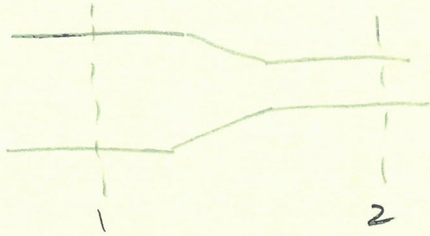
move to A

Conservation of mass



for steady conditions

$$\dot{m}_1 = \dot{m}_2$$

in terms of flow rates

$$Q_1 \rho_1 = Q_2 \rho_2$$

for incompressible flow $\rho_1 = \rho_2$ thus

$Q_1 = Q_2$ flow rate constant along the pipe

$$Q = \int V \, dA \quad \text{so} \quad \int V_1 \, dA_1 = \int V_2 \, dA_2$$

V is velocity normal to cross section

if inviscid velocity profile is uniform - no wall effects

so

$$V_1 \int dA_1 = V_1 A_1 \;\; \& \;\; \int V_2 \, dA_2 = V_2 A_2$$

thus

$$\boxed{V_1 A_1 = V_2 A_2}$$

Note: $Q = VA$ if $V = V_{avg}$ even if V varies across the cross section

conservation of mass for steady incompressible flow.

With conservation of mass we can relate V_1 & V_2

$$V_1 = V_2 \frac{A_2}{A_1} = V_2 \frac{\pi \frac{D_2^2}{4}}{\pi D_1^2/4}$$

thus

$$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 \qquad (3)$$

h related to V_1
h related to V_2
V_1 related to V_2

substitute (3) into (1) to eliminate V_1

$$h = \frac{P_1 - P_0}{\gamma} + \frac{V_2^2}{2g} \left(\frac{D_2}{D_1}\right)^4 \qquad (4)$$

substitute (2) into (4)

$$h = \frac{P_1 - P_0}{\gamma} + h\left(\frac{D_2}{D_1}\right)^4$$

check units – each term has dimensions of length

solve for h

$$h\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{P_1 - P_0}{\gamma}$$

so

$$h = \left(\frac{P_1 - P_0}{\gamma}\right)\left[\frac{1}{1 - (D_2/D_1)^4}\right]$$

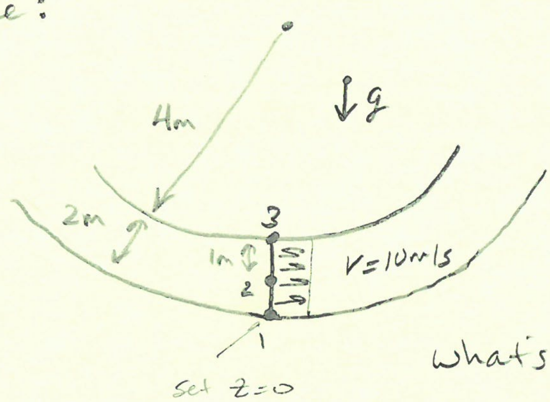
substituting numerical values

$$h = 1.98 \text{ ft}$$

Note: For a fixed $\frac{P_1 - P_0}{\gamma}$

- increasing D_2 will increase the flowrate from the tank. This will cause a higher velocity and hence lower pressure at ①
  - thus increasing D_2 will decrease the h required to cause cavitation
- increasing D_1 will not change the flow rate but it will reduce the velocity at ①. A decreased velocity will increase the pressure at ① reducing the tendency for cavitation
  - increasing D_1 will increase required h for cavitation.

Example:



Water flows around a 2-D bend with circular streamlines & a uniform velocity profile

If the pressure at pt 1 is 40kPa

what's the pressure at pts 2 & 3

Assume flow is steady, incompressible, inviscid (slip at wall => velocity is uniform)

Applying
Momentum balance across streamlines

$$P + \rho \int \frac{V^2}{R} dn + \gamma z = \text{constant}$$

can't neglect - circular path

need relationship between n & z and limits to integrate.

points of interests lie on a vertical line so $z = n$ in region of interest $dn = dz$

