

Tank Draining Exercise

Before coming to the lab,
read sections 1 through 5
of this document.

Engineering of Everyday Things
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1 Introduction

Before coming to the lab, read sections 1 through 5 of this document.

This exercise is an investigation of transient fluid behavior and a study in the applicability of the Bernoulli equation. The exercise also builds on the experience gained during the tank filling exercise. The measurements are designed to answer the following questions.

- To what extent can the hydrostatic principle be applied to a tank that is slowly being drained?
- What is the equation for the shape of a free jet of water?
- How does the shape of a tank affect the rate at which water drains from a hole in the tank that is open to the atmosphere?
- How does head loss affect the tank draining phenomena?

2 Learning Objectives

After completing this lab exercise students will be able to

1. Apply mass conservation to a control volume with a time-varying mass.
2. Determine the pressure at depth in a tank with a small hole in its side;
3. Use digital photographs to measure geometric features, specifically the trajectory of the water jet emerging from a hole in the side of the tank.
4. Apply the Bernoulli and Energy Equations to compute the velocity of a free jet emerging from a hole in a tank;
5. Explain the key similarities and key differences between the draining of two tanks with different shapes.

3 Apparatus

Figure 1 shows the equipment for this laboratory exercise. The key components are

1. Two cylindrical tanks, one with straight walls (single diameter) and another with a step-shaped wall (two diameters).
2. A pressure transducer mounted on the side of each tank at a distance H from the base.

Not shown in the sketches are

3. A digital camera and tripod used to capture images of the free jet of water.
4. A power supply to provide electrical energy to the transducer.
5. A data acquisition device (DAQ) for digitizing the transducer output.
6. A computer to record and display the digitized output of the transducer

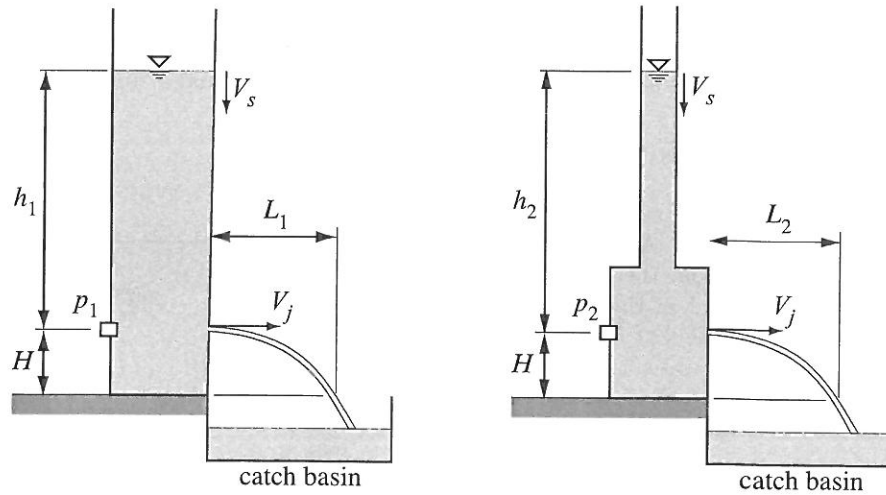


Figure 1: Apparatus for the tank-draining experiment.

4 Background and Review

Read this section before coming to the laboratory.

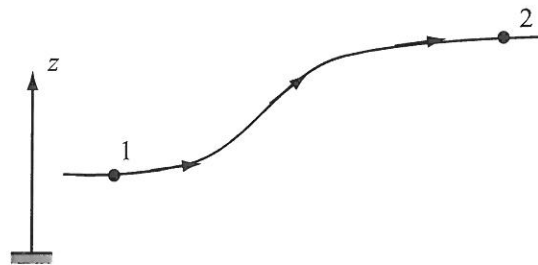
4.1 Bernoulli and Energy Equations

The Bernoulli equation is

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad (1)$$

where p is the static pressure, γ is the specific weight of the flowing fluid, V is the velocity magnitude, and z is the elevation above a fixed datum. The Bernoulli equation applies *only* along a streamline *and only if* the following conditions are met.

