

Dimensions & Units

- A dimension is a qualitative measure of a physical quantity without reference to a value (magnitude) for that quantity.



4 primary dimensions

- M = mass amount
- L = length physical size
- t = time duration
- Θ/T = temperature thermal equilibrium

- A unit is the specific value of the dimension
- different systems exist

ex. the skis are 205 cm long
 value unit dimension

the balloon contains 3.0×10^{-4} slug of air
 value unit mass

accepted scientific notation
↓
Unit Systems

	SI	USCS (EE)	BG (British gravitational)	
M	kg	lb _m	slug	
L	m	ft	foot	
T (t)	s	s	s	book focus on B.G.
Θ (T)	K	$^{\circ}R$	$^{\circ}R$	

Secondary Dimensions:

- combination of primary dimension

	SI	USCS	BG
$F = \text{force}$ $\left[\frac{ML}{T^2} \right]$	N	lbf	lbf
$V = \text{volume}$ $[L^3]$	m ³	ft ³	ft ³
$P = \text{pressure}$ $\left[\frac{F}{L^2} = \frac{M}{LT^2} \right]$	Pa	psi $\frac{\text{lbf}}{\text{in}^2}$	psi $\frac{\text{lbf}}{\text{in}^2}$

Confusion with BG: USCS^(EE) → the term "pound"

Force unit is lbf

Mass in BG → slug

Mass in EE → lbm

$$1 \text{ lbm} \neq 1 \text{ slug}$$

1 lbm exerts 1 lbf in a standard earth gravity ($g = 32.174 \text{ ft/s}^2$)

Newton's law $F = ma$

SI units $1 \text{ N} = (1 \text{ kg})(1 \text{ m/s}^2)$

BG units $1 \text{ lbf} = (1 \text{ slug})(1 \text{ ft/s}^2)$ defn. 1 lbf required to accelerate 1 slug 1 ft/s^2

EE units = ? correction term necessary

$$F = \frac{ma}{g_c}$$

↑
constant of proportionality

EE

$$1 \text{ lbf} = \frac{(1 \text{ lbm})(32.174 \text{ ft/s}^2)}{g_c}$$

thus $g_c = 32.174 \frac{\text{lbm ft}}{\text{lbf s}^2}$

alternately $1 \text{ slug} = 32.174 \text{ lbm}$

1 lbm & 1 lbf are numerically equal on earth

1 lbm defined by what you could measure on earth

See Tables 1.3 & 1.4 for unit conversions

Note: conversion of units is commonplace

- you need to be able to work between systems
- you can not interchange units feet can't be added to meter

Regardless of units:

you need Dimensional Homogeneity

higher level • all terms in an equation must have the same dimensions

ex. horizontal trajectory

$$x(t) = x_0 + v_{x,0} t + a_x t^2$$

$\underbrace{\quad}_{[L]} \quad \underbrace{\quad}_{\left[\frac{L}{s}\right][s]} \quad \underbrace{\quad}_{\left[\frac{L}{s^2}\right][s^2]}$

Dimensionless terms include π

$$A = \pi r^2$$

$[L^2] \quad [L][L]$

Later we will discuss dimensionless variables in much more detail

IMPORTANT NOTE:

if you are working a problem and your units or dimensionality don't work out you made a mistake! 1st check

useful in judging the relative importance of forces on fluid motion

$$Re = \frac{\rho V L}{\mu}$$



So what are fluids?

3 phases of materials - solid, liquid, gas

- Solids: tightly packed molecules that oscillate
 brick, ice around fixed positions (related to temperature)
 • long range molecular order

- liquids: molecules freely move/rotate
 water, gasoline, milk • short range molecular order but no long range

- Vapor (gas): no short range or long range order
 large separation between molecules. Only molecular interactions by collisions. Random motion of molecules

Definition of a fluid: a fluid is a material that cannot sustain a shearing stress in the absence of motion

- a solid can deform until it reaches an equilibrium or it fails
- liquids & gases continuously deform or flow to an applied shear stress
 - although microscopic descriptions are different
 - macroscopic properties are analogous

Because

In our analysis we will assume fluids are a continuum

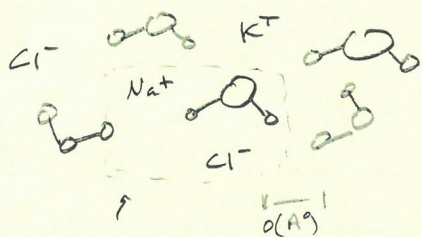
Continuum Hypothesis assumes that the mathematical limits for volumes tending to zero are reached over a scale that remains large compared to molecular dimensions.



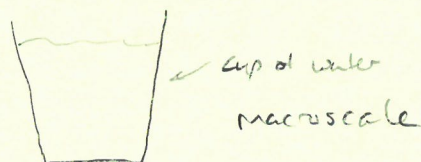
what does this mean? Variations on a molecular scale can be ignored since they average out over all length scales of interest

For example: H_2O

at molecular length scale



on small scale we see variation



at macroscale H_2O looks like a continuous substance

In the class, observations (measurements) are made over dimensions that are much larger than size of a molecule or their spacing

— in our use of calculus, a mathematical pt will contain a huge # of fluid molecules

We assume that a fluid has intrinsic macroscopic properties that are unique and well-defined down to arbitrarily small dimensions

Note: Results derived using the continuum hypothesis should not be expected to apply when the distances between molecules are large (rarefied gases) or or dimensions are at molecular scales

2 most important fluid properties.

