# Conservation of Linear Momentum

linear momentum = mass × velocity = $mV$

thus
velocity is momentum per unit mass.

Applying the Reynolds Transport Theorem –

$$\frac{D}{Dt}\int_{sys} \rho b \, d\forall = \frac{\partial}{\partial t}\int_{CV} \rho b \, d\forall + \int_{CS} \rho b \, \vec{V}\cdot\hat{n} \, dA$$

let $b = V$

$$\underbrace{\frac{D}{Dt}\int_{sys} \rho \vec{V} d\forall}_{\text{time rate of change of linear momentum of a system}} = \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho \vec{V} d\forall}_{\text{time rate of change of linear momentum in the control volume}} + \underbrace{\int_{CS} (\rho\vec{V})\vec{V}\cdot\hat{n} \, dA}_{\text{net rate of linear momentum through the control surface}}$$

From Newton's 2nd Law of motion the

time rate of change of linear momentum of a system = sum of all external forces acting on the system

$$\frac{D}{Dt}\int_{sys} \rho \vec{V} d\forall = \sum F_{sys} = \sum F_{CV}$$ when the system & control volume are coincident at a instant in time

thus.

$$\boxed{\sum F_{CV} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{V} d\forall + \int_{CS} (\rho \vec{V})\vec{V}\cdot\hat{n} \, dA}$$

Fixed non-deforming CV

Conservation of Linear momentum

Gravity
EM
electrophoretic
Body forces
& Surface forces

pressure
viscous drag
friction.
surface tension, Electroosmotic

Working with the momentum Eqn.

1. Draw system & identify the Control Volume
2. Locate coordinate axis
3. Sum the forces
   - identify forces due to structural supports
     - the control volume must cut through the support* to expose the force
   - pressure force in fluid are exposed when the C.V cuts through the fluid : Pressure forces always act inward
4. Momentum is a vector eqn.: Balances must apply in each direction

$$\Sigma F_x = \frac{\partial}{\partial t}\int_{CV} \rho V_x \, d\forall + \int_{CS} \rho V_x (\vec{V}\cdot n)\, dA$$

$$\Sigma F_y = \frac{\partial}{\partial t}\int_{CV} \rho V_y \, d\forall + \int_{CS} \rho V_y (\vec{V}\cdot n)\, dA$$

$$\Sigma F_z = \frac{\partial}{\partial t}\int_{CV} \rho V_z \, d\forall + \int_{CS} \rho V_z (\vec{V}\cdot n)\, dA$$

5. Conservation of mass generally relevant to the analysis

do example →

Conservation of Linear Momentum

- Moving (uniform velocity), non-deforming control volume

  relative velocity $\vec{W} = \vec{V} - \vec{V}_{cv}$

$$\Sigma \vec{F} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{V} \, dv + \int_{CS} \rho \vec{V} (\vec{W}\cdot\hat{n})\, dA$$

W represent actual velocity across surface

Moving cv work with $\vec{W}$

5.38

**5.38** A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in **Video V5.96** and Fig. P5.38. Determine the minimum volume flowrate needed to tip the block.

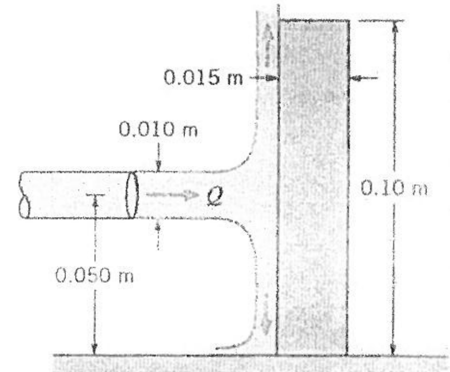
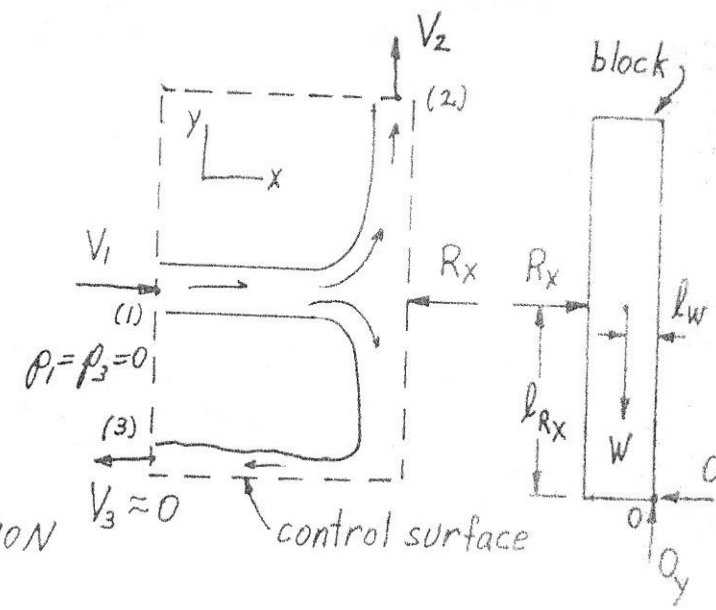



FIGURE P5.38

From the free body diagram of the block when it is ready to tip $\Sigma M_o = 0$, or $R_x \ell_{R_x} = W \ell_w$ where $R_x$ is the fore that the water puts on the block.