1. The flow is steady.
2. The flow is incompressible.
3. There is no heat or work interactions between stations 1 and 2.



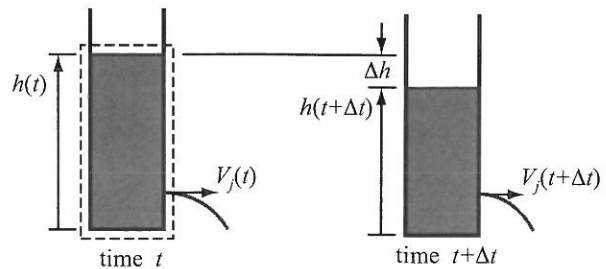
The steady flow energy equation is

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 - h_p + h_f \quad (2)$$

where h_p is the head increase due to shaft work by a pump and h_f is the head loss. The steady flow energy equation is more generally applicable than the Bernoulli Equation. The energy equation applies to steady, incompressible flow in a conduit with a single inlet and a single outlet.

4.2 Mass Conservation

The following diagram represents a cylindrical tank with a hole in its side. A jet of water with velocity V_j issues from the hole. The left diagram shows the tank at time (t) . The right diagram shows the tank at a later time $t + \Delta t$. The depth of the water is $h(t)$. The cross-sectional area of the tank is A_t , and the cross-sectional area of the jet is A_j .



The following boxes contain two alternative derivations of the mass conservation principle for the tank draining problem.

Reynold Transport Theorem

Apply the Reynolds Transport Theorem to mass conservation for the dashed control volume.

$$\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho(\mathbf{V} \cdot \hat{\mathbf{n}}) dA = 0 \quad (3)$$

For the tank, the differential element of volume is $dV = A_t dh$. The first term simplifies to

$$\frac{\partial}{\partial t} \int_{CV} \rho dV = \frac{\partial}{\partial t} \int_0^h \rho A_t dh = \rho A_t \frac{dh}{dt}.$$

The integral over the control surface can be simplified by assuming that the water velocity is uniform over the control surface

$$\int_{CS} \rho(\mathbf{V} \cdot \hat{\mathbf{n}}) dA = \rho V_j A_j.$$

Substituting the pieces back into the Reynolds Transport Theorem yields

$$\rho A_t \frac{dh}{dt} + \rho V_j A_j = 0$$

Canceling the ρ and rearranging this equation gives

$$\frac{dh}{dt} = -\frac{A_j}{A_t} V_j. \quad (4)$$

System Balance Formulation

Compare the fluid volume at two different times: t and $t + \Delta t$:

$$A_t \Delta h = \text{change in fluid volume during } \Delta t$$

$$V_j A_j \Delta t = \text{volume of fluid leaving via the jet}$$

Equating those volumes and rearranging slightly gives

$$A_t \frac{\Delta h}{\Delta t} = V_j A_j$$

In the limit as $\Delta t \rightarrow \infty$, the ratio $\Delta h/\Delta t$ becomes dh/dt . Since the depth of the water is decreasing the derivative is negative. Therefore the preceding equation becomes

$$\frac{dh}{dt} = -\frac{A_j}{A_t} V_j. \quad (5)$$

which is identical to Equation (4).

4.3 Study Questions

1. Although the fluid in the tank is moving, can the hydrostatic equation be used to related the pressure measured by the transducer to the depth of the water in either tank?
2. If the tank drains slowly, then the upper surface of the tank can be assumed to have zero velocity. Under this circumstance, the flow would be considered to be quasi-steady. In order for the tank flow to be considered quasi-steady, what must be the size relationship between the area of the tank and the area of the jet?
3. Regardless of whether the velocity at the upper free surface is negligible, can the steady form of the mass conservation equation be used to determine the relationship between V_0 , the velocity at the upper free surface, and V_j , the velocity of the jet?

4.4 Pressure versus Depth

In a stationary liquid, the pressure at depth h below the free surface is $p = \gamma h$, where γ is the specific weight of the liquid. Can that formula be applied when water is flowing out through the hole in the side of the tank?