- density (familiar concept)
- viscosity (intuitive but detailed consideration necessary)
honey is more viscous than water \Rightarrow pours more slowly

Density

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} = \frac{M}{L^3}$$

specific weight

$$\gamma = \rho g$$

$$\gamma_{\text{water}} = 62.4 \frac{\text{lb}_f}{\text{ft}^3} = 980 \frac{\text{N}}{\text{m}^3}$$

$$\gamma_{\text{air, 120}^\circ\text{C}} = 7.52 \times 10^{-2} \frac{\text{lb}_f}{\text{ft}^3} = 11.8 \frac{\text{N}}{\text{m}^3}$$

Specific gravity:

ratio of the density of a fluid to some reference material

gases \Rightarrow reference is air $SG_{\text{gas}} = \frac{\rho}{\rho_{\text{air}}}$

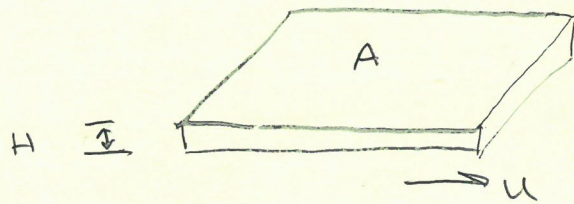
liquids \Rightarrow reference is water $SG_{\text{liq}} = \frac{\rho}{\rho_{\text{water}}}$

Viscosity

An operational definition is one that defines a quantity in terms of an experiment that can be performed to measure the value of the quantity;

Viscosity can be defined by ^{or with} one experiment

- Assume we have 2 very large parallel plates separated by a uniform thickness $H \frac{1}{2}$
bottom plate is moved at a constant velocity U



infinite parallel plates

after initial startup - Force to move plate approaches a constant value

observations:

- 1) For constant $u : H$, the force is directly proportional to area, A
- 2) For constant A , force is a unique, monotonically increasing function of the ratio u/H

convenient to define

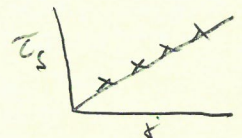
$$\text{shear stress} \quad \tau_s = \frac{F}{A} \quad \frac{\text{force}}{\text{area}}$$

$$\text{shear rate} \quad \dot{\gamma} = \frac{u}{H}$$

observations state

$$\tau_s = \tau_s(\dot{\gamma})$$

$$\frac{d\tau_s}{d\dot{\gamma}} > 0$$



For many fluids a plot of τ_s vs $\dot{\gamma}$ yields a straight line passing through origin. \Rightarrow motivation for the definition of viscosity

$$\text{viscosity} \equiv \mu = \frac{\tau_s}{\dot{\gamma}} \quad \text{units of Pa}\cdot\text{s}$$

For the above case (straight line through origin) these fluids are called Newtonian

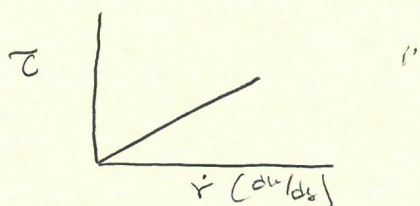
Newtonian viscosity independent of shear rate but
function of temperature; pressure (weak)
generally low-molecular weight fluids

High molecular weight fluids are generally non-Newtonian
note: viscosity alone may not describe a non-Newtonian fluid

Newton's Law of Viscosity

$$\tau = \mu \frac{du}{dy}$$

$\frac{du}{dy}$ is the shear rate for our parallel plates



$$u = U_y/h \text{ so } \frac{du}{dy} = \frac{U}{h}$$

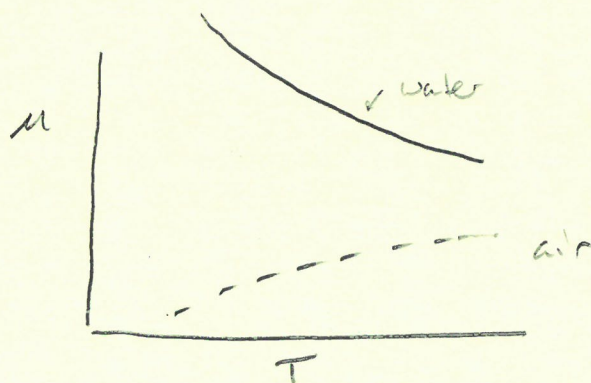
Common combination: Kinematic viscosity (often show up together)

$$\nu = \mu/\rho$$

Viscosities of Common Newtonian liquids given in Table 1.6

$$\mu_{H_2O} = 1.12 \times 10^{-3} \quad \mu_{oil, 30 \text{ out}} = 0.38 \text{ Pa}\cdot\text{s}$$

Newtonian viscosity behavior with Temperature



liquids $\mu \downarrow T \uparrow$

gases $\mu \uparrow T \uparrow$

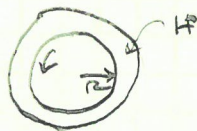
Note: Newton's law of viscosity provides a means to calculate the shear stress from a known velocity profile if $u = U(y)$ you can calculate du/dy

In our experiment to define viscosity, we had 2 "infinite plates". $H \ll \sqrt{A}$

- It would be exceedingly difficult to (1) keep 2 large plates at a constant uniform gap width, and (2) after a short time the plates would have moved relatively to one another a sufficient distance to introduce significant error into the determination of the surface area

How do we get around this?

