Experiment 4

Impact of a Jet

Purpose

The purpose of this experiment is to demonstrate and verify the integral momentum equation. The force generated by a jet of water deflected by an impact surface is measured and compared to the momentum change of the jet.

Apparatus

The experimental apparatus consists of a water nozzle, a set of impact surfaces, a spring scale connected to a balance beam, a flow meter, and plumbing for recirculating the water. Figure 4.1 is a schematic of these components. The pump draws water from the collection tank and provides sufficient head for the water to flow through the nozzle and the flow meter. The jet of water from the nozzle impinges on the impact surface. The balance beam attached to impact surface allows measurement of the force necessary to deflect the water jet.

Theory

A theoretical model for the force necessary to hold the impact surface stationary is obtained by applying the integral forms of the continuity and momentum equations. The details of the model depend on whether or not the fluid stream leaving the impact surface is symmetric relative to the vertical axis of the surface.

Symmetric Jet

The geometric and fluid parameters for this experiment are identified in the sketch in Figure 4.2. A stream of water with average velocity V flows upward from the nozzle. It impinges on the impact surface and turns to flow radially outward from the axis of the impact surface.

The control volume, bounded by the dashed lines, is chosen so that it crosses the jet streams at right angles. To proceed with the analysis make the following assumptions

- friction between the impact surface and the water jet is negligible

- the magnitude of the jet velocity does not change as the jet is turned
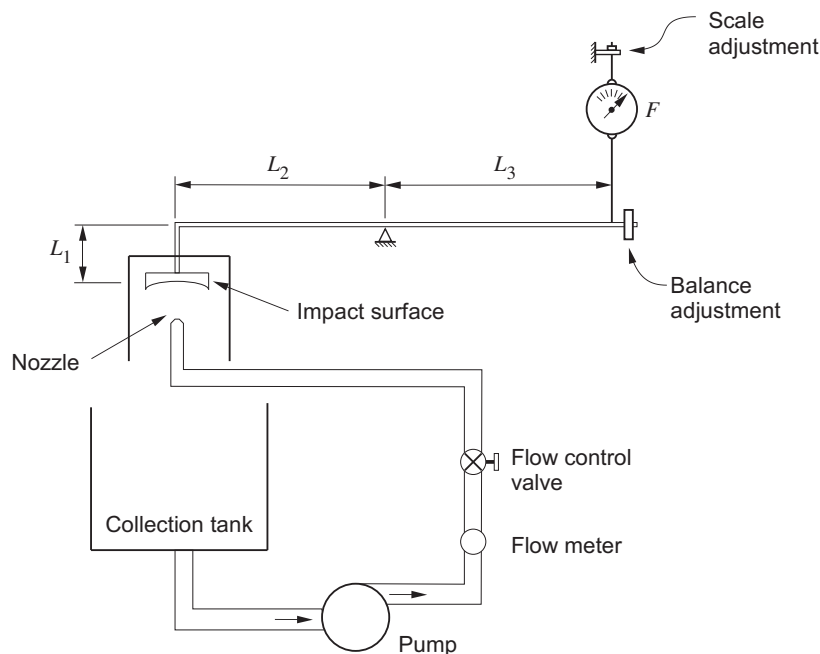
Figure 4.1: Apparatus used in the jet impact experiment.

- velocity profiles are uniform where the flow crosses the control surface

- the jet exit is circumferentially symmetrical

If any of the impact surfaces used in the experiment cause flows that violate these assumptions, the formulas for reaction forces given below will not match the measured reaction forces.

Applying the conservation of mass to the jet streams gives

$$V_1 A_1 - V_2 A_2 = 0 \tag{4.1}$$

where V is the average velocity at a given cross-section, and A is the cross-sectional area normal to the direction of the average velocity. The subscripts 1 and 2 refer to the inlet and outlet of the control volume, respectively. Since the magnitude of the velocity is assumed to not change, Equation(4.1) simplifies to

$$A_1 = A_2 = A \tag{4.2}$$

The integral equation for momentum conservation in the x-direction is

$$\Sigma F_x = \int_{CS} \rho V_x \left(\vec{V} \cdot \hat{n} \right) \, dA$$

$$\implies \quad R_h = \rho V_2 \cos\theta V_2 A_2 + \rho(-V_2)\cos\theta V_2 A_2 = 0 \tag{4.3}$$

