

# Sudden Expansion Exercise

Before coming to the lab,  
read sections 1 through 4  
of this document.

Engineering of Everyday Things  
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# 1 Introduction

Before coming to the lab, read sections 1 through 4 of this document.

This exercise is an investigation of the relationships between pressure drop, velocity profile, and area change for flow through a sudden expansion in a duct. The lab measurements are designed to answer the following questions:

- What is the velocity profile downstream of the sudden expansion?
- What is the relationship between the velocity profile and the flow rate?
- What are the qualitative and quantitative relationships between the velocity and pressure as the air crosses the expansion?
- What are the appropriate engineering models to describe the pressure and velocity changes across the sudden expansion?

## 1.1 Learning Objectives

After completing this lab exercise students will be able to

1. Sketch the velocity profile downstream of a sudden expansion;
2. Sketch the velocity profile at two flow rates;
3. Apply mass conservation to a measurement of velocity profile in a duct;
4. Use the conservation of mass to calculate the inlet velocity from the measured velocity downstream of the sudden expansion;
5. Explain why or why not the Bernoulli equation can be used to compute the pressure change across the sudden expansion;
6. Apply the Energy Equations to calculate the head loss across a sudden expansion.

## 1.2 Sudden Expansion Device

Air and water transport systems consist of long straight sections of duct, as well as elbows, tees, valves and area changes. A sudden expansion occurs whenever a smaller duct is joined to a larger duct. In this lab, you will study the behavior of air flowing through a sudden expansion.

Figure 1 depicts the main components of the laboratory apparatus. A blower draws air through a duct constructed from acrylic tubing of two diameters. The inlet end of the duct has diameter  $d_1$  and length  $L_1$ . A longer section of tubing with diameter  $d_2 > d_1$  connects the inlet section to the blower. The transition from  $d_1$  to  $d_2$  is abrupt.

The flow rate through the duct is controlled by adjusting the blower speed switch (either high or low) and a sliding damper, also called a *blast gate*. Local air velocity downstream of the sudden expansion is measured by a velocity sensor mounted on a moveable stage. The pressure change across the sudden expansion is measured with an inclined manometer and a differential pressure transducer.

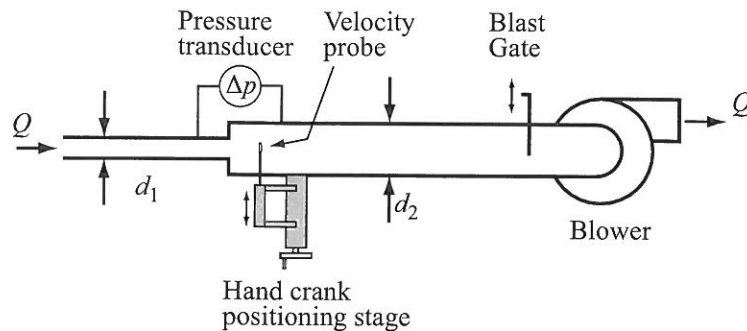


Figure 1: Apparatus for measuring flow downstream of a sudden expansion.

## 2 Background and Review

Read this section before coming to the laboratory.

### 2.1 Bernoulli and Energy Equations

The Bernoulli equation is applied between stations 1 and 2 along a streamline

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad (1)$$

where  $p$  is the static pressure,  $\gamma$  is the specific weight of the flowing fluid,  $V$  is the velocity magnitude, and  $z$  is the elevation above a fixed datum. The Bernoulli equation only applies along a streamline *if* (1) the flow is steady, (2) the flow is incompressible, and (3) there is no heat or work interactions (including flow losses) between stations 1 and 2.

The steady flow energy equation is

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 - h_p + h_L \quad (2)$$

where  $h_p$  is the head increase due to shaft work by a pump and  $h_L$  is the head loss. The steady flow energy equation is more generally applicable than the Bernoulli Equation. The energy equation applies to steady, incompressible flow in a conduit with a single inlet and a single outlet.

### 2.2 Mass Conservation: Numerical Integration of a Velocity Profile

The average velocity at any axial location in the duct is

$$V = \frac{1}{A} \int_A u \, dA \quad (3)$$

where  $u$  is the velocity component along the duct axis, and  $A$  is the cross-sectional area of the duct. Values of  $u$  at different radial locations are obtained when the velocity transducer is traversed across the duct. Because the  $u(r)$  data consists of discrete measurements, Equation (3) must be evaluated numerically.