Approximate it with concentric cylinders where the gap H is $\ll \ll$ than the radius



inner cylinder turned at angular velocity Ω outer cylinder fixed

linear velocity $u = R\Omega$

shear rate $\dot{\gamma} = \frac{R\Omega}{H}$

$$\text{Area} = 2\pi R L$$

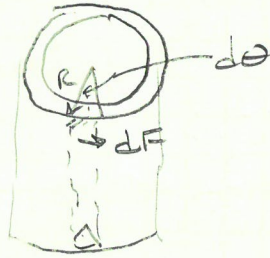
note outer cylinder area = $2\pi(R+H)L = 2\pi R$

but $\frac{H}{R} \ll \ll 1$

$(1 + \frac{H}{R})L$

Force around cylinder is zero \rightarrow need to examine torque for cylindrical system

Consider a differential area $dA = R L d\theta$



differential force acting on the surface

$$dF = \tau_s dA = \mu \dot{\gamma} dA = \mu \left(\frac{R\Omega}{H} \right) R L d\theta$$

direction of force tangential therefore

differential torque

$$dT = R dF = \frac{\mu R^3 \Omega}{H} L d\theta$$

sum over all of the area \Rightarrow integrate

$$T = \int dT = \int_0^{2\pi} \frac{\mu R^3 \Omega}{H} L d\theta$$

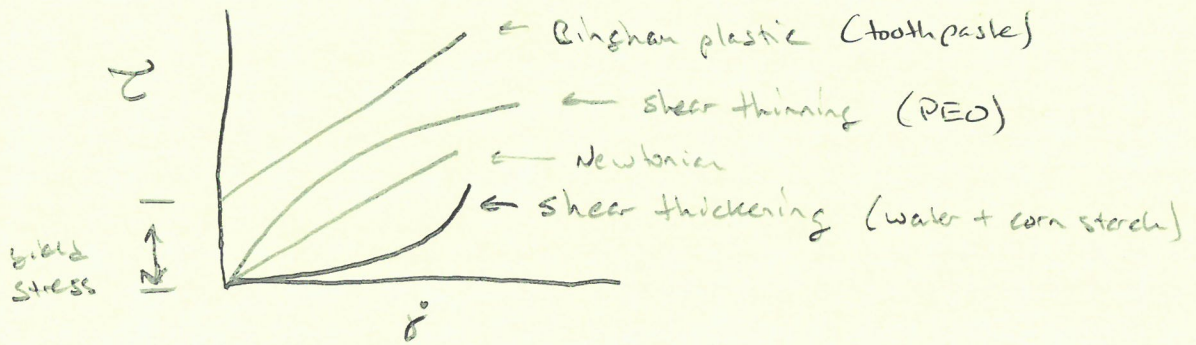
$$T = \frac{2\pi \mu R^3 \Omega}{H}$$

rearrange

$$\mu = \frac{TH}{2\pi R^3 L \Omega}$$

μ in terms of easily
measured quantities with an
"effectively" infinite area
between the "plates"

Non-Newtonian Fluids do not have simple τ vs $\dot{\gamma}$ behavior
 slurries, high MW liq. suspensions (blood)



Non-Newtonian beyond scope of introductory class

- Newtonian case always starting point for non-Newtonian studies

* Couette viscometer example

Additional Properties

Interfacial tension (surface tension)

- property of liquid-liquid or liquid-gas interface
- work is required to create interfacial area

differential change in free energy = σdA

σ = interfacial tension { energy/interarea or $\frac{\text{force}}{\text{length}}$ }

Interfacial tension causes a pressure difference across a static curved surface equal to

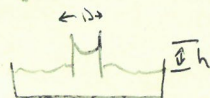
$\Delta P = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ R_1, R_2
radii of curvature

for a spherical bubble or drop $R_1 = R_2$

$\Delta P = \frac{2\sigma}{R}$

Interfacial tension can be quite significant in 2 phase flows

Responsible for capillary rise



$h = \frac{4\sigma \cos\theta}{\rho g}$

static contact angle

gravity & surface tension balance

Compressibility

- if the fluid density can be considered to be uniform at all points in the flow, the fluid is incompressible
- if the fluid density varies significantly point to point, the fluid is compressible
- When compressibility is a concern we will need an equation of state to relate density to fluid pressure & possibly temp.

Ideal Gas Law

- EOS for simple compressible substances
 - intermolecular forces are small
 - volume occupied by gas particles is small

$$P = \frac{nRT}{V}$$

n = # of moles
 T = absolute temperature
 V = volume

$R_u = \text{universal gas constant} = \frac{8.314 \text{ kPa} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}} = \frac{1545 \text{ ft} \cdot \text{lb}_f}{\text{lb-mol} \cdot \text{R}} = \frac{10.73 \text{ psia} \cdot \text{ft}^3}{\text{lb-mol} \cdot \text{R}}$

