

# Experiment 1

## Viscosity Measurement

### Purpose

The purpose of this experiment is to measure the viscosity of a glycerin-water mixture with a Thomas-Stormer viscometer.

### Apparatus

Figure 1.1 is a schematic of the viscometer. A weight,  $W$ , is used to drive a rotor that is partially submerged in a sample of liquid. The torque exerted by viscous shear on the rotor is balanced by the work input of the falling weight. The experiment involves measurement of the time it takes for a known number of revolutions of the rotor.

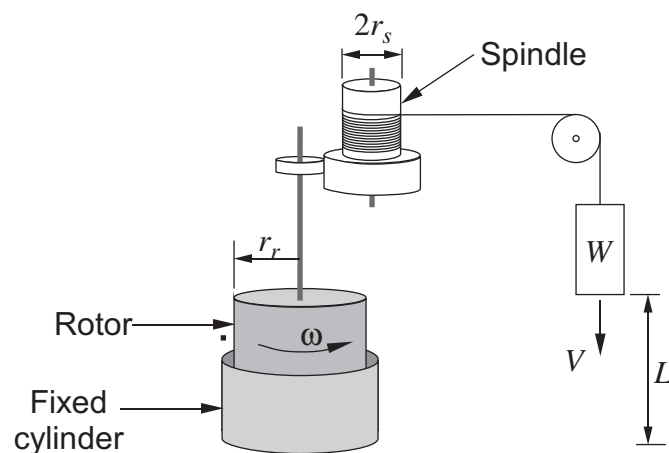


Figure 1.1: Thomas-Stormer viscometer.

## Theory

The operation of the viscometer relies on a linear velocity profile in the gap between the rotor and the fixed cylinder. If the velocity profile is linear the viscous shear stress on the surface of the rotor can be written

$$\tau = \mu k \omega \quad (1.1)$$

where  $\tau$  is the viscous shear stress,  $\mu$  is the fluid viscosity,  $k$  is a constant that depends only on the geometry of the viscometer, and  $\omega$  is the angular velocity of the rotor. Given the shear stress from Equation (1.1), the torque exerted by the rotor on the fluid is

$$T_f = (\tau A) r_r \quad (1.2)$$

where  $A$  is the wetted surface area of the rotor, and  $r_r$  is the radius of the rotor. The area,  $A$ , accounts for the inner and outer surfaces of the rotor. Since the fluid is in contact with both surfaces  $r_r$  is an effective radius.

Neglecting any friction in the pulleys and bearings, the power dissipated by viscous stresses in the fluid,  $P_f$ , is equal to the power input of the falling weight,  $P_w$ .

$$P_f = P_w \implies T_f \omega = W V \quad (1.3)$$

where  $W$  is the magnitude of the weight, and  $V$  is the velocity of the falling weight. Combining Equations (1.1) through (1.3) gives

$$\mu k \omega^2 A r_r = W V \quad (1.4)$$

The velocity of the weight falling a distance  $L$  in time  $t$  is

$$V = \frac{L}{t} = \frac{2\pi r_s n_s}{t} \quad (1.5)$$

where  $n_s$  is the number of revolutions of the spindle, and  $r_s$  is the radius of the spindle. The angular velocity of the rotor is

$$\omega = \frac{2\pi n_r}{t} \quad (1.6)$$

where  $n_r$  is the number of revolutions of the rotor in time  $t$ .

Substitution of Equations (1.5) and (1.6) into Equation (1.4) and rearranging yields

$$\mu = \frac{1}{k \omega \frac{2\pi n_r}{t} A r_r} W \frac{2\pi r_s n_s}{t} = \frac{r_s \beta}{k A r_r} \frac{W}{\omega} \quad (1.7)$$

where  $\beta = n_s/n_r$  is the overall gear ratio between the spindle and the rotor. Defining the viscometer constant as

$$C = \frac{r_s \beta}{k A r_r} \quad (1.8)$$

Equation (1.7) can be written

$$\mu = C \frac{W}{\omega} \quad (1.9)$$

Assuming that the model of viscous shear in Equation (1.1) is valid, the constant  $C$  depends only on the geometry of the device.