Approximate the velocity profile as a series of straight lines. The integral in Equation (3) is then a sum of trapezoidal areas as shown in the sketch to the right.

$$\int_{r_a}^{r_b} f(r) \, dr \approx \sum a_i$$

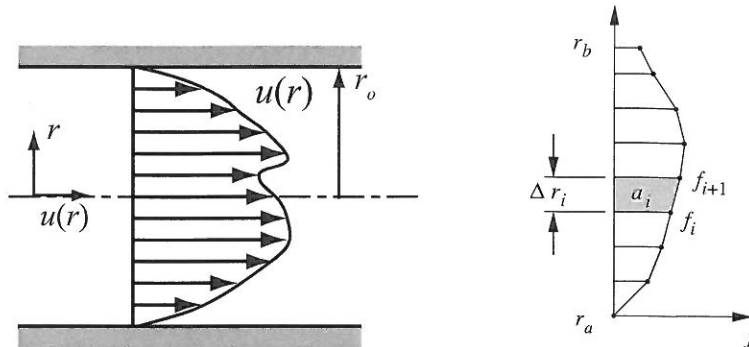


Figure 2: Left: non-uniform velocity profile in a duct. Right: Trapezoid rule for numerical integration of the velocity profile.

The area of a typical trapezoid is

$$\begin{aligned}
 a_i &= (r_{i+1} - r_i)f_i + \frac{1}{2}(r_{i+1} - r_i)(f_{i+1} - f_i) \\
 &= \frac{1}{2}(f_{i+1} + f_i)(r_{i+1} - r_i) \\
 &= \frac{1}{2}(f_{i+1} + f_i)\Delta r_i
 \end{aligned}$$

where  $\Delta r_i = r_{i+1} - r_i$ . The numerical approximation to the integral is known as the trapezoid rule

$$\int_{r_a}^{r_b} f(r) dr \approx \sum \frac{1}{2}(f_{i+1} + f_i)\Delta r_i \quad (4)$$

Note that the  $a_i$  in the trapezoid rule is the area of a trapezoid, *not* an element of duct area in Equation (3). For the numerical integration of Equation (3),

$$f_i = 2\pi r_i u_i.$$

Therefore,

$$V = \frac{1}{A} \int_A u dA = \frac{1}{2\pi R} \int_0^R u 2\pi r dr = \frac{1}{R} \int_0^R u r dr = \frac{1}{R} \sum \frac{1}{2} (u_{i+1} + u_i) r_i \Delta r_i$$

### 3 Apparatus

Read this section before coming to the lab.

#### 3.1 Duct Geometry

Before you make any measurements of the flow, you will need to obtain some critical physical dimensions of the device. You will measure the internal diameters of the duct, and some features of the velocity probe that will allow you to locate the position of the probe in the duct. Making these measurements will also help you to become familiar with the apparatus.

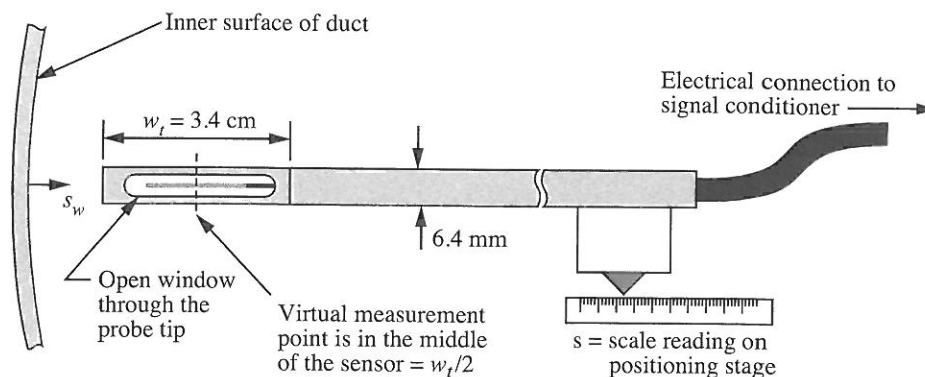


Figure 3: Detail of the velocity probe tip when it is near the far wall of the duct.