$\left[\frac{\text{kJ}}{\text{mol} \cdot \text{K}} \right]$

$= \frac{49709 \text{ ft} \cdot \text{lb}_f}{\text{slug-mol} \cdot \text{R}}$

can also be written

$$P = \frac{mRT}{V} = \rho RT$$

in this case R is gas specific

$M = nMW$
 ↑ molecular weight

$$R = \frac{R_u}{MW}$$

$MW_{air} = 28.96 \frac{\text{kg}}{\text{kmol}}$

$R_{air} = 0.287 \text{ kJ/kg} \cdot \text{K}$

Compressibility of Liquids

- generally negligible i.e. liquids are incompressible

important phenomena caused by compressibility

- water hammer
- transmission of sound

Compressibility of Liquids requires empirical data for isothermal compressibility & Bulk modulus

isothermal compressibility.

$$\beta = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

Bulk Modulus

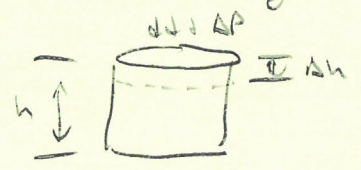
$$K = \frac{1}{\beta} = -V \left(\frac{\partial P}{\partial V} \right)_T = E_v$$

} thermodynamic properties

For quasi-static isothermal processes

$$K = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)}$$

Interpretation: compress cylinder of liquid



what is $\frac{\Delta h}{h}$ for applied ΔP

$$V = \frac{\pi}{4} d^2 h$$

$$\Delta V = \frac{\pi}{4} d^2 \Delta h$$

$$\frac{\Delta V}{V} = \frac{\Delta h}{h}$$

thus

$$\frac{\Delta h}{h} = -\frac{\Delta P}{K}$$

how much water is compressed if $h_0 = 25\text{cm}$ $d = 3\text{cm}$
 piston mass is 2 kg; additional mass $m = 100\text{kg}$.



$$\Delta P \sim \frac{\text{force}}{\text{area}} = \frac{(M_p + M)g}{(\pi/4)d^2}$$

$$\Delta P = \frac{(2 + 100)(9.8 \text{ m/s}^2)}{(\pi/4)(0.03)^2} = 1.41 \times 10^6 \frac{\text{N}}{\text{m}^2}$$

$$\Delta h = -h \frac{\Delta P}{K} = -(0.25\text{m}) \left(\frac{1.41 \times 10^6 \text{ N/m}^2}{2.15 \times 10^9 \text{ N/m}^2} \right) \quad \text{Table 1.6}$$

$$\Delta h = -0.16 \text{ mm}$$

measurable but small height decreases with mass

Repeat with air

2 methods A) ideal gas eqn.

B) Bulk modulus

follows A)

$$P = \frac{MRT}{V} \quad T_2 = T_1 \quad M_2 = M_1$$

thus. $P_1 V_1 = P_2 V_2 = M_1 R T_1$

so $V_2 = \frac{V_1 P_1}{P_2}$ or $h_2 = \frac{h_1 P_1}{P_2}$

with absolute pressures (I.G. requires absolute)

$$P_2 = P_1 + \text{weight of piston \& mass}$$

$$= P_1 + \frac{(M_p + M)g}{(\pi/4)d^2}$$

$$P_1 = \text{atmospheric} = 1.01325 \times 10^5 \frac{\text{N}}{\text{m}^2}$$

thus
$$P_2 = 1.01 \times 10^5 \frac{N}{m^2} + 1.41 \times 10^6 \frac{N}{m^2}$$

$$= 1.51 \times 10^6 \text{ N/m}^2$$

thus

$$h_2 = h_1 \frac{P_1}{P_2} = (0.25 \text{ m}) \left(\frac{1.01 \times 10^5 \text{ N/m}^2}{1.51 \times 10^6 \text{ N/m}^2} \right) = 0.017 \text{ m}$$

huge change 93% reduction

For this application the compressibility of air cannot be ignored

so within the scope of this class.

density, viscosity, interfacial tension, compressibility
basically covers the properties we need to know.

There's other properties important in more advanced topics

- permittivity
- emissivity
- ionic strength / conductivity
- zeta potential
- dynamic contact angle
- normal stresses / elasticity
- capillary phenomena
- microfluidics.

But we aren't going there. & the 4 above are sufficient

Before we start with hydrostatics let's do a little math review
 - terminology of functions

Scalar function $y = f(x_1, x_2, x_3, \dots)$ is a map that associates single values (scalars) with each set of input values (x_1, x_2, x_3, \dots) the independent variable

Vector function

$\vec{v} = g(x_1, x_2, x_3, \dots)$ is a map that associates sets of values (vectors) with each set of input values (independent)

Let \vec{x} be the position vector

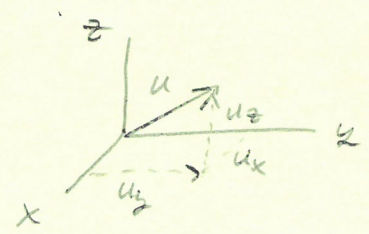
$$\vec{x} = \hat{e}_x x + \hat{e}_y y + \hat{e}_z z$$

A scalar field assigns a scalar function to \vec{x}

A vector field assigns a vector field to each \vec{x}

Velocity is a vector field

$$U = \hat{e}_x u_x + \hat{e}_y u_y + \hat{e}_z u_z$$



each velocity component can be a function of space & time

$$u_x = u_x(x, y, z, t) \quad u_y = u_y(x, y, z, t) \quad u_z = u_z(x, y, z, t)$$

Gradient operator ∇ converts a scalar field f to a vector field.