Figure 2 depicts two streamlines that converge at the hole at Station 3 in the side of the tank¹. Station 1 is at the free surface. Station 2 is adjacent to the pressure transducer attached to the tank wall. Station 4 is on the streamline immediately outside of the tank. The following analysis shows that under some very reasonable assumptions, the pressure at Station 2 is determined only by the depth of water in the tank.

Write the Bernoulli Equation for the streamline from Station 1 to Station 3.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad (6)$$

Since Station 1 is a free surface: $p_1 = 0$, $V_1 = 0$. Choose $z_3 = 0$. The vertical datum is arbitrary, but its location cannot be changed once it is specified for the analysis.

$$\Rightarrow p_3 = \gamma z_1 - \frac{1}{2}\rho V_3^2 \quad (7)$$

Write the Bernoulli Equation for the streamline from Station 2 to Station 3.

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad (8)$$

Since Station 2 is adjacent to the wall, $V_2 = 0$. Also $z_1 = z_3 = 0$.

$$\Rightarrow p_2 = p_3 + \frac{1}{2}\rho V_3^2 \quad (9)$$

Combine Equation (7) and Equation (9) to obtain a formula for p_2 as a function of z_1 .

$$p_2 = \left(\gamma z_1 - \frac{1}{2}\rho V_3^2 \right) + \frac{1}{2}\rho V_3^2$$

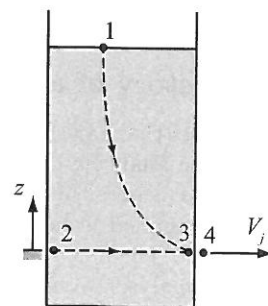


Figure 2: Streamlines used to determine pressure at tank side wall during tank draining.

¹Technically, these two streamlines cannot end at the same point. However, we can assume that near point 3, the two streamlines are separated by a negligible vertical distance.

Therefore,


$$p_2 = \gamma z_1 \quad (10)$$

Your Summary:

Based on the preceding analysis, list two conditions under which $p = \gamma h$ can be used to measure the instantaneous tank depth with the pressure transducer attached to the side wall of the tank.

1.

2.

	<p>Before continuing, show your lab manual to the instructor. It's important at this point to make sure you are on the right track.</p>	INSTRUCTOR APPROVAL <input type="checkbox"/>
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4.5 Trajectory of a Free Jet

Figure 3 depicts the trajectory of the jet of water emanating from the hole in the side of the tank. The goal of the following derivation is to obtain a formula for L_{jet} as a function of the velocity V_j of the fluid as it leaves the hole. The water jet is assumed to retain the same diameter as it flows. The water is assumed to have a purely horizontal velocity component V_j when it leaves the hole at $x_p = 0$, $y_p = H$. The downward motion of the water is determined by the acceleration of gravity. In the following analysis we neglect the drag on the surface of the water as it moves through the air.

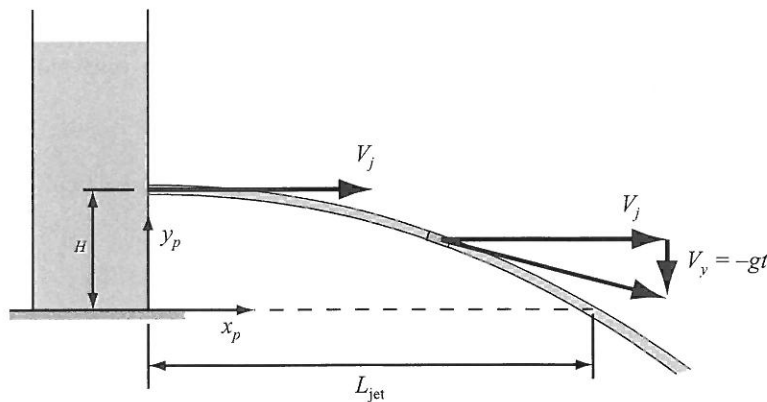


Figure 3: Arc of the water jet. The path of the fluid is assumed to be the same as that of a rigid particle falling under the influence of only gravity, and having an initial horizontal velocity V_j .