### 3.2 Velocity Sensor

Air velocity is measured with a TSI Model 8455 Air Velocity Transducer<sup>1</sup>. The velocity transducer is a thermal anemometer. A small electrical current is passed through the probe tip and the rate of cooling is calibrated to the oncoming velocity. The velocity sensor is mounted on a manual positioning stage that allows the sensor to be moved radially across the larger duct.

Figure 3 depicts the location of the sensing element in the probe tip. The sensing element looks like a small cylinder suspended in the center of an opening in the otherwise solid probe assembly. For the purpose of this laboratory exercise, the velocity is assumed to be measured at a *virtual sensor* location at the center of the opening in the probe tip. In other words the velocity measurement is assumed to occur at a distance  $w_t/2$  from the physical end of the probe.

The location of the velocity sensor in the duct is controlled by a manual positioning stage. The user changes the location of the probe by turning the crank on the positioning stage. The position is indicated by a linear scale on the side of the positioning stage. Figure 3 shows the schematic relationship between the probe position in the duct and the position indicated on the positioning stage. The indicator on the positioning stage defines the value of  $s$ . When the probe tip is touching the inner surface of the duct wall the indicator on the positioning stage has a value of  $s$  called  $s_0$ .

### 3.3 Pressure Measurement

The pressure change across the sudden expansion is measured manually and with electronic instruments. The manual readings are taken with a Dwyer Model 102.5 Durablock portable inclined manometer<sup>2</sup>. The electronic system consists of an Omega PX653-0.5D5V differential pressure transducer connected to the data acquisition system. The two pressure taps on the duct wall are located 7.6 cm (3 in.) upstream and 15.2 cm (6 in.) downstream of the sudden expansion, respectively.

### 3.4 Data Acquisition

Figure 4, is a schematic of the data acquisition (DAQ) system. The primary components are the velocity sensor and its signal conditioner, the pressure transducer, a DC power supply, and a data

<sup>1</sup>TSI Incorporated, Environmental Measurements and Controls Division, 500 Cardigan Road, Shoreview, Minnesota, [www.tsi.com](http://www.tsi.com).

<sup>2</sup>Dwyer Instruments Inc., Michigan City, Indiana, [www.dwyer-inst.com](http://www.dwyer-inst.com).

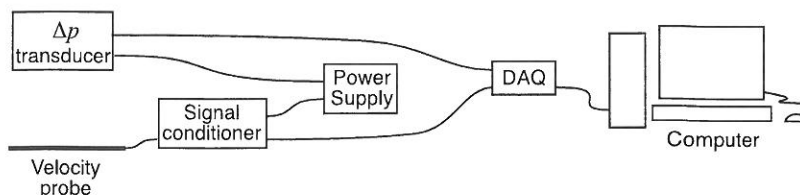


Figure 4: Connection of the sensors to the acquisition system.

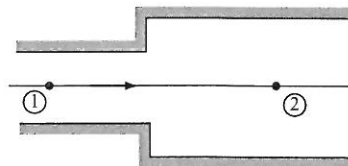
acquisition DAQ unit. The signal conditioning unit of the velocity probe controls the power to the sensing tip and provides an output voltage that is linearly related to the air velocity. The DC power supply provides the electrical energy to the velocity signal conditioner and the pressure transducer. Both sensors require a DC voltage input of 12 V.

Output of the velocity and pressure sensors is digitized with a USB-6008 DAQ from National Instruments<sup>3</sup>. The LabVIEW software system (also from National Instruments) is used to control and obtain readings from the DAQ device. A LabVIEW *virtual instrument* (VI) provides a user interface for the data collection.

## 4 Pre-Lab Study Questions

Before coming to the lab, answer the following questions on your own. Before starting any measurements, discuss your answers with your lab partners.

1. Under what limited set of conditions can the Bernoulli equation be applied?
2. Under what limited set of conditions can the steady flow energy equation be applied?
3. What is fully-developed flow in a pipe?
4. What does the velocity profile look like for fully-developed *laminar* flow in a pipe?
5. What does the velocity profile look like for fully-developed *turbulent* flow in a pipe?
6. Do you expect the velocity profile immediately downstream of the sudden expansion to look like a fully-developed velocity profile?
7. One part of the lab exercise involves measuring the velocity profile at a constant flow rate. As the velocity probe is moved across the duct, will the pressure measurement change? If so, what variation in pressure measurement do you expect?
8. Consider steady, frictionless flow through the sudden expansion in the schematic to the right. Use the Bernoulli equation to derive a formula for  $p_1 - p_2$ . Do you expect the Bernoulli equation to apply (to be valid for) an analysis of the pressure change across the sudden expansion?



