ECE 510 Lecture 8
Acceleration, Maximum Likelihood

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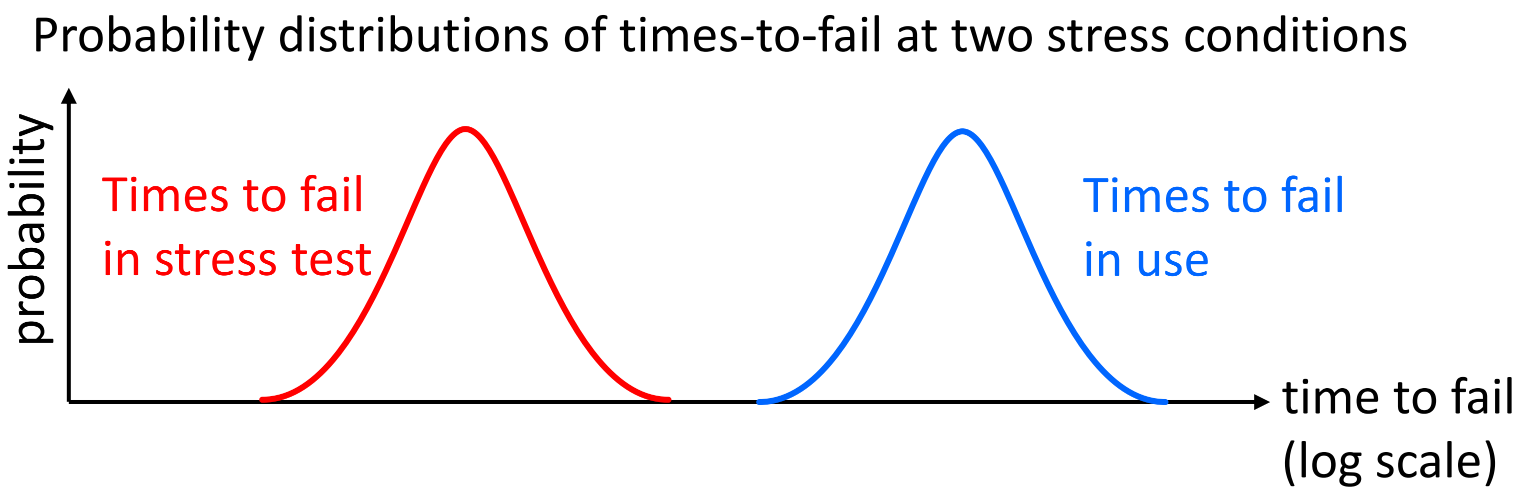
Acceleration Concept

Stress and Failure

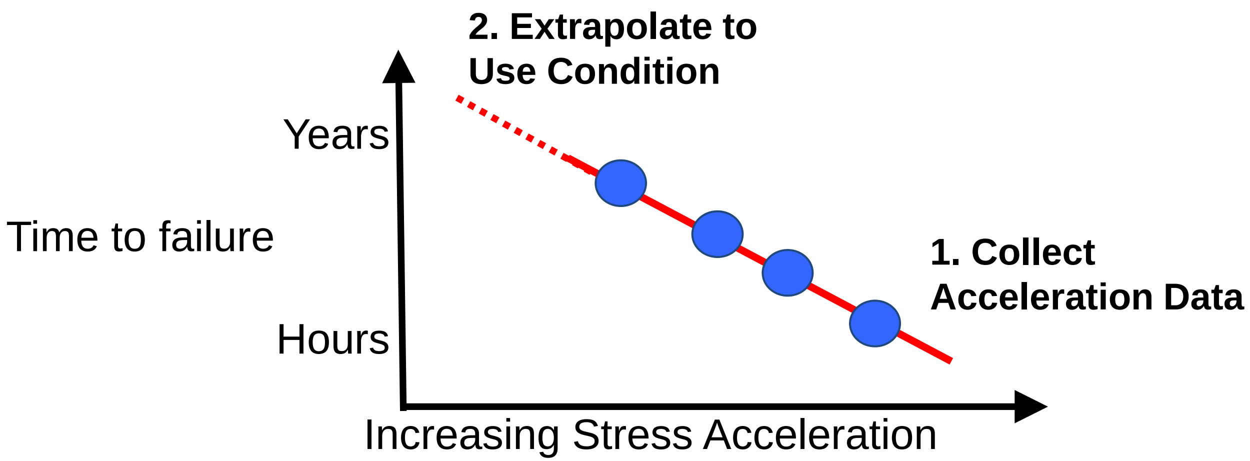
- How long is our product going to last?
- We can’t wait until it fails to see – that takes too long!
- We need to identify the stresses that cause it to fail
  - …and then apply them harder to make our parts fail in a reasonable amount of time
- Our stresses include
  - Voltage
  - Temperature
  - Current
  - Humidity
  - Mechanical stress
  - …and others