Choose $z = 0$ at pt 1 & $R_3 < R < R_1$

$$P + \rho \int_0^z \frac{V^2}{R} dz + \gamma z = \text{constant}$$

at $z = 0$ $P = P_1$ thus the constant $= P_1$

$$P(z) + \rho \int_0^z \frac{V^2}{R} dz + \gamma z = P_1$$

or

$$P(z) = P_1 - \rho \int_0^z \frac{V^2}{R} dz - \gamma z$$

V is independent of z - pull out of integral

R is related to z $\Rightarrow$ $z = R_o - R$ where R_o is outer radius

substitute into integral or $R = R_o - z$

$$\int_0^z \frac{V^2}{R} dz = \int_0^z \frac{V^2}{R_o - z} dz$$

integrate

$$V^2 \int \frac{1}{R_o - z} dz = -V^2 \ln\left(\frac{R_o - z}{R_o}\right) = V^2 \ln\left(\frac{R_o}{R_o - z}\right)$$

therefore

$$P(z) = P_1 - \rho V^2 \ln\left(\frac{R_o}{R_o - z}\right) - \gamma z$$

with the numerical values given

$P_2 = 12.0$ kPa

$P_3 = -20.1$ kPa

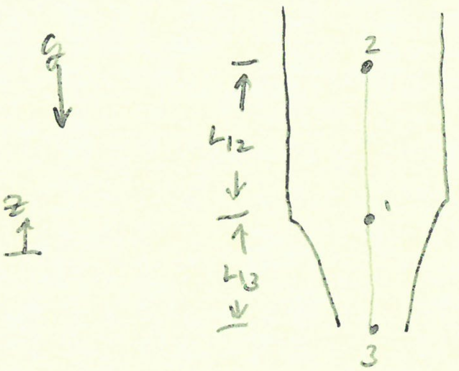
Note: pressure increases in the radial position

the radial pressure gradient must balance the acceleration of gravity and the tendency of the fluid particles to travel in a straight line

– pressure gradient responsible for curved streamlines

Example:

We have a vertical nozzle



Known:

$L_{12} = 2ft$

$L_{13} = 3ft$

$d_1 = d_2 = 0.12 ft$

$d_3 = 0.1 ft$

$P_1 = \phi$ psig.

A) What's the flow rate?

B) what's the pressure in the pipe? (at pt 2)

Assume - no viscous effects, steady incompessible flow

- Apply Bernoulli's Eqn between 1 & 3 and 1 & 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \qquad (1)$$

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \qquad (2)$$

How are V_1, V_2 & V_3 related?

at steady state $Q_1 = Q_2 = Q_3$

if incompressible & uniform velocity $V_1A_1 = V_2A_2 = V_3A_3 = Q$

Simplifying Eqn (1)

$P_3 = 0$ free jet $\quad z_1 - z_3 = L_{13}$

$$\frac{P_1}{\gamma} + z_1 - z_3 = \frac{V_3^2 - V_1^2}{2g}$$

$$\frac{P_1}{\gamma} + L_{13} = \frac{(Q/A_3)^2 - (Q/A_1)^2}{2g}$$ since $Q = V_3 A_3$
$= V_1 A_1$

thus

$$\frac{Q^2}{2g}\left(\frac{1}{A_3^2} - \frac{1}{A_1^2}\right) = \frac{P_1}{\gamma} + L_{13}$$

$$Q = \sqrt{\frac{2\left(\frac{P_1}{\rho_w} + L_{13} g\right)}{\frac{1}{A_3^2} - \frac{1}{A_1^2}}}$$

substituting values

$$Q = 0.152 \frac{ft^3}{s}$$

B) To find P_2 apply Eqn (2)

$$\frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + z_2 = \overset{0 \text{ given}}{\cancel{\frac{P_1}{\gamma}}} + \cancel{\frac{V_1^2}{2g}} + z_1$$

0 since $Q_1 = V_1 A_1 = Q_2 = V_2 A_2$ and $A_1 = A_2$

thus

$$\frac{P_2}{\gamma} = z_1 - z_2 \Rightarrow P_2 = \frac{z_1 - z_2}{1/\gamma} = -\gamma L_{12}$$

substituting #'s

$$P_2 = -124 \frac{lbf}{ft^2} = -0.87 \frac{lbf}{in^2}$$

Bernoulli Example

What pump pressure is required to produce an exit velocity of 20 m/s

apply Bernoulli

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

assume
- inlet velocity small compared to exit velocity $\Rightarrow V_1 \approx 0$
- pump inlet & outlet at approximately same height $\Rightarrow z_1 = z_2$
- exit is at atmospheric pressure $\Rightarrow P_2 = 0$

thus

$$\frac{P_1}{\gamma} = \frac{V_2^2}{2g} \Rightarrow P_1 = \frac{1}{2}\rho V^2$$

general relationship

for water

$$P_1 = \frac{1}{2}(1000)\frac{kg}{m^3}(20^2)\frac{m^2}{s^2} \cdot \frac{N\,s^2}{kg\,m} \times \frac{kPa\,m^2}{10^3\,N}$$

$$P_1 = 200\ kPa$$