4.5.1 Jet Shape Determined by Equations for Particle Motion

Consider the trajectory of a particle of water emerging from the hole in the side of the tank. Rearranging Newton's law of motion $F = ma$ as $a = F/m$, and writing the differential form of the

acceleration in the x and y direction gives.

$$\frac{d^2 x_p}{dt^2} = 0 \quad \frac{d^2 y_p}{dt^2} = -g$$

where $x_p(t)$ and $y_p(t)$ are the instantaneous coordinates of the fluid particle as it follows the arc of the jet. Integrate the equations of motion with the initial conditions ($t = 0$)

$$\frac{dx_p}{dt} = V_j, \quad x_p = 0, \quad \frac{dy_p}{dt} = 0, \quad y_p = H.$$

The particle crosses the $y_p = 0$ plane when $\frac{1}{2}gt_0^2 = H$ or at time $t_0 = \sqrt{2H/g}$. At time t_0 the particle has travelled a horizontal distance $L_{\text{jet}} = V_j t_0$. Therefore,

$$L_{\text{jet}} = V_j t_0 = V_j \sqrt{\frac{2H}{g}} \quad (11)$$

4.5.2 Analytical Model of Jet Velocity

Apply the Bernoulli Equation between Station 1 and Station 4 in Figure 2 on page 5.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4$$

At the free surface, $p_1 = 0$, and $V_1 \approx 0$. At the jet exit, $p_2 = 0$. Making these assumptions the Bernoulli equation simplifies to

$$z_1 = \frac{V_4^2}{2g} + z_4$$

or

$$\frac{V_4^2}{2g} = z_1 - z_4. \quad (12)$$

Since V_j is the same as V_4 , and $h = z_1 - z_4$, we obtain


$$V_j = V_4 = \sqrt{2gh}. \quad (13)$$

Substituting Equation (13) into Equation (11) gives

$$L_{\text{jet}} = \sqrt{2gh} \sqrt{\frac{2H}{g}} \implies \boxed{L_{\text{jet}} = 2\sqrt{Hh}} \quad (14)$$

4.6 Study Questions

1. What physical effect(s) could cause the jet to not follow the trajectory of a rigid particle?
2. Suppose you had a series of measurements of L_{jet} and h at different times. After making a plot of L_{jet} versus h , how would you make a *quantitative* (i.e., numerical) comparison between the data in the plot and Equation (14)? *Hint:* If $L = ch^{1/2}$ is a model for the horizontal distance traveled by the jet as a function of water depth above the hole in the tank, is there a computational procedure for finding the constant, c ?

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5 Overview of Measurements

This section marks the beginning of the physical experiments. Figure 4 is a schematic representation of the data collection and data reduction process for the lab exercise. Data is recorded while the tank drains.

- (a) A digital camera records images of the jet.
- (b) A pressure transducer records $p(t)$ on the wall opposite of the hole.

After the experiment, a MATLAB program provided by the instructors, is used to convert the $p(t)$ and $L(t)$ data to a table of L versus h values, and plot L versus h .

- (c) Students examine the digital images to extract the jet distance $L(t)$. The L versus t data is entered into a table in the MATLAB program.
- (d) The pressure transducer output is converted from $p(t)$ to $h(t)$, where h is the instantaneous depth of the water in the tank.
- (e) The t values for the images are used to interpolate in the measured $h(t)$ data from the pressure transducer. The result is a table of L values and h values for each of the digital images, i.e., at each time a digital image was captured.
- (f) The MATLAB program plots the $L(h)$ data and obtains a curve fit to $L = c\sqrt{h}$, where c is a constant obtained by the least squares method.