$$\nabla f = \hat{e}_x \frac{\partial f}{\partial x} + \hat{e}_y \frac{\partial f}{\partial y} + \hat{e}_z \frac{\partial f}{\partial z}$$

$|\nabla f|$ is the maximum rate of increase in f at a point

∇f is a vector that points in the direction in which f increases most rapidly

Example:

$$f(x, y, z) = 2xy - 3z + 4(x + y + z)$$

evaluate derivatives

$$\frac{\partial f}{\partial x} = 2y + 4$$

$$\frac{\partial f}{\partial y} = 2x + 4$$

$$\frac{\partial f}{\partial z} = 1$$

put back together

$$\nabla f = \underbrace{\hat{e}_x (2y + 4)}_{f_x} + \underbrace{\hat{e}_y (2x + 4)}_{f_y} + \underbrace{\hat{e}_z}_{f_z}$$

Taylor Series Expansion

Given a "sufficiently smooth" $f(x)$ the behavior of f in the neighborhood of $x = x_0$ can be approximated by

$$f(x) = f(x_0) + (x - x_0) \left. \frac{\partial f}{\partial x} \right|_{x_0} + \frac{(x - x_0)^2}{2} \left. \frac{\partial^2 f}{\partial x^2} \right|_{x_0} + \dots$$

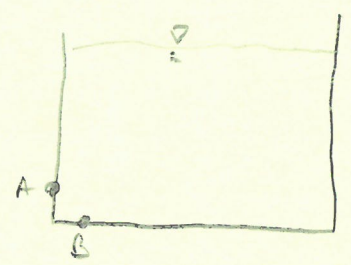
Pressure

Pressure is a scalar $P = P(x, y, z, t)$

the value of P may vary in space & time but P does not have different components in each coordinate direction

$P(x, y, z, t)$ variation may or may not be expressible as a mathematical function

$P = F/A$ force, F is always normal to a surface



- It's a Normal Stress
- if pts A & B are close together the $P_A \approx P_B$ independent of the surface orientation

MYO Section 2.1 proves pressure is independent of direction for a stationary fluid.
 ↳ at a point it is isotropic

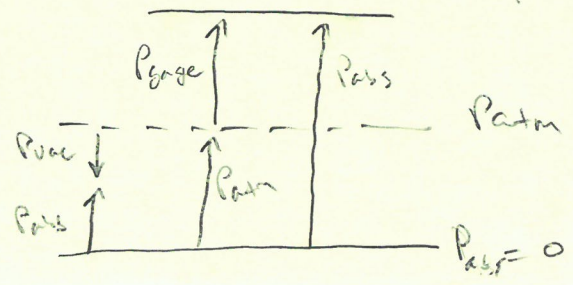
Pressure Scales:

Absolute pressure — measured relative to absolute vacuum
always positive

gauge pressure — measured relative to current atmospheric

pressure — can be positive or negative ⇒ vacuum pressure

$$P_{abs} = P_{gauge} + P_{atm}$$



* if multiplying / dividing pressure must use absolute pressure *
 add / subtract P can use either

Units F/A $[F/L^2]$

SI $Pa = 1 \frac{N}{m^2}$

BG $psi = \frac{lbf}{in^2}$
 psia - absolute
 psig - gage

relations between units

$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa}$

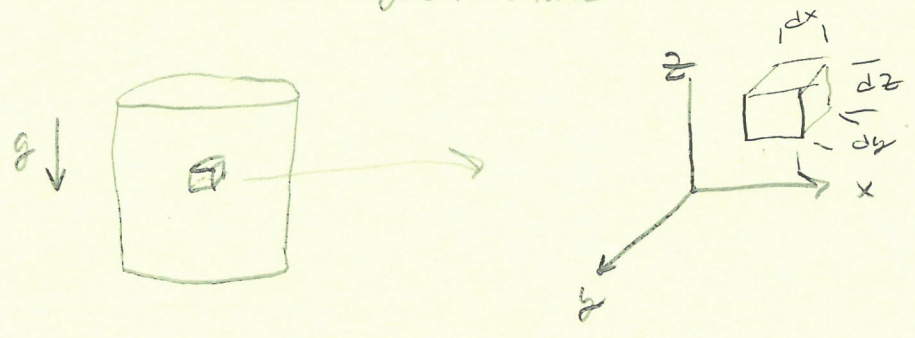
$1 \text{ atm} = 101.3 \text{ kPa} = 1.013 \text{ bar} = 14.7 \text{ psia}$

Derivation of the Hydrostatic Pressure Egn

water stationary - good for any stationary fluid

consider a control volume of fluid

- volume of gas in a tank or in the atmosphere



If the fluid is stationary

Note: may derive eqn with constant acceleration w/o shear

Apply force balance $\sum \vec{F} = m\vec{a}$ for the control volume on fluid element

$\sum \vec{F} = \vec{F}_{weight} + \vec{F}_{pressure} + \vec{F}_{viscous} = 0$
 $m\vec{a} = 0$
 no motion

no relative fluid motion
 \Downarrow
 no shear

So
$$\vec{F}_w + \vec{F}_{\text{pressure}} = 0 \quad (1)$$

Weight

$$\vec{F}_w = -\hat{e}_z \rho dV \cdot g$$

\nearrow acts in negative z direction
 \nwarrow mass
 \nwarrow gravitational acceleration

