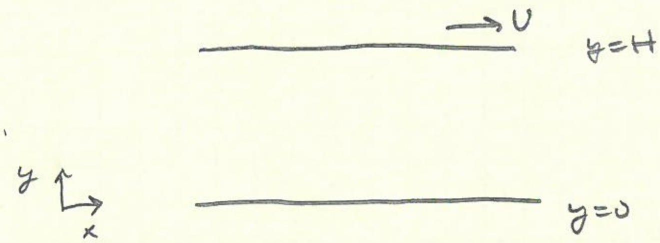
2. Exam #2 11/20

3. No class Wednesday

4. No lab Wednesday

Example

Plane Couette Flow –



previously we've stated that the velocity in this gap is linear.

Why is it? we took it as given.

Find the fluid velocity between the gap.

Assume we have a Newtonian incompressible fluid. Steady flow

use

Navier Stokes Eqn & continuity

Continuity (incompressible fluid)

$$\underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{\substack{0\\ \text{steady}}} + \rho \frac{\partial v_x}{\partial x} + \underbrace{\rho \cancel{\frac{\partial v_y}{\partial y}}}_{v_y = \text{constant} = 0} + \underbrace{\rho \cancel{\frac{\partial v_z}{\partial z}}}_{v_z = \text{constant} = 0} = 0$$

infinite direction no variation in z

boundary condition

thus $\dfrac{\partial v_x}{\partial x} = 0$

Navier Stokes

x component

Equivalent pressure $P = p + \rho g x$

$$\rho\Big(\underbrace{\cancel{\frac{\partial v_x}{\partial t}}}_{0 \text{ s.s}} + \underbrace{v_x \cancel{\frac{\partial v_x}{\partial x}}}_{0 \text{ continuity}} + \underbrace{\cancel{v_y} \frac{\partial v_x}{\partial y}}_{v_y = 0} + \underbrace{\cancel{v_z} \frac{\partial v_x}{\partial z}}_{v_z = 0}\Big) = -\underbrace{\cancel{\frac{\partial P}{\partial x}}}_{\substack{\text{no}\\ \text{imposed}\\ \text{pressure}\\ \text{gradient}}} + \mu\Big(\underbrace{\cancel{\frac{\partial^2 v_x}{\partial x^2}}}_{\substack{0\\ \frac{\partial v_x}{\partial x} = 0}} + \frac{\partial^2 v_x}{\partial y^2} + \underbrace{\cancel{\frac{\partial^2 v_x}{\partial z^2}}}_{\substack{\text{no}\\ z \text{ dependence}}}\Big)$$

thus.

$$\frac{\partial^2 V_x}{\partial y^2} = 0$$

y component

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right)$$

entire eqn is zero. If we didn't know $P(y)$ the eqn tells us $P \neq P(y)$

$\frac{dP}{dy} = 0 \Rightarrow P(y) = \text{constant}$

z component $\Rightarrow$ replace v_y with v_z same result entire eqn is zero

thus fluid motion is governed by

$$\frac{\partial^2 V_x}{\partial y^2} = 0 \quad \text{but since } V_x = V_x(y) \quad \frac{d^2 V_x}{dy^2} = 0$$

2nd order ordinary differential eqn.

Need 2 B.Cs. to solve the eqn.

B.C. 1 at $y = 0$ $V_x = 0$

B.C. 2 at $y = H$ $V_x = U$

<u>Solve.</u>

$$\frac{d^2 V_x}{dy^2} = 0 \rightarrow \frac{d}{dy}\left(\frac{dV_x}{dy}\right) = 0$$

derivative of something equals zero something is a constant

seperate & integrate

$$\frac{dV_x}{dy} = C_1$$

seperate & integrate. $V_x = C_1 y + C_2$

apply B.Cs

$y = 0$ $V_x = 0 \Rightarrow C_2 = 0$

$y = H$ $V_x = U \Rightarrow C_1 = \frac{U}{H}$ thus $\boxed{V_x = \frac{U}{H} y}$

linear velocity