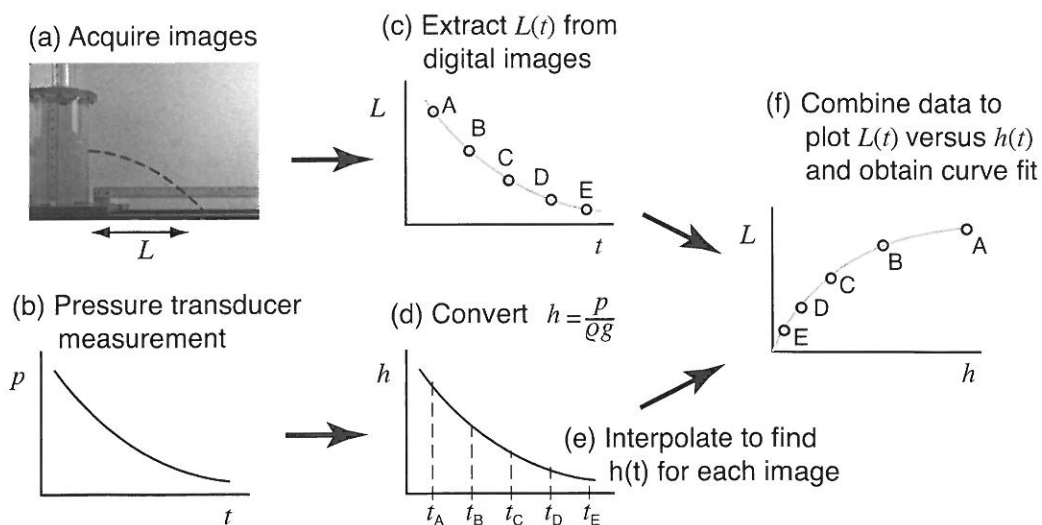


Figure 4: Schematic of data collection and analysis process for the tank draining exercise.

6 Tank Draining Transient

In this exercise, you will use LabVIEW software to record the pressure of the water in the tank as it drains. At the same time you use a digital camera to capture images of the water jet issuing from the side of the tank.

When the experiment is running, you have two main tasks:

1. Properly configure and start the VI (*virtual instrument*) that controls the recording of the $p(t)$ data.
2. Take several picture of the water jet as the tank drains.

After each run, you will analyze the images to extract L , the distance traveled by the jet, at each time (i.e., for each image). After extracting the $L(t)$ data from the images, you will enter that data into a MATLAB program. You will also load the $p(t)$ data recorded during the experiment. Using menus and push-button selections with the MATLAB program, you will create plots that will allow you to reason about the data.

6.1 Uniform Tank Draining

Estimate how long it will take the uniform tank to drain through the hole in the side of the tank. Will it take 3 seconds, 30 seconds, 3 minutes, or 30 minutes?

1. Make sure that the plug is securely in the hole on the side of the tank. Do not use excessive force with the plug. You want to be able to remove the plug with your fingers.
2. Fill the tank with water up to the black line near the top.
3. At least one member of your group should volunteer to take the pictures. That person should set up the camera with the following configuration
 - Turn the flash off.
 - Make sure the camera is set to focus on the ruler.
 - Frame the shot so that the LED time display is visible on the left edge of the frame, and the end of the ruler is just off the right edge of the frame. If necessary, use the zoom setting on the camera and move the tripod closer or farther away from the tank.
 - Prepare to capture images by pressing the shutter switch without jostling the camera. The tripod should help stabilize the camera, but it's still possible to jiggle the camera as the image is taken, especially at slower shutter speeds.

Be prepared to repeat the experiment if the camera and its operator are not configured properly. Do not expect any experiment to work the first time you try it.

4. Prepare to run the VI. Refer to Figure 5 for the four main steps necessary to run the data recording software.

Note: Values of parameters are set before the data acquisition is started. Therefore, in some sense the VI is already running when it is in the foreground window. Your first set of tasks is to make sure the correct parameters are entered into the controls: Steps 1 through 4 in Figure 5. To start the recording of data, you push the start button: Step 4 in Figure 5.