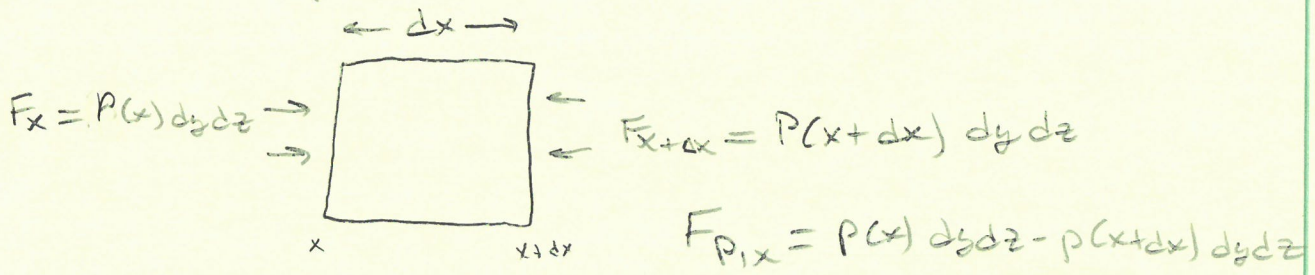
assuming gravity acts in \downarrow direction

$dV = \text{differential volume} = dx dy dz$

thus,

$$\vec{F}_{\text{weight}} = -\hat{e}_z \rho g dx dy dz \quad (2)$$

x direction of pressure force



Use Taylor's series to express $P(x+dx)$ in terms of $P(x)$

$$P(x+dx) = P(x) + dx \left. \frac{\partial P}{\partial x} \right|_x + \frac{(dx)^2}{2} \left. \frac{\partial^2 P}{\partial x^2} \right|_x + \dots$$

$$F_{P,x} = P(x) dy dz - \left[P(x) + dx \left(\frac{\partial P}{\partial x} \right)_x + \frac{(dx)^2}{2} \left(\frac{\partial^2 P}{\partial x^2} \right)_x \right] dy dz$$

neglect higher order term

$$F_{P,x} = - \left. \frac{\partial P}{\partial x} \right|_x dx dy dz$$

repeat for y & z directions

$$F_{p,y} = -\frac{\partial p}{\partial y} \Big|_y dx dy dz$$

$$F_{p,z} = -\frac{\partial p}{\partial z} \Big|_z dx dy dz$$

combining

$$\vec{F}_p = -\left(\hat{e}_x \frac{\partial p}{\partial x} + \hat{e}_y \frac{\partial p}{\partial y} + \hat{e}_z \frac{\partial p}{\partial z}\right) dx dy dz$$

or
$$\vec{F}_p = -\nabla p dx dy dz$$

from Eqn (1)

$$\vec{F}_w + \vec{F}_p = 0$$

substituting

$$-\hat{e}_z \rho g dx dy dz - \left(\hat{e}_x \frac{\partial p}{\partial x} + \hat{e}_y \frac{\partial p}{\partial y} + \hat{e}_z \frac{\partial p}{\partial z}\right) dx dy dz = 0$$

divide by $dx dy dz$

$$\hat{e}_x \frac{\partial p}{\partial x} + \hat{e}_y \frac{\partial p}{\partial y} + \hat{e}_z \frac{\partial p}{\partial z} = -\hat{e}_z \rho g$$

or

$$\nabla p = -\rho g = -\gamma \quad \text{more general vector notation form}$$

Component eqns must be true

$$x: \quad \frac{\partial p}{\partial x} = 0$$

$$y: \quad \frac{\partial p}{\partial y} = 0$$

$$z: \quad \frac{\partial p}{\partial z} = -\rho g$$

with $\frac{\partial P}{\partial x} = 0$! $\frac{\partial P}{\partial y} = 0$ $P = P(z)$ only

replace $\frac{\partial P}{\partial z}$ with $\frac{dP}{dz}$

so $\frac{dP}{dz} = -\rho g$ or $\frac{dP}{dz} = -\gamma$

To calculate $P(z)$ we need to know how ρ varies with z

Incompressible fluids $\Rightarrow \rho = \text{constant}$ γ is uniform

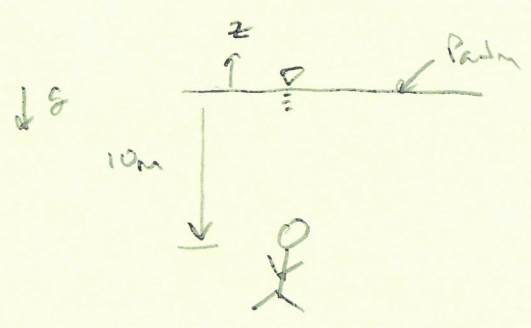
$$dP = -\rho g dz$$
$$\int_{P_1}^{P_2} dP = \int_{z_1}^{z_2} -\rho g dz = \int_{z_1}^{z_2} -\gamma dz = -\gamma \int_{z_1}^{z_2} dz$$

$$P_2 - P_1 = -\gamma (z_2 - z_1)$$

Alternate derivation $\left. \frac{\Delta h}{\Delta z} \right|_{\frac{P_1}{P_2}} \downarrow \Delta z$

force on surface z $P_2 \cdot A = P_1 \cdot A + mg$
 $P_2 \cdot A = P_1 \cdot A + \rho A \Delta h g \Rightarrow P_2 = P_1 + \rho g \Delta h$
 $\Delta h = z_1 - z_2$

