

Experiment: Viscosity Measurement B

The Falling Ball Viscometer

Purpose

The purpose of this experiment is to measure the viscosity of an unknown polydimethylsiloxane (PDMS) solution with a falling ball viscometer.

Learning Objectives

This laboratory exercise involves measurements and analysis related to fluid density, fluid viscosity, and hydrodynamic interactions.

After successfully completing this exercise students should be able to

- Determine the density of a spherical particle.
- Determine the density and viscosity of an unknown fluid.
- Determine the uncertainty in the density and viscosity measurements
- Identify any discrepancies within the experimental results and provide a plausible explanation for the observed discrepancies.

Apparatus

Figure 1 is a schematic of a falling ball viscometer. A sphere of known density and diameter is dropped into a large reservoir of the unknown fluid. At steady state, the viscous drag and buoyant force of the sphere is balanced by the gravitational force. In this experiment, the speed at which a sphere falls through a viscous fluid is measured by recording the sphere position as a function of time. Position is measured with a vertical scale (ruler) and time is measured with a stopwatch.

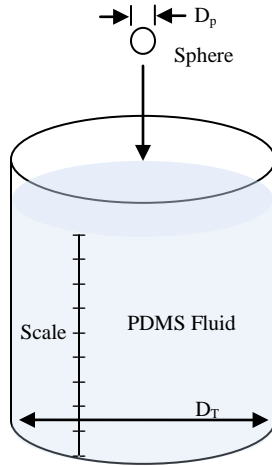


Figure 1. Schematic of a falling ball viscometer where a sphere of diameter, D_p , is dropped into a tank of diameter, D_T . The sphere position is measured with the vertical scale at known times.

Theory

When a sphere is placed in an infinite incompressible Newtonian fluid, it initially accelerates due to gravity. After this brief transient period, the sphere achieves a steady settling velocity (a constant terminal velocity). For the velocity to be steady (no change in linear momentum), Newton's second law requires that the three forces acting on the sphere, gravity (F_G), buoyancy (F_B), and fluid drag (F_D) balance. These forces all act vertically and are as follows:

$$\text{gravity:} \quad F_G = -\frac{\pi}{6} \rho_p D_p^3 g \quad (1)$$

$$\text{buoyancy:} \quad F_B = +\frac{\pi}{6} \rho D_p^3 g \quad (2)$$

$$\text{fluid drag:} \quad F_D = \frac{\pi}{8} \rho V_p^2 D_p^2 C_D \quad (3)$$

where ρ_p is the density of the solid sphere, ρ is the density of the fluid, D_p is the diameter of the solid sphere, g is the gravitational acceleration (9.8 m/s^2), V_p is the velocity of the sphere, and C_D is the drag coefficient. The gravitational force is equal to the weight of the sphere, and the sign is negative because it is directed downward. The buoyancy force acts upwards and is equal to the weight of the displaced fluid. The drag force acts upwards and is written in terms of a dimensionless drag coefficient. The drag coefficient is a unique function of the dimensionless Reynolds number, Re . The Reynolds number can be interpreted as the ratio of inertial forces to viscous forces. For a sphere settling in a viscous fluid the Reynolds number is

$$Re = \rho V_p D_p / \mu \quad (4)$$

where μ is the viscosity of the fluid. If the drag coefficient as a function of Reynolds number is known the terminal velocity can be calculated. For the Stokes regime, $Re < 1$, the drag coefficient can be determined either analytical (as will be shown later in the course) or empirically. Under these conditions $C_D = 24/Re$ and the settling velocity is

$$V_p = g D_p^2 (\rho_p - \rho) / 18\mu \quad (5)$$

In the intermediate region, $1 < Re < 1000$ the drag coefficient is roughly calculated empirically as $C_D = Re^{-0.6}$ with the corresponding settling velocity

$$V_p \approx \left[\frac{2g}{27} \left(\frac{\rho_p}{\rho} - 1 \right) \right]^{5/7} D_p^{8/7} \left(\frac{\rho}{\mu} \right)^{3/7} \quad (6)$$