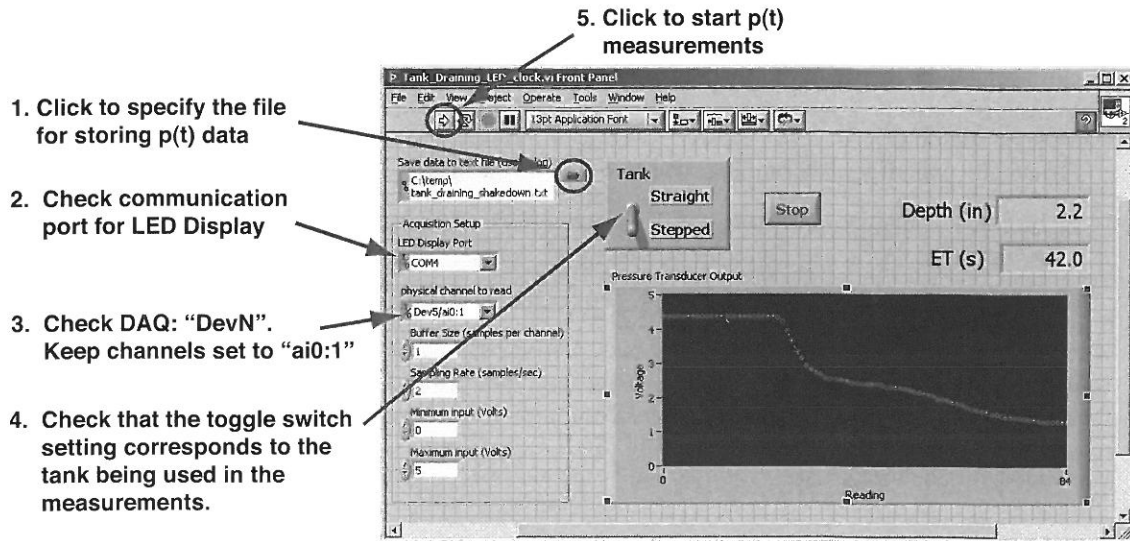


Figure 5: Annotated screenshot of the LabVIEW VI for the tank draining exercise.

5. Enter an appropriate file name for the data the DAQ will record during the experiment. It is *important* that you choose a file name and location that you can find later.
6. There are at least two USB devices connected to the lab computer. One is the LED display, which shows the elapsed time for the experiment. The other is the DAQ that records the pressure data. Steps 2 and 3 in Figure 5 involve setting the communication parameters for these two devices.
Chances are good that both devices are already properly configured. If the VI gives you an error message, you will likely need to change the communication parameters. Ask your TA for assistance.
7. Step 4 in Figure 5: Check to see that the toggle switch is set for the tank being used. Adjust as necessary.
8. Step 5 in Figure 5: Start the VI. Wait a few seconds, and then pull the plug in the side of the tank. While the tank is draining, the team photographer should take several pictures.
9. To stop the experiment, press the large “Stop” button on the VI. *Do not* press the stop button on the LabVIEW menu bar.

By completing the preceding steps you will have a series of images recorded on the camera, and a $p(t)$ data file recorded on the hard drive of the lab computer.