Application: Scuba diving



we can rewrite eqn with gauge pressure if we set z_1 where $P = P_{atm}$

$$P_1 = P_{atm} \text{ at } z_1 = 0$$



$$P_2 = -\gamma z_2 = \gamma h$$

so

$$P_{gauge} = \gamma h = 1000 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2} \times 10m = 98100 \frac{N}{m^2} = 98.1 kPa$$

$$P_{diver} = P_{atm} + P_{gauge} = 101.3 kPa + 98.1 kPa = 199.4 kPa = 28.94 \text{ PSI}$$

you can feel this pressure - that's why you have to clear your ears



Pressure head (definition)

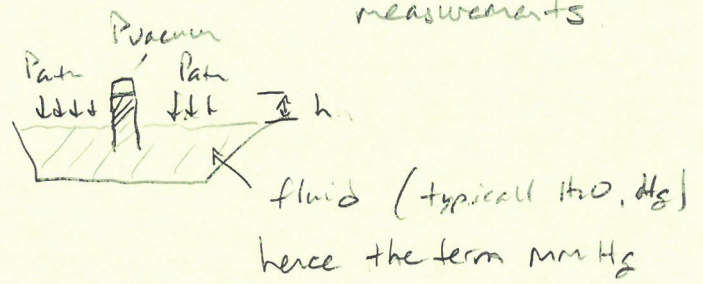
$$P = \rho h \quad \text{rearrange} \quad h = \frac{P}{\rho}$$

therefore, any pressure value can be expressed as a height of a column of an incompressible fluid.

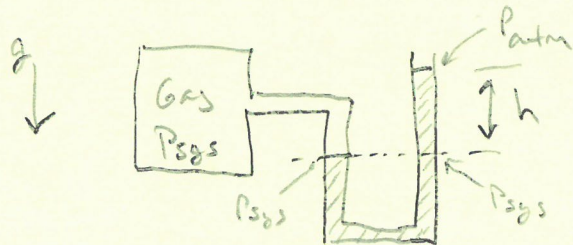
The increase in pressure with depth due to weight of fluid above it, is the basis of a manometer - simplest way to make pressure measurements.

- atmospheric pressure

$$P_{atm} = \rho g h + P_{hg}$$



- System pressure



$$P_{sys} = P_{atm} + \rho g h$$

Note: we generally neglect the height of the gas column

Since $\rho_{gas} \ll \rho_{liq}$ or $P_{gas} \ll P_{liq}$ } not always true
 if not a gas can't neglect it

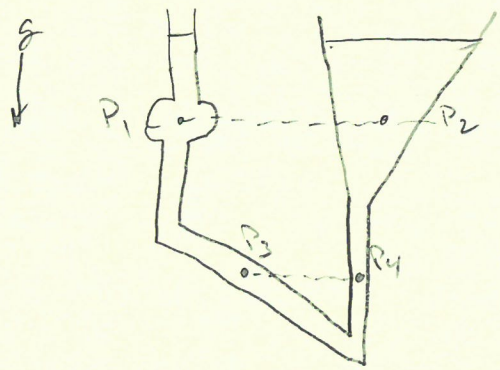
In calculating the system pressure we applied Pascal's law

Pascal's Law: The pressure is the same for any two points at the same elevation in a continuous mass of fluid

(point out P_{sys} above)



Shape & dimensions of container is irrelevant



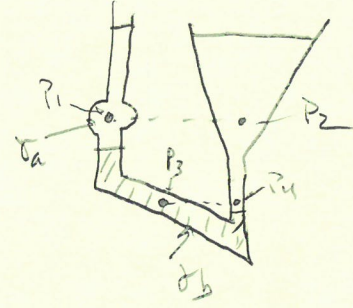
$$P_1 = P_2$$

$$P_3 = P_4$$

But the fluid has to be continuous

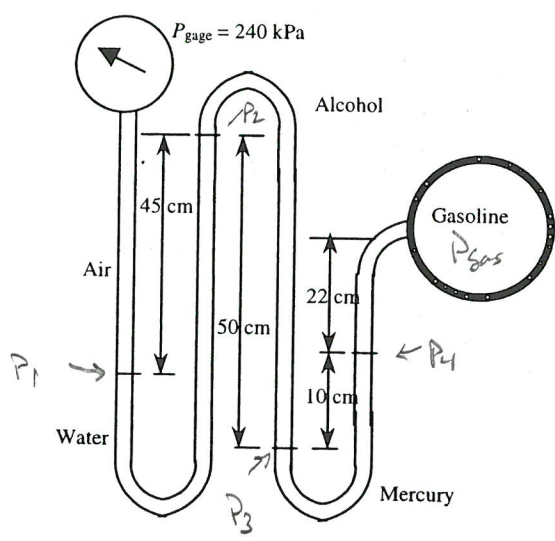
$$P_1 \neq P_2, P_3 \neq P_4$$

since $\sigma_a \neq \sigma_b$



Calculating pressure with manometers

A gas line is connected to a pressure gage through a double U tube manometer. What's the gas pressure if the gage pressure reads 240 kPa.



