**Experiment: Hydrostatic Force on a Submerged Surface**

**Purpose**

The purpose of this experiment is to experimentally locate the center of pressure of a vertical submerged surface and compare the measurements to theoretical predictions.

**Learning Objectives**

The viscosity exercises involve measurements and analysis related to hydrostatics.After successfully completing the exercise students should be able to

* Measure moments about a pivot point
* Calculate the experimental and theoretical center of pressure
* Identify any discrepancies within the experimental results and provide a plausible explanation for observed discrepancies.

**Apparatus**

Figure 1 is a schematic of the Armfield hydrostatic pressure apparatus. It consists of an annular sector of a solid material attached to a balance beam. When the device is properly balanced the vertical face of the sector is directly below (coplanar with) the pivot axis. The solid sector and the balance beam are supported above a tank of water. The key geometrical parameters of the device are:

d: the submersion depth (distance from the bottom of the surface to the free surface)

h’: the depth of the center of pressure from the free surface

h”: the distance of the center of pressure below the pivot position

B: the width of the surface

D: the height of the surface

W: the weight (mg) of the hangar

L: is the distance from the pivot point to the hangar weight

 H: is the distance from the pivot point to the bottom of the vertical surface

 h: either ½ the distance from the bottom of the face to the free surface if the surface is partially submerged or equal to D/2 if the surface is fully submerged.



Figure 1. Diagram of the Armfield hydrostatic pressure apparatus with the key geometrical parameters defined with the surface partially submerged. *F* is the hydrostatic thrust and *mg* is the hangar weight

**Theory**

When the quadrant is immersed in water, it is possible to analyze the forces acting on the surfaces of the quadrant as: 1) the hydrostatic force at any point on the curved surfaces is normal to the surface and therefore resolves through the pivot point because the pivot point is located at the origin of the radii. Hydrostatic forces on the upper and lower curved surfaces therefore have no net torque effect; 2) the forces on the sides of the quadrant are equal and opposite horizontal forces; 3) the hydrostatic force on the vertical submerged face is counteracted by the balance weight. At equilibrium, the sum of the moments about the pivot point is zero. Thus,
where *m* is the hanging mass, *g* is the acceleration due to gravity, *L* is the distance from the pivot point to the hanging mass, and *h”* is the distance from the pivot point to the center of pressure. With the mass, balance length, and hydrostatic force determined, the location of the center of pressure on the end face may be determined for either a partially submerged vertical face or a fully submerged vertical face.

[1]

 **Partially Submerged Vertical Face**

For the case of a partially submerged face the hydrostatic thrust is defined as

[2]

where *A* is the wetted surface area of the vertical face (*B∙d*),  is the density of water, and h is the mean depth of immersion (*h = d*/2). Therefore the hydrostatic thrust is

[3]

Substituting Eqn. 3 into Eqn. 1 and solving for *h”* yields the experimentally determined distance between the pivot point and center of pressure

[4]

The theoretical depth of pressure below the free surface is

[5]

where *Ix* is the 2nd moment of area of immersed section about an axis in the free surface. Applying the parallel axes theorem yields

[6]

Substituting Eqn. 6 into Eqn. 5 yields

[7]

From geometry, the theoretical depth of center of pressure below the pivot point is

[8]

Combining Eqn. 7 and Eqn. 8 yields the theoretical depth of center of pressure

[9]

 **Fully Submerged Vertical Plane**

For the case where the vertical face of the quadrant is fully submerged, the hydrostatic thrust is

[10]

Substituting Eqn. 10 into Eqn. 1 the experimental distance between the pivot point and the center of pressure is

The theoretical depth of pressure below the free surface is given by Eqn. 5. Applying the parallel axes theorem for the fully submerged surface yields

[11]

[12]

Inserting Eqn. 12 into Eqn. 5 yields

[13]

Substituting Eqn. 13 into Eqn. 8 the theoretical depth of center of pressure below the pivot point is

[14]

**Procedure**

1. Measure all relevant geometric parameters
2. With the apparatus on a level surface, adjust the screwed feet until the built-in circular spirit level indicates that the tank is level in both planes.
3. Position the balance arm on the knife edges and check that the arm is free to swing.
4. Move the counter-balance weight until the balance arm is horizontal as indicated by the central index mark on the beam level indicator.
5. Add a small amount of weight to the weight hangar.
6. Slowly add water to the tank until the balance arm is horizontal as indicated by the central index mark on the beam level indicator. Note: avoid wetting the top or side surfaces of the quadrant.
7. Record the depth of immersion from the scale on the face of the quadrant or a ruler if necessary. Record the hanging weight.
8. Repeat steps 5, 6, and 7 as necessary. Try to achieve 10 measurements for the vertical surface for both the partially and fully submerged cases.

**Data Analysis**

Calculate the experimental and theoretical distances between the pivot point and center of pressures. Compare these values as a function of the water depth by plotting each against depth, and plotting the difference between the two as a function of depth. Which plot provides more insight into the discrepancy between experimental and theoretical distances? Note, a difference plot is a good way to compare two quantities that have nearly the same value. Is there an another relative measure to indicate the discrepancy? Conduct an error analysis of the measured distance and the theoretical distance. Are the results within error?

**Possible Discussion Pts**

Does the design of the apparatus significantly affect the measurement of the force on the vertical surface? Are any significant forces neglected? Explain any discrepancies observed? What is the effect of a measurement error? What is the dominant error in the measurements and theoretical prediction? Are the experimental and theoretical measurements consistent?