where R_h is the reaction force in the x-direction necessary to hold the impact surface stationary, and θ is the angle between the horizontal and the velocity vector of the fluid leaving the control volume. Equation (4.3) shows that $F_h = 0$ if the flow leaving the impact surface is symmetric about the vertical axis of the impact surface. If there is any disruption to the symmetry, e.g., variations in V_2 or θ around the periphery of the exit, R_h will not be zero.
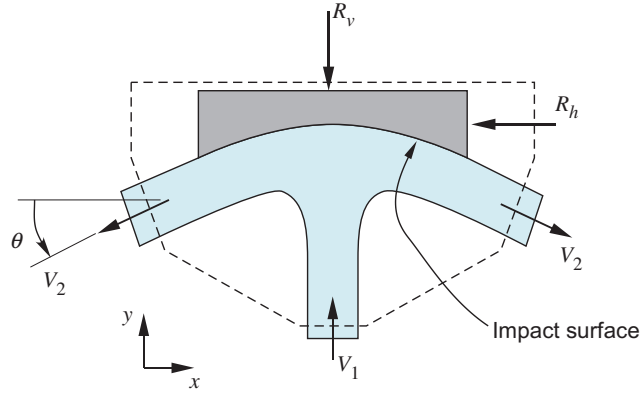
Figure 4.2: Nomenclature for control volume analysis of the jet. Ideally, the apparatus and jet are symmetric about the centerline of the jet.

Applying the y-direction integral momentum equation gives

$$\Sigma F_y = \int_{CS} \rho V_y \left(\vec{V} \cdot \hat{n} \right) dA$$

$$\implies \quad -R_v = \rho V_1 \left(-V_1 \right) A_1 + \rho (-V_2 \sin \theta) V_2 A_2 \tag{4.4}$$

where F_v is the reaction force in the y-direction. Using the simplifications $A_1 = A_2 = A$ and $V_1 = V_2 = V$, Equation(4.4) reduces to

$$R_v = \dot{m} V (1 + \sin \theta) \tag{4.5}$$

where $\dot{m} = \rho V_1 A_1 = \rho V_2 A_2$.

Equation (4.5) is the theoretical model for predicting the vertical force on the impact surface. The experimental apparatus is designed to measure F_v. A moment balance about the point O in Figure 4.3 yields

$$F_v L_2 + F_h L_1 - F_s L_3 = 0 \tag{4.6}$$

where F_v and F_h are the vertical and horizontal forces transmitted from the impact surface to its support, and F_s is the force measured by the spring balance. Solving this equation for F_v allows a
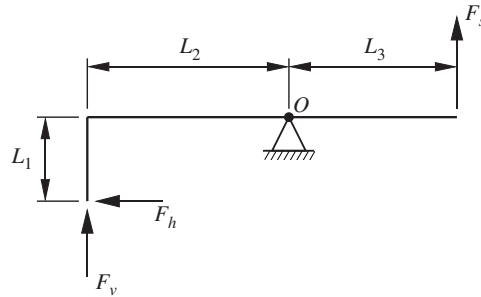

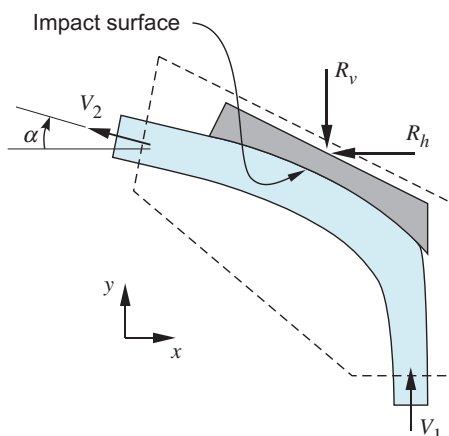Figure 4.3: Moments arising from forces in the jet experiment.

Figure 4.4: Forces in the jet experiment when the impact surface is not symmetric.

comparison between the theoretical and measured reaction forces caused by the fluid jet. Note that $F_v = |R_v|$ and $F_h = |R_h|$. Also note that $R_h = F_h = 0$ for a symmetric jet.

Asymmetric Jet

Now consider the case depicted in Figure 4.4 where the impact surface is not symmetric about its vertical axis. The fluid stream leaving the surface will cause a nonzero horizontal reaction force. Applying the momentum integral equation in the x direction yields

$$-R_h = \rho(-V_2 \cos \alpha)V_2 A_2$$

which simplifies to

$$R_h = \dot{m}V_2 \cos \alpha \tag{4.7}$$

where $\dot{m} = \rho V_2 A_2$. Applying the momentum integral equation in the y direction gives

$$-R_v = \rho V_1(-V_1)A_1 + \rho(V_2 \sin \alpha)V_2 A_2$$

or

$$R_v = \dot{m}(V_1 - V_2 \sin \alpha) \tag{4.8}$$

where $\dot{m} = \rho V_1 A_1 = \rho V_2 A_2$ has been used to simplify the expression.