<sup>3</sup>National Instruments, Austin, Texas, [www.ni.com](http://www.ni.com).

## 5 Measurement of the Velocity Profile

The purpose of this section is to measure the axial velocity as a function of position across the duct.

### 5.1 Device Dimensions

Before turning on the blower, obtain physical dimensions of the apparatus.

1. Use instruments available in the lab to measure the physical dimensions of the duct. Record your answers in the table to the right.

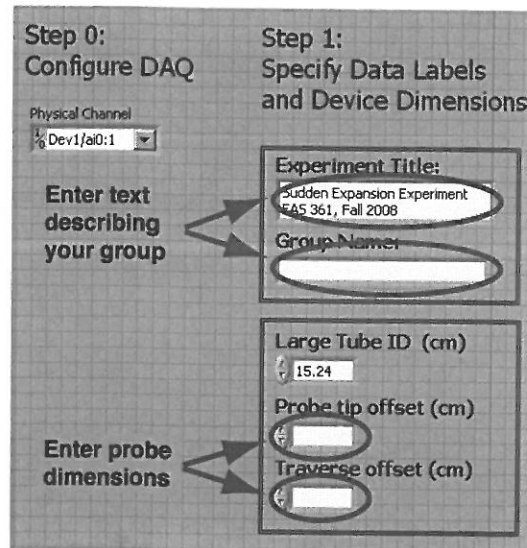
Feature	Dimension (cm)
Small Duct ID	
Wall Thickness	
Large Duct OD	
Large Duct ID	

2. Start LabVIEW by double-clicking on the `SuddenExpansion.vi` icon on the desktop of the lab computer. Do not *run* the VI yet. Inspect VI by scrolling left and right on the screen. Scroll to the left of the VI so that the “Step 0”, “Step 1”, labels are visible, as shown in the image to the right.

3. Inspect the text in the *Experiment Title* box, and make modifications as desired.

4. Add a descriptive name for your group in the *Group Name* text box.

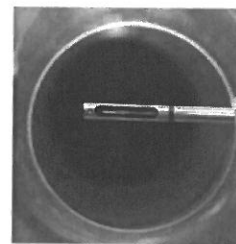
The data in the *Experiment Title* and *Group Name* boxes will help you identify your data later in the lab session, and when you take your data home.



5. Turn the crank on the linear stage until the probe is near the centerline. Inspect the probe tip by looking axially along the duct from the open end. Refer to Figure 3. In the box marked *Probe tip offset* record the distance from the free end of the probe to the center of the sensor.

**Probe tip offset:** \_\_\_\_\_ cm

Enter the value of the *Probe tip offset* in the input box on the VI.



6. Move the probe tip fully into the duct so that it *gently* touches the far wall. Enter the reading on the physical scale in the *Traverse offset* box on the left side of the LabVIEW screen. Record the value of **Traverse offset** in the space below.

**Traverse offset:** \_\_\_\_\_ cm

Enter the value of the *Traverse offset* in the input box on the VI.

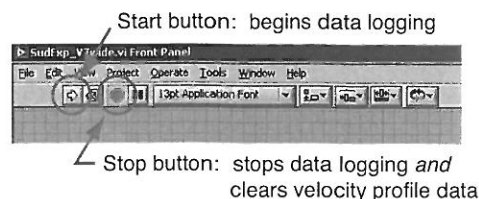
The *Probe tip offset* and the *Traverse offset* will not need to be entered again for the remainder of the lab exercise as long as you do not quit and restart the VI.