## Procedure

Use of the Thomas-Stormer viscometer requires determination of  $C$  in Equation (1.9) by calibrating the instrument with a fluid having a known viscosity. With  $C$  known from the calibration step, measurements of  $W$  and  $\omega$  can be used with Equation (1.9) to compute the viscosity of an unknown fluid. The overall procedure may be divided into three phases: (1) setting up the viscometer, (2) adjusting the weight in preparation for the tests, and (3) running the tests.

### Set Up the Viscometer

1. Fill the test cup to the top of the side vanes. Make sure that the depth of the fluid is the same for all tests.
2. Replace the test cup in the viscometer
3. Raise the platform that supports the bath and test cup until the fluid is at least 0.6 cm (0.25 inch) above the top of the rotor. Add fluid if necessary. Make sure that the platform is raised so that it touches the stop.
4. Place the thermometer in the holder. Allow the system to come into thermal equilibrium and record the temperature. Note that most samples have been sitting overnight in the lab. Unless the fluid temperature has been disturbed, e.g., by excessive handling, it should be in reasonable thermal equilibrium with the viscometer and the surroundings.

### Adjust Weight in Preparation for Tests

The viscosity measurement is based on the assumption that the flow on the surface of the rotor is laminar. After placing a new sample of liquid in the test cup, and raising the cup into position, adjust the weight until 100 revolutions on the counter takes at least 20 seconds. Shorter run times will cause turbulent flow, and result in erroneously high viscosity values. The preliminary weight adjustment will also allow you to become familiar with the measurement procedure.

All data runs should be taken over a range of weights no greater than the weight that gives 100 revolutions in no less than 20 seconds. You will also need to allow about 20 revolutions for the rotor to attain steady state velocity. Make sure, therefore, that the weight can fall far enough to cause at least 120 revolutions of the rotor.

### Running Tests

Once the instrument is set up, the calibration runs and the viscosity measurement runs use the following procedure.

1. Turn the brake on and raise the driving weight by turning the handle of the rewinding drum counter-clockwise until the weight nearly touches the pulley.
2. Release the brake one quarter turn and allow the weight to slowly descend until the pointer on the dial is located between 80 and 90. Reset the brake. The time for the weight to fall 100 revolutions will be measured. The starting time is when the revolution counter crosses 0, and the stopping time is when the revolution counter crosses 100. Setting the starting position so that the revolution counter is between 80 and 90 allows several revolutions of the rotor to be completed before the beginning of the measured time interval. This guarantees that the rotor is rotating at steady angular velocity during the interval of the time measurement.

3. Reset the stopwatch.
4. Fully release the brake. Start the stopwatch when the counter passes 0 and stop it when the counter passes 100. Record the time and mass of the weight box.
5. Repeat the preceding steps for several weights (at least 5) by adding or removing shot from the weight box.

## Data Reduction

Equation (1.9) can be rearranged as

$$\omega = C \frac{W}{\mu} \quad (1.10)$$

Data from the calibration runs allows  $C$  to be determined from a least squares curve fit of  $\omega$  versus  $W/\mu$ . Note that many software packages provide the capability for finding curve fits of the form

$$y = a_0 + a_1x$$

but Equation (1.10) requires  $a_0 = 0$ . If the curve fit does not pass through  $(W/\mu, \omega) = (0, 0)$  the value of the slope (and hence  $C$ ) will be in error.

A simple formula for the least squares fit to

$$y = a_1x \quad (1.11)$$

is not hard to derive. Given a set of measured  $(x, y)$  data pairs the least squares fit to Equation (1.11) is

$$a_1 = \frac{\sum x_i y_i}{\sum x_i^2}$$

## Report

1. Plot the data for the calibration run and report the value of  $C$ . Be sure to include the point  $(W/\mu, \omega) = (0, 0)$  to verify that the fit is reasonable.
2. Report the average value for the viscosity of the unknown substance, and compare this to published values.
3. Discuss discrepancies and anomalies in your data.
4. Which measurements (data points) are most reliable? Which measurements are most limited by your ability to measure time? Justify your answers.
5. How well does the  $\omega$  versus  $W/\mu$  data fit the model of a line with zero intercept?
6. What other methods are available for measuring the viscosity of liquids?