1st method

Label drawing with undetined pressure at each interface

isolate individual fluid columns.

$$P_1 = P_{base} + \rho_{air} g h_{air}$$

$$P_1 = P_2 + \rho_w g h_w \Rightarrow P_2 = P_1 - \rho_w g h_w$$

$$P_3 = P_2 + \rho_{alc} g h_{alc}$$

$$P_3 = P_4 + \rho_{Hg} g h_{Hg} \Rightarrow P_4 = P_3 - \rho_{Hg} g h_{Hg}$$

$$P_4 = P_{gas} + \rho_{gas} g h_{gas} \Rightarrow P_{gas} = P_4 - \rho_{gas} g h_{gas}$$

combining/solving for P_{gas}

$$P_{gas} = P_{base} - \rho_w g h_w + \rho_{alc} g h_{alc} - \rho_{Hg} g h_{Hg} - \rho_{gas} g h_{gas}$$

in terms of specific gravity $SG_{alc} = 0.79$ $SG_{Hg} = 13.6$ $SG_{gas} = 0.70$

$$P_{gas} = P_{base} - \rho_w g (h_w - SG_{alc} h_{alc} + SG_{Hg} h_{Hg} + SG_{gas} h_{gas})$$

with #'s in drawing! SG of fluids

$$P_{gas} = 240 \text{ kPa} - (1000 \frac{\text{kg}}{\text{m}^3})(9.81 \frac{\text{m}}{\text{s}^2}) [(0.45 \text{ m} - 0.79(0.5 \text{ m}) + 13.6(0.1 \text{ m}) + 0.70(0.22 \text{ m})] \times$$

$$P_{gas} = 224.6 \text{ kPa}$$

$$\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}} \right) \times \left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right)$$

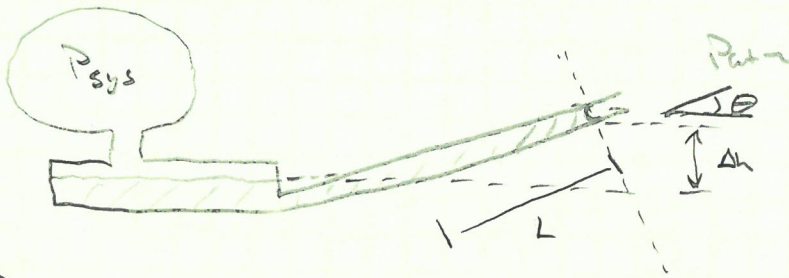
Alternate/with experience

if fluid column goes down subtract if it goes up add it.

Inclined manometers

When pressure differences are small Δh may not be measured well enough to provide sufficient measurement accuracy

Low pressure application \rightarrow use inclined manometer



$$P_1 = P_{atm}$$

$$P_{sys} = P_1 + \gamma \Delta h$$

relate Δh to $L \Rightarrow$ geometry / trig



$$\sin \theta = \frac{\Delta h}{L} \Rightarrow \Delta h = L \sin \theta$$

so

$$P_{sys} = P_1 + \gamma L \sin \theta$$

θ can be small but accurately measured

L is long enough.

In our analysis of the scuba diver, manometers we have assumed our fluids are incompressible

- good assumption for liquid if height not outrageous
- may or may not be good for gases.

Compressible Fluids : (fluids with variable density
salt water)

hydrostatic Eqn

$$\frac{dP}{dz} = -\gamma = -\rho g$$

still applies but we need to know how γ varies
with either P or z .

For a gas we have an eqn of state that relate ρ to P

I.6. $P = \rho R T \Rightarrow \rho = \frac{P}{R T}$

substituting into hydrostatic Eqn.

$$\frac{dP}{dz} = -\rho g = -\frac{P g}{R T}$$

separate variables

$$\frac{dP}{P} = -\frac{g}{R T} dz$$

integrate between two locations.

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$$

what's $T = T(z)$?

Case a) $T = \text{uniform} \Rightarrow T = T_0$

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{g}{R T_0} \int_{z_1}^{z_2} dz = -\frac{g(z_2 - z_1)}{R T_0}$$

thus
$$\frac{P_2}{P_1} = \exp\left[-\frac{g(z_2 - z_1)}{R T_0}\right]$$

Case b) linear variation of T with z (good approx for atmosphere)
 10°F for every 3000 ft)

$$T = T_0 - \beta z$$

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T_0 - \beta z} = \frac{g}{R\beta} \ln\left[\frac{T_0 - \beta z_2}{T_0 - \beta z_1}\right]$$

if $P_1 = P_{atm}$ at $z = 0$ (sea level)

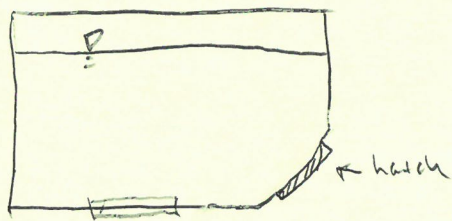
$$\frac{P_2}{P_{atm}} = \left(1 - \frac{\beta z_2}{T_0}\right)^{g/R\beta}$$

So pressure varies with depth, how it varies depends on if it's compressible or incompressible

- depend on T, P , even concentration (salt water)

So if pressure varies w/ depth then the force exerted on an object would vary depending not only on where it's located in a fluid media but how it's oriented.

For example, say we have a tank with a hatch to get in or out of the tank. to clean it



easiest case if the hatch is on the bottom

What's the force necessary to hold the hatch in place?

Maybe a better question

Why would we care?