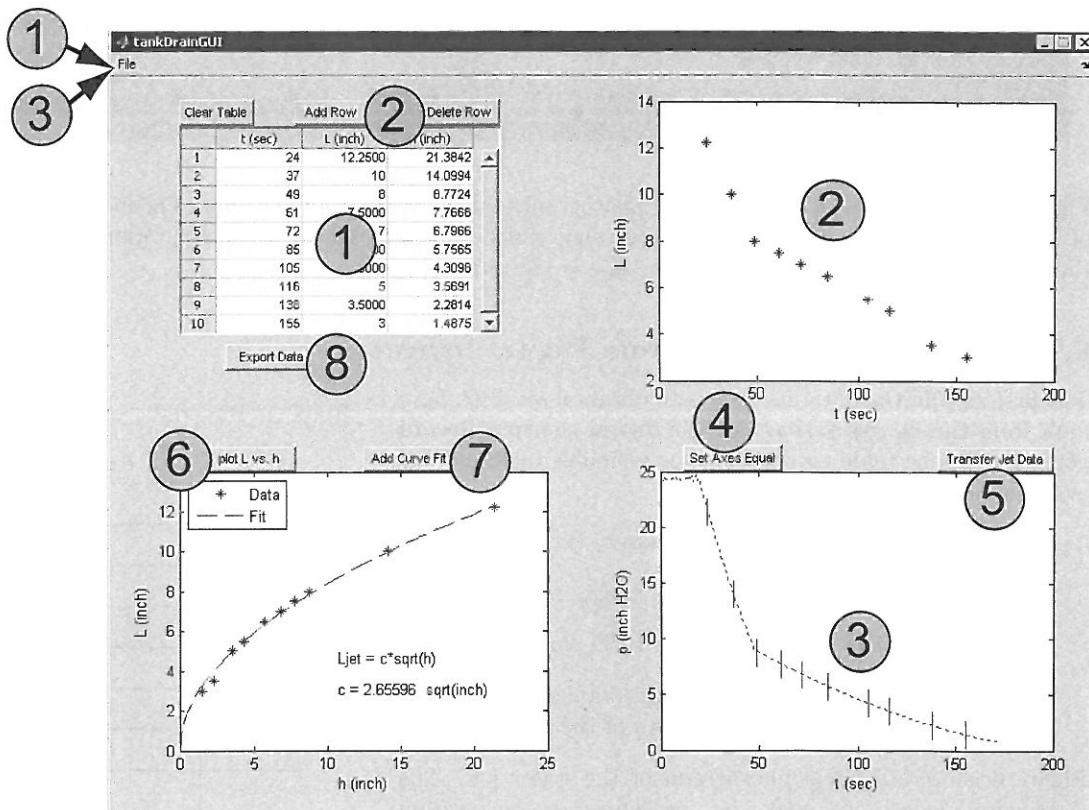


Figure 7: Annotated screen shot of the tankDrainGUI program.

6.3 Automated Analysis of Jet and Transducer Data

Start MATLAB and run the `tankDrainGUI` program by typing

```
>> tankDrainGUI
```

at the command prompt. Use the following steps to analyze the data. The numbers of the items in the list correspond to the large numbers in circles in Figure 7.

1. Enter the (t, L_{jet}) data in the table in the upper left quadrant of the GUI. Alternatively, if you have already saved some (t, L_{jet}) data from previous work with `tankDrainGUI`, then load it by selecting `File→Open Jet Data` from menu at the top of the GUI window.
2. Data entered in the table is automatically plotted in the figure pane in the upper right quadrant of the GUI. You can directly edit data in the first two columns of the table. Row and table-level actions are achieved with the three buttons at the top of the table.

Clear Table replaces all values in the table with zeros.

Add Row inserts a new row at the bottom of the table.

Delete Row removes the row that is currently selected. To delete a row, first select it by clicking in a cell in the t or L columns.

Note that data in the h column is calculated by the program and cannot be edited manually.

3. Load pressure versus time data by selecting `File→Open Pressure Data` from the menu at the top of the GUI window. Locate the file that was saved by the LabVIEW VI. After it is loaded, the (t, p) data is displayed in the figure pane in the lower right quadrant of the GUI.
4. The t axis scaling for the $L_{\text{jet}}(t)$ and $p(t)$ plots are unlikely to be the same. Click the `Set Axes Equal` button to use the same scale for t in both plots.
5. After $L_{\text{jet}}(t)$ and $p(t)$ have been loaded, click the `Transfer Jet Data` button. This causes the t values from the $L_{\text{jet}}(t)$ data set to be transferred to the $p(t)$ plot. Those t values are represented by short vertical lines that cross the $p(t)$ data.

For each of the t values in the $L_{\text{jet}}(t)$ data set, a new pressure value, call it \hat{p} , is obtained by interpolating in the measured pressure data set. The interpolated \hat{p} values are taken as the pressure (and hence tank depth) for each of the $L_{\text{jet}}(t)$ values. Note that when the pressure is given in units of head, the pressure value is h , the depth of water measured from the location of the pressure transducer.

6. Click the `plot L vs. h` button to combine the jet length and pressure measurements in a single plot in the lower left quadrant of the GUI.
7. Click the `Add Curve fit` button to compute and plot a curve fit of the form $L_{\text{jet}} = c\sqrt{h}$. The curve fit to the data appears as a dashed red line. The curve fit coefficient c is displayed on the plot.