## 5.2 Velocity Profile Measurement

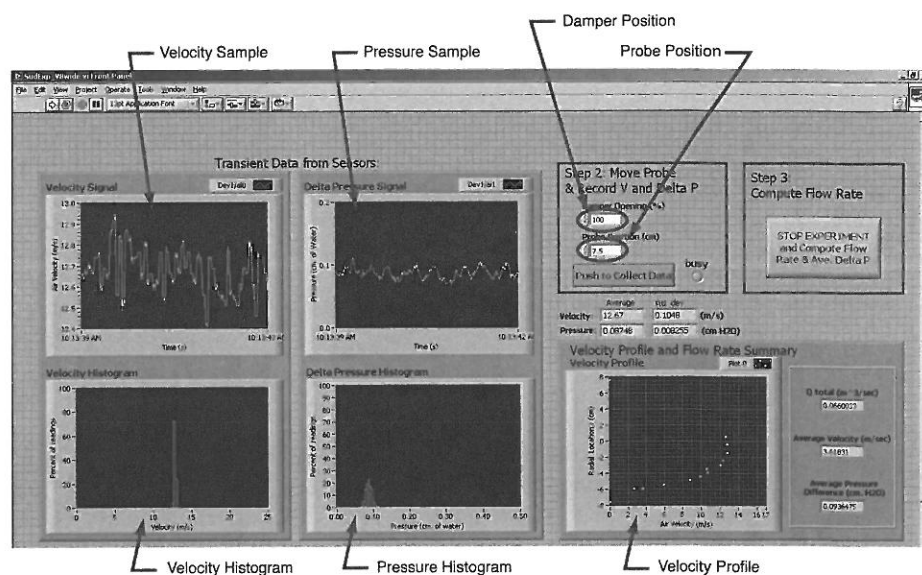
1. Turn the blower on and set the air speed by adjusting motor speed (high/low) and the blast gate. **Keep the blower speed and damper position fixed for the remainder of the velocity profile measurements.**

2. Start data logging with the LabVIEW VI by clicking the start button.

*Do not* use the *Stop* button on the menu bar after you take the last profile measurement. Pressing the *Stop* button on the menu bar will stop the data logging *and* purge the accumulated profile data.



3. Scroll the LabVIEW VI to the right so that the four plots labeled *Transient Data from Sensors* and *Step 2: Move Probe & Record V and Delta P* are visible in the middle of the screen.



4. Repeat the following steps at least 10 times. Take data at radial increments of 1 cm or less, with closer spacing near the walls. Ideally you should randomize the order of the radial positions.

- a. Rotate the handle on the positioning stage to move the probe to a new radial location.
- b. Read the value on the scale of the positioning stage, and enter that value as the probe position in the VI.
- c. Watch the plots of instantaneous velocity and pressure signals. After two or three bursts of data acquisition, the histograms of the velocity and pressure signals will settle. When the mean of the data in the histogram appears to be stable, press the green *Push to Collect Data* button. When the *busy* light has turned from red to green, it is OK to return to step 4a.  
*Do not* move the positioning stage until the busy light turns green.

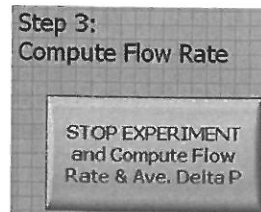


- d. After recording several velocity values across the duct, press the *Stop Experiment...* button. *Do not* push the stop button on the menu bar.

**Be patient:** The computer may pause a few seconds while the last burst of measurements are retrieved and the summary calculations are performed. *Do not* click the *Stop Experiment...* button more than once.

- e. Record the summary data

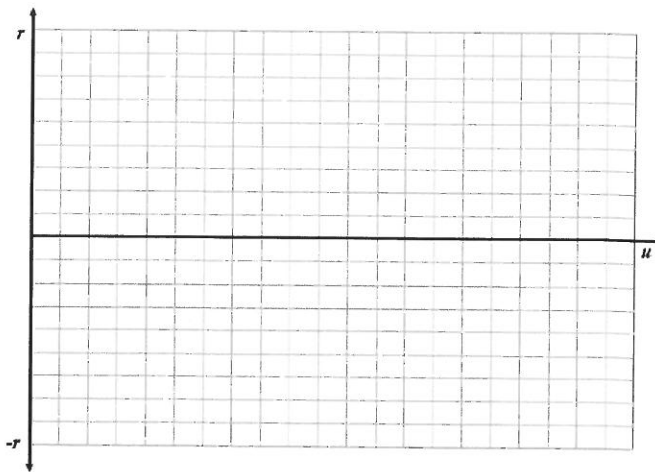
Diameter ratio,  $d_2/d_1$  \_\_\_\_\_  
 $Q_{\text{total}}$  \_\_\_\_\_  $\text{m}^3/\text{s}$   
 Average velocity \_\_\_\_\_  $\text{m}/\text{s}$   
 Average  $\Delta p$  \_\_\_\_\_  $\text{cm H}_2\text{O}$



### 5.3 Qualitative Analysis

On the following mesh, sketch the velocity profile by transferring the data from the screen of the LabVIEW VI.