Reliability Models



- Knowledge-based qual based on a *reliability model*
  - Model is built at one test condition
  - It can be scaled ("*accelerated*") to other use conditions
- Models are built from data from reliability tests

Accelerated Test



- Accelerated test increases one or more conditions (e.g., T, V, etc.) to reduce times to failure

  Life Test (years) → Accelerated Test (hours)

- Intention is to accelerate a mechanism without inducing new mechanisms

Semiconductor Failure Mechanisms

Category	**Mechanism**	**Cause**	**Stress**
Constant	Electrical Overstress	ESD and Latchup	V, I
IM	Infant Mortality	Extrinsic Defects	V, T
Wearout	Hot Carrier	e- Impact ionization	V, I
Wearout	Neg. Bias-T Instability	Gate dielectric damage	V, T
Wearout	Electromigration	Atoms move by e- wind	I, T
Wearout	Time-Dep Diel. B'down	Gate dielectric leakage	V, T
Wearout	Stress Migration	Metal diffusion, voiding	T
Wearout	Interlayer Cracking	Interlayer stress	ΔT
Wearout	Solder Joint Cracking	Atoms move w/ stress	ΔT
Wearout	Corrosion	Electrochemical reaction	V, T, RH
Constant	Soft Error	n & α e-h pair creation	radiation

V = Voltage, I = Current, T = Temperature, ΔT = Temp cycle, RH = Relative Humidity

Reliability Tests

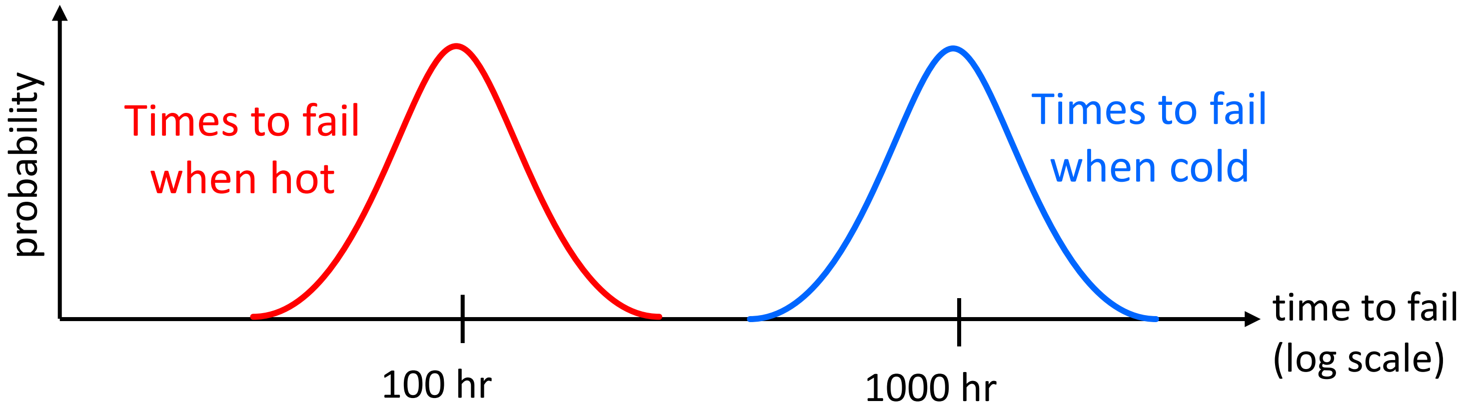
Name	**Count**	**Time and Stress**	**Mechanisms**
Infant Mortality Experiment	~10,000 units	48 hr at hi-V, hi-T	Latent reliability defects (IM)
Extended Life Test	~300 units	500 hr at hi-V, hi-T	Wearout (oxide, PBT, Fmax, Vccmin)
Test structure stress tests	100's of devices	Hours at hi-V, hi-T	Oxide breakdown, PMOS bias-temp, electromigration, other silicon mechs
Bake	~300 units	100's of hours at hi-T	TIM degradation, cracking and delaminating
Highly Accelerated Stress Test (HAST)	~300 units	50-150 hr at hi-T, hi-RH	Metal migration, adhesion fail
Temperature Cycling	~300 units	~1000 cycles -55C to 125C	Cracks anywhere, TIM degradation