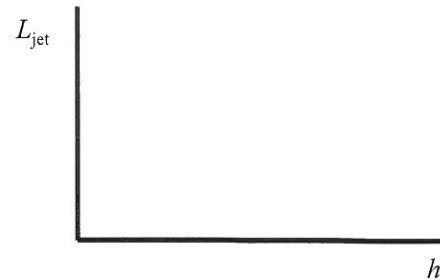
Record the curve fit coefficient for future reference.

$c =$ _____ **units** = _____

8. Click the `Export Data` button at the bottom of the data table in the upper left quadrant of the GUI. This opens a dialog box for specifying the file name and location of the data file. The file consists of the three columns of data shown in the upper left quadrant of the GUI. Save the data on your MCECS account on the file server.

6.4 Qualitative Summary for the Straight Tank

On the axes to the right, sketch the relationship of L_{jet} versus h for the straight-walled tank. Use an arrow to indicate the direction that the data on the curve follows as time increases during the tank draining experiment.

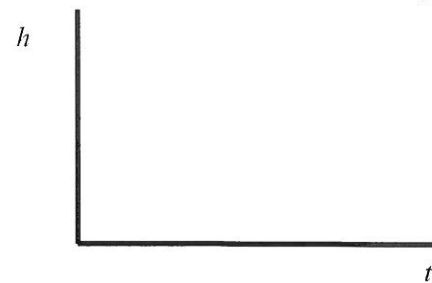


6.5 Anticipation of Results for the Step-shaped Tank

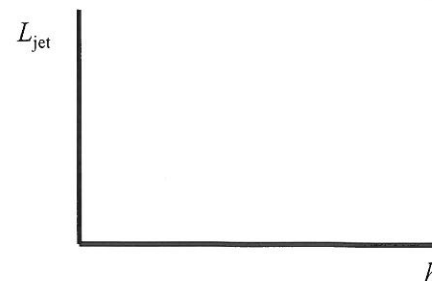
On the axes to the right, sketch the relationship of L_{jet} versus t for the step-shaped tank. Put a circle on the curve to identify the time where the free surface passes the change in area of the tank.



On the axes to the right, sketch the relationship of pressure transducer output versus t for the step-shaped tank. The pressure recorded by the transducer is proportional to the depth of water in the tank so the vertical axis is labeled h . Put a circle on the curve to identify the time where the free surface passes the change in area of the tank.



On the axes to the right, sketch the relationship of L_{jet} versus h for the step-shaped tank. Use an arrow to indicate the direction that the data on the curve follows as time increases during the tank draining experiment. Add a dashed curve to indicate the $L_{\text{jet}}(h)$ relationship for the *straight* tank.



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7 Comparison of the Two Tanks

7.1 Qualitative Results

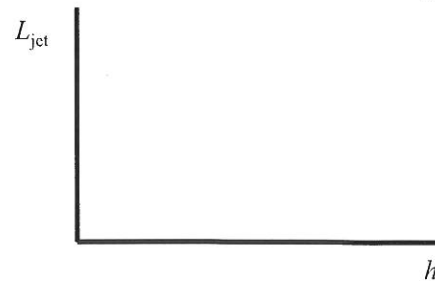
On the axes to the right, sketch the L_{jet} versus t curves for the two tanks.



On the axes to the right, sketch the water depth (pressure) versus t curves for the two tanks.



On the axes to the right, sketch the relationship of L_{jet} versus h relationship for both tank.



7.2 Conceptual Analysis and Conclusions

1. When the distance, h , between the free surface and the hole in the side of the tank is the same for the straight and and the step-shaped tanks, which tank has the higher V_j ?
2. For the same h , which tank has the larger L_{jet} ?
3. At the start of the experiment the depth of the step-shaped tank decreases more rapidly than the straight-walled tank. Mass conservation would seem to imply that a higher rate of decrease in h would correspond to a greater exit velocity V_j . What is the error in that reasoning? Equation (4) and Equation (5) on page 4 may be useful.
4. Consider two straight-walled tanks that have holes with different diameters, say d_1 and d_2 , with $d_1 > d_2$. At the same h , which tank would have the larger L_{jet} ? At the same h , which tank would have the larger V_j ?