1. Which variable should be aligned along the horizontal ( $x$ ) axis?
2. Use a solid line to identify the location of the duct walls in your experiment.
3. Use dashed lines to sketch two additional velocity profiles: one for a higher average velocity, and one for a lower average velocity. Label those profiles "higher" and "lower".



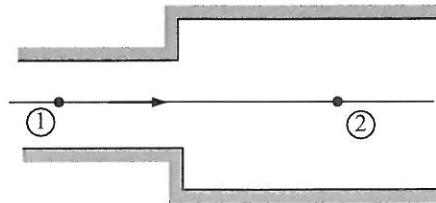
Before continuing, show your lab manual to the instructor. It's important at this point to make sure you are on the right track.

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## 6 Prediction and Measurement of Pressure Change

The purpose of this set of measurements is to determine how the pressure change across the sudden expansion varies with flow rate through the system. The flow rate will be estimated from the velocity on the centerline of the duct.

### 6.1 Predict the Sign of the Pressure Change



1. Turn off the blower.
2. Consider the streamline along the centerline of the duct as represented in the preceding sketch. The pressure difference from station 1 to station 2 is (circle one)
  - a.  $p_1 - p_2 > 0$
  - b.  $p_1 - p_2 < 0$
  - c.  $p_1 - p_2 = 0$  (exactly)
  - d.  $p_1 - p_2 < 0$  or  $p_1 - p_2 > 0$  depending on the flow rate and  $d_2/d_1$  ratio.
3. What theoretical model or equation can you use to predict the answer to the previous question?
4. What evidence from the experiment can you use to determine which answer to Question 2 is correct?
5. Turn on the blower and perform a simple experiment to determine the sign of  $p_1 - p_2$ .
  - a. Describe your experiment in one or two sentences.
  - b. What is the result of your experiment? Use the notation from Question 2, above.



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### 6.3 Theoretical Model of the Pressure Change versus Flow Rate

1. Write a theoretical model (a basic formula) that can be used to predict the pressure change  $p_2 - p_1$ .
2. Rearrange the formula from the preceding step to the form  $p_2 - p_1 = \dots$
3. Use the formula from the preceding step to complete the last column of Table 1. Show your calculations for at least one row of the table in the space below.

### 6.4 Comparison of Measured and Computed Pressure Change

1. How well do the measured and theoretical values of pressure difference compare?
2. Give at least one reason for any discrepancy between the measured and computed values of  $\Delta p$ ?
3. Can you suggest a better theoretical model (a better formula)? What is it? How might that formula account for the true physics in the experiment?



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## 6.5 Calculation of Head Loss

- Rearrange the steady-flow energy equation to the form  $h_L = \dots$ , i.e., with all terms except  $h_L$  moved to the right hand side. Simplify the right hand side by canceling any terms that are zero or negligible.
- What are the units of  $h_L$ ?
- Use the data from Table 1 and the formula from Question 1 to fill in the  $h_L$  column in Table 2, below. What is the general trend of  $h_L$  versus  $V_2$ ?
- For sudden expansions and other *minor loss* elements, the head loss is reported as a *minor loss coefficient*

$$K_L = \frac{h_L}{V^2/(2g)} \quad (5)$$

For the sudden expansion,  $V$  is the upstream velocity, ( $V_1$ ). Use Equation (5) to fill in the last column in Table 2. What is the general trend of  $K_L$  versus  $V_2$  and  $V_1$ ?

Table 2: Head loss versus velocity.

$V_2$ (m/s)	$V_1$ (m/s)	$h_L$ units: _____	$K_L$ units: _____



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## 7 Conceptual Analysis and Conclusion

1. Is there a general rule about the *sign* of the pressure change with distance for steady flow of an incompressible fluid in a straight tube? If so, what is that rule?
2. Is there a general rule about the *sign* of the pressure change with distance for steady flow of an incompressible fluid through a sudden expansion? If so, what is that rule?
3. Discuss your results with at least one other group in the lab. Do the trends in your data and your conclusions agree with their trends and conclusions?
4. In analyzing the flow across a sudden expansion, is it more correct to use the Bernoulli equation or the steady flow energy equation? Or are both equations equally valid? Provide at least one reason for your answer.
5. Transfer your data to your MCECS account on the file server. Discuss with your lab partners a plan to share the data and any additional analysis you might want to perform with the data.



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