The falling ball viscometer is one of the practical applications of either Eqn.(5) or Eqn. (6). The falling ball viscometer requires the measurement of a sphere's terminal velocity, usually by measuring the time required for sphere to fall a given distance. Falling sphere viscometers are in common use as are viscometers that use the rise time of a bubble. In this experiment we measure the position of a sphere as a function of time and determine the steady state settling velocity. From this, we can calculate the viscosity from either Eqn. (5) or Eqn. (6) depending on the Reynolds number. For $Re < 1$ the viscosity would be

$$\mu = g D_p^2 (\rho_p - \rho) t_p / 18L \quad (7)$$

where t_p is the time required for a sphere to fall a distance, L . If $1 < Re < 1000$, the viscosity could be determined from

$$\mu = \left[\frac{2g}{27} \left(\frac{\rho_p}{\rho} - 1 \right) \right]^{5/3} D_p^{8/3} \left(\frac{L}{t_p} \right)^{-7/3} \rho \quad (8)$$

Note: The proceeding analysis assumes that the fluid reservoir is an incompressible Newtonian fluid of infinite size. We considered the fluid to be infinite to neglect hydrodynamic interactions between the sphere and the container wall. For practical purposes, the surroundings can be considered infinite if the nearest wall is at least 20 sphere diameters away. In any real situation we may have to account for possible influence of the container walls. From dimensional analysis (to be discussed later in lecture), we would expect the drag coefficient to be corrected by a function which would depend on the ratio of the container diameter to the sphere diameter.

Procedure

Regardless of the Re , the settling velocity depends on the sphere diameter, the sphere density, the fluid density, and the gravitational constant. For each sphere examined

1. Measure the diameter of the sphere. Measure it multiple times to gain an accurate measurement and to determine the relative error in the measurement.
2. Measure the weight of the sphere
3. Calculate the sphere density

Since only one fluid will be examined, measure the fluid density by weighing a known volume fluid. With the fluid density and the particle density determined, the viscosity can be determined by measuring the position of the sphere as a function of time as it settles through the unknown fluid. For each sphere

1. Place the sphere near the top of the fluid reservoir. Try to get the sphere as close as possible to the air-fluid interface.
2. Release the sphere and begin timing.
3. As the sphere settles, record its position as a function of time. Note: some spheres may settle faster than others and may be difficult to measure without knowledge of its settling velocity. For the denser spheres drop a similar sphere to initially estimate its settling rate. In addition, it may be more efficient to have one person drop the sphere, one person run the stopwatch, and the third to read the time off the stopwatch.

A number of spheres are available to measure the viscosity. For this experiment measure the settling velocity for the range of sphere diameters provided. For each sphere size selected run multiple spheres (2-4) to provide sufficient data for the terminal velocity of each sphere type.

Data Reduction

1. For each sphere examined, plot the position as function of time and determine the terminal velocity of the sphere from the slope (assuming the data is linear).
2. From the terminal velocity, calculate the viscosity of the unknown fluid for each sphere using Eqn. (7). Is the equation valid for the measured velocities and calculated viscosity?
3. Are the measured viscosities consistent? If the measured viscosities aren't consistent, plot the measured viscosity for each sphere as a function of sphere diameter. Does the data approach a constant value at large or small sphere diameters?

Report

1. Include a figure showing the sphere position as a function of time and discuss the valid range useful for measuring the terminal velocity.
2. Report the viscosity of the PDMS fluid.

3. Which measurements (data points, results) are most reliable? Which measurements are most limited by your ability to measure time, diameter, or mass? Justify your answers.
4. Discuss discrepancies and anomalies in your data.
5. Does the measured viscosities depend on the size of the sphere? Is it independent of size for a given size range? If the viscosity data shows a size dependence, what is plausible cause for the observed dependence?
6. Compare the measured viscosity to viscosity measured with the Thomas-Stormer viscometer. Within error, are there any discrepancies? What are the measurement errors associated with each technique?