Accelerated Testing Pitfalls

- Different mechanisms might accelerate differently
- No universal test:
  - Stress tests are idealizations of real life
  - Some mechanisms might get too much acceleration
  - Single stress does not stimulate all relevant behaviors
  - May not comprehend effects like materials creep
- The most accurate data is the most difficult or unrealistic to acquire:
  - Long test times are required at low acceleration conditions

Acceleration Calculation

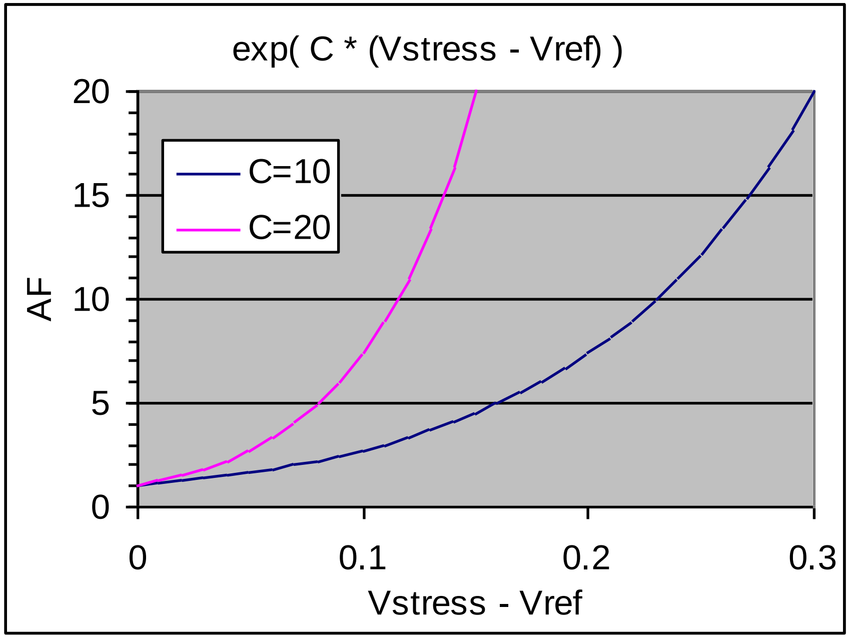
Acceleration Factor



$$AF = \frac{t_{cold}}{t_{hot}} = \frac{1000\text{hr}}{100\text{hr}} = 10$$

- An acceleration factor describes how much a particular stress accelerates degradation or failure

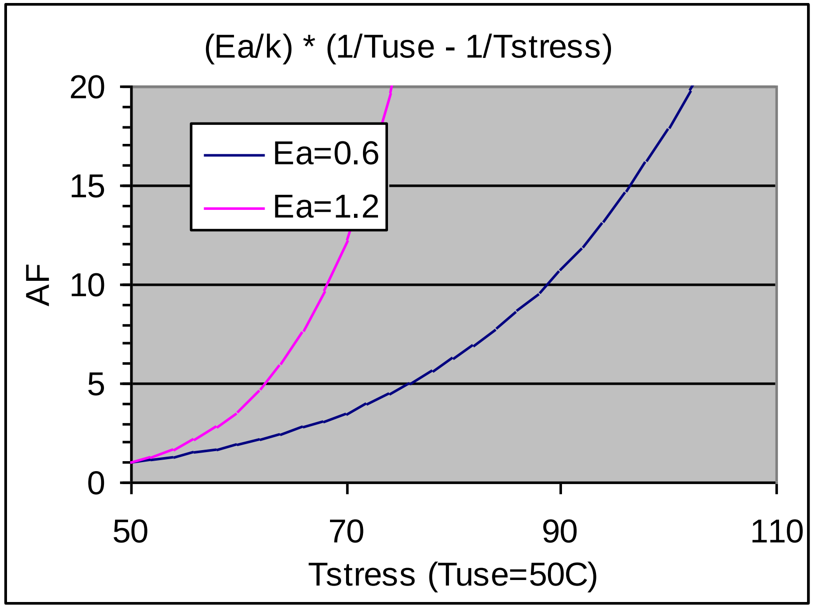
Voltage Acceleration Model



$$AF = \exp\left\{C \times \left(V_{stress} - V_{use}\right)\right\}$$

- Acceleration models are determined empirically
- Voltage acceleration is usually exponential, like this example