Equations (4.7) and (4.8) can be simplified further if we know the cross sectional area of the jets entering and leaving the control volume. Without approximation we can use the incompressible mass conservation relationship

$$V_1 A_1 = V_2 A_2. \tag{4.9}$$

Using Equation (4.9) to eliminate V_2 from Equations (4.7) and (4.8) gives

$$R_h = \dot{m}V_1 \frac{A_1}{A_2} \cos \alpha \tag{4.10}$$

$$R_v = \dot{m}V_1 \left(1 - \frac{A_1}{A_2} \sin \alpha\right) \tag{4.11}$$

Unfortunately, there is no easy way to measure A_1/A_2. In the absence of additional information, assume $A_1/A_2 = 1$. Under this assumption, the formulas for the horizontal and vertical reaction forces simplify to

$$R_h = \dot{m}V_1 \cos\alpha \tag{4.12}$$

$$R_v = \dot{m}V_1(1 - \sin\alpha) \tag{4.13}$$

Note the difference in signs for Equation (4.5) and Equation (4.13). Rearranging Equation (4.13) gives

$$\dot{m}V_1 = \frac{R_h}{1 - \sin\alpha}$$

and substituting this result into Equation 4.12 and simplifying gives

$$R_h = R_v \frac{\cos\alpha}{1 - \sin\alpha} \tag{4.14}$$

Finally, substituting Equation (4.13) and (4.14) into Equation (4.6) and solving for F_v gives

$$F_v = F_s \frac{L_3/L_2}{1 + \dfrac{L_1}{L_2}\left(\dfrac{\cos\alpha}{1 - \sin\alpha}\right)} \tag{4.15}$$

This equation is the theoretical model for predicting the vertical force on the impact surface when the impact surface is not symmetric about its vertical axis. Note that Equation (4.15) is based on the assumption that $A_1/A_2 = 1$. This is consistent with an assumption that the fluid velocity does not decrease in magnitude as the fluid impinges on and leaves the impact surface.

Procedure

In the theoretical calculation of the force on the impact surface it is assumed that the jet exit is symmetric around the impact surface. For the flow to be symmetric the balance beam must be horizontal. Two adjustments are necessary to keep the balance beam horizontal: one on the balance beam and one on the scale.

Balance adjustment: For each impact surface, adjust the knurled knob on the balance beam so that *with no flow and with no load on the scale* the balance beam is horizontal. Do not make further adjustments to this knob unless the impact surface is changed.

Scale adjustment: The mechanism inside the spring scale stretches as the force on it is increased. This causes the balance beam to tip as the flow rate is changed. The new equilibrium orientation of the balance beam is established when the reaction moment from the scale balances the moment exerted by the water on the impact nozzle. To get a proper force reading you have to adjust the scale so the balance beam is restored to horizontal. This is achieved by turning the nut on the rod that passes through the support for the scale.

Step-by-step Instructions

To perform the experiment:

1. Measure the length of the lever arms of the balance beam.

2. Install an impact surface and adjust the balance knob as described above.

3. Set the desired flow rate with the flow control valve.

4. Adjust the scale as described above so that the balance beam is horizontal.

5. Record the flow rate and the force on the scale.

6. Repeat steps 4 through 6 for a total of six flow rates

7. Repeat steps 2 through 7 for at least two different impact surfaces.

Analysis

Analysis of the data for the symmetric impact surface is almost identical to the analysis to the analysis for the asymmetric impact surface. The only differences are in the formula used to compute the theoretical reaction force and the formula used to compute the reaction force from the measured spring force. In the following steps, the references to the formulas for the asymmetric impact surface are given in parenthesis.

1. For each setting of the control value:

   - Convert the flow rate to average velocities for the jet.
   - Use Equation (4.5) (or Equation (4.13)) to compute the theoretical reaction force given V computed from the flow rate measurement.
   - Use Equation (4.6) (or Equation (4.15)) to compute F_v from F_s and that apparatus dimensions.

2. On the same axes, plot R_v and F_v versus V for each impact surface. (Create a separate plot for each surface, but compare R_v and F_v on each plot.)

3. Plot the discrepancy $R_v - F_v$ versus V. (Create a separate plot for each surface.)

Report

1. Discuss the trends in reaction forces versus jet velocity. Is the trend consistent with the theory? Does it make sense?

2. How well does the theoretical model predict the measured force for the symmetric and asymmetric impact surfaces?

3. Do any of the measured values point to differences between the assumptions used to develop the theoretical models and the actual flow conditions? Be specific.