Thus,

$$R_x = \frac{W \ell_w}{\ell_{R_x}} = \frac{6\,N\left(\frac{0.015}{2}\,m\right)}{0.050\,m} = 0.90\,N$$




For the control volume shown the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_x$$

becomes

$$V_1 \rho (-V_1) A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

$$V_1 = \sqrt{\frac{0.9\,N}{\left(999\,\frac{kg}{m^3}\right)\frac{\pi}{4}(0.01\,m)^2}} = 3.39\,\frac{m}{s}$$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.01\,m)^2\left(3.39\,\frac{m}{s}\right) = \underline{2.66 \times 10^{-4}\,\frac{m^3}{s}}$$

5.47

5.47 A converging elbow (see Fig. P5.47) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m³ between sections (1) and (2). The water volume flowrate is 0.4 m³/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal ($x$ direction) and vertical ($z$ direction) anchoring forces required to hold the elbow in place.

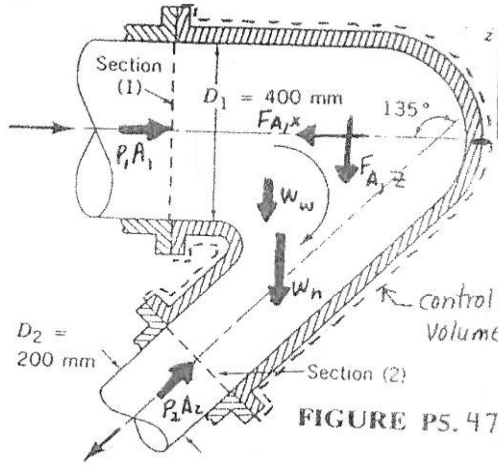



FIGURE P5.47

A control volume that contains the elbow and the water within the elbow between sections (1) and (2) as shown in the sketch above is used. Application of the horizontal or $x$ direction component of the linear momentum equation yields $\sum F_{cv} = \frac{\partial}{\partial t}\int_{cv} \vec{V}\rho\, dV + \int_{s.s.} \rho\vec{V}(\vec{V}\cdot\hat{n})\,dA$

$$-u_1 \rho u_1 A_1 - V_2 \cos 45^\circ \rho V_2 A_2 = p_1 A_1 - F_{A,x} + p_2 A_2 \cos 45^\circ$$

From conservation of mass

$$\dot{m} = \rho u_1 A_1 = \rho V_2 A_2 = \rho Q \qquad (1)$$

Thus

$$F_{A,x} = \frac{\rho Q^2}{A_1} + \frac{\rho Q^2 \cos 45^\circ}{A_2} + p_1 A_1 + p_2 A_2 \cos 45^\circ = \frac{\rho Q^2}{\frac{\pi D_1^2}{4}} + \frac{\rho Q^2 \cos 45^\circ}{\frac{\pi D_2^2}{4}} + p_1 \frac{\pi D_1^2}{4} + p_2 \frac{\pi D_2^2}{4}\cos 45^\circ$$

$$F_{A,x} = \left(999 \frac{kg}{m^3}\right)\left(0.4 \frac{m^3}{s}\right)^2 \frac{1}{\left(\frac{\pi}{4}\right)} \left[\frac{\left(1000 \frac{mm}{m}\right)^2}{(400\ mm)^2} + \frac{\cos 45^\circ \left(1000 \frac{mm}{m}\right)^2}{(200\ mm)^2}\right]\left(\frac{1\ N}{kg\ \frac{m}{s^2}}\right)$$

$$+ \frac{\pi}{4}\frac{\left(1000 \frac{N}{kPa\ m^2}\right)}{\left(1000 \frac{mm}{m}\right)^2}\left[(150\ kPa)(400\ mm)^2 + (90\ kPa)(200\ mm)^2 \cos 45^\circ\right]$$

$$F_{A,x} = \underline{25{,}700\ N}$$

Application of the vertical or $z$ direction component of the linear momentum equation leads to

$$-V_2 \sin 45^\circ \rho V_2 A_2 = p_2 A_2 \sin 45^\circ - F_{A,z} - W_w - W_e$$

which when combined with Eq. 1 gives

$$F_{A,z} = \frac{\rho Q^2 \sin 45^\circ}{A_2} + p_2 A_2 \sin 45^\circ - W_w - W_e = \frac{\rho Q^2 \sin 45^\circ}{\frac{\pi D_2^2}{4}} + p_2 \frac{\pi D_2^2}{4} \sin 45^\circ - \rho g V_w - m_e g$$

(con't)

5.47 (con't)

$$F_{A,z} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(0.4\,\frac{m^3}{s}\right)^2 \sin 45^\circ}{\frac{\pi}{4}\frac{(200\,mm)^2}{\left(1000\,\frac{mm}{m}\right)^2}}\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) + \frac{(90\,kPa)\,\pi\,(200\,mm)^2 \sin 45^\circ}{4\left(1000\,\frac{mm}{m}\right)^2}$$

$$-\left(999\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(0.2\,m^3\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) - (12\,kg)\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)$$

$$F_{A,z} = \underline{\underline{8920\ N}}$$