Temperature Acceleration Model

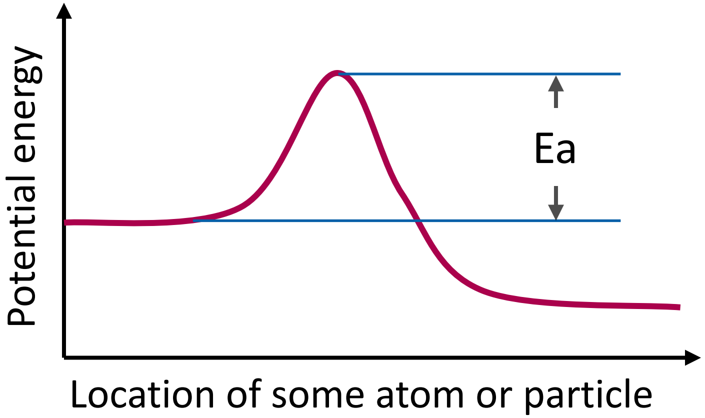


Activation energy

$$AF = \exp\left\{\left(\frac{E_a}{k}\right)\times\overbrace{\left(\frac{1}{T_{use}}-\frac{1}{T_{stress}}\right)}^{\text{Must be Kelvin}}\right\}$$

Boltzmann constant

$k=8.62\text{x}10^{-5}$ eV/K



- Temperature acceleration is usually like a chemical reaction
  - Arrhenius model with an energy barrier

Exercise 8.1

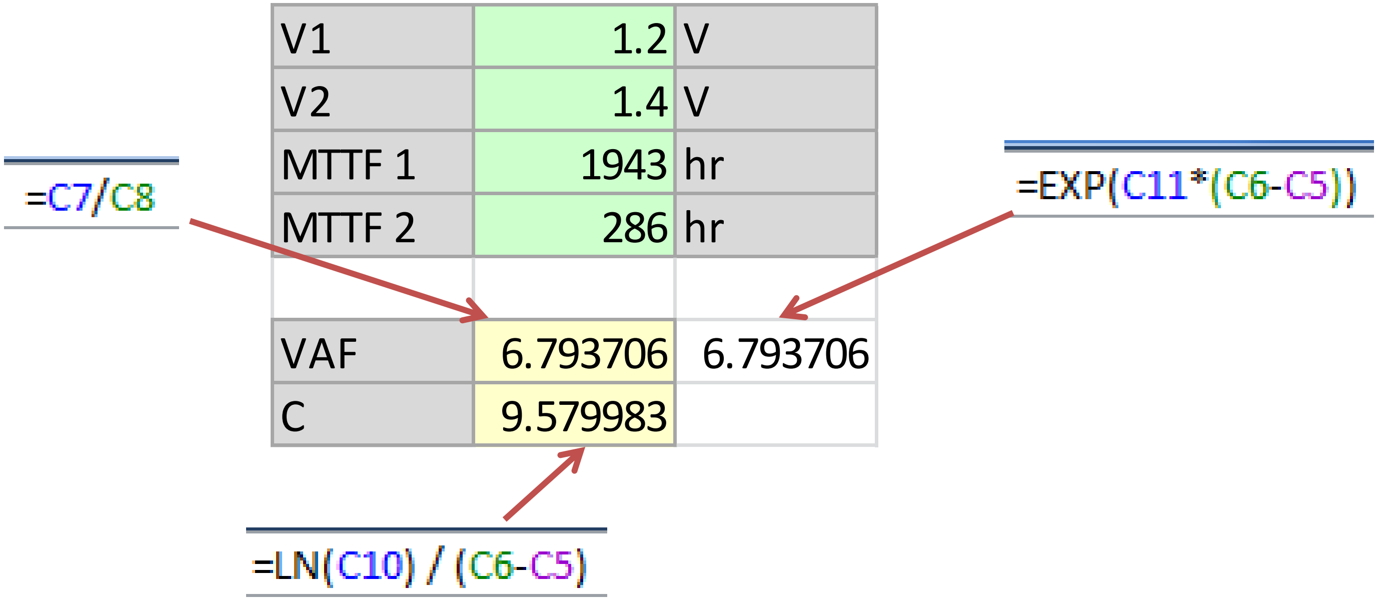
If two samples of devices give these MTTFs:

- 1943 hours at 1.2V
- 286 hours at 1.4 V

find the

- Voltage Acceleration Factor (VAF)
- Constant C in the an exponential voltage acceleration model

Solution 8.1



V1	1.2	V
V2	1.4	V
MTTF 1	1943	hr
MTTF 2	286	hr
VAF	6.793706	6.793706
C	9.579983	

=C7/C8

=EXP(C11*(C6-C5))

=LN(C10) / (C6-C5)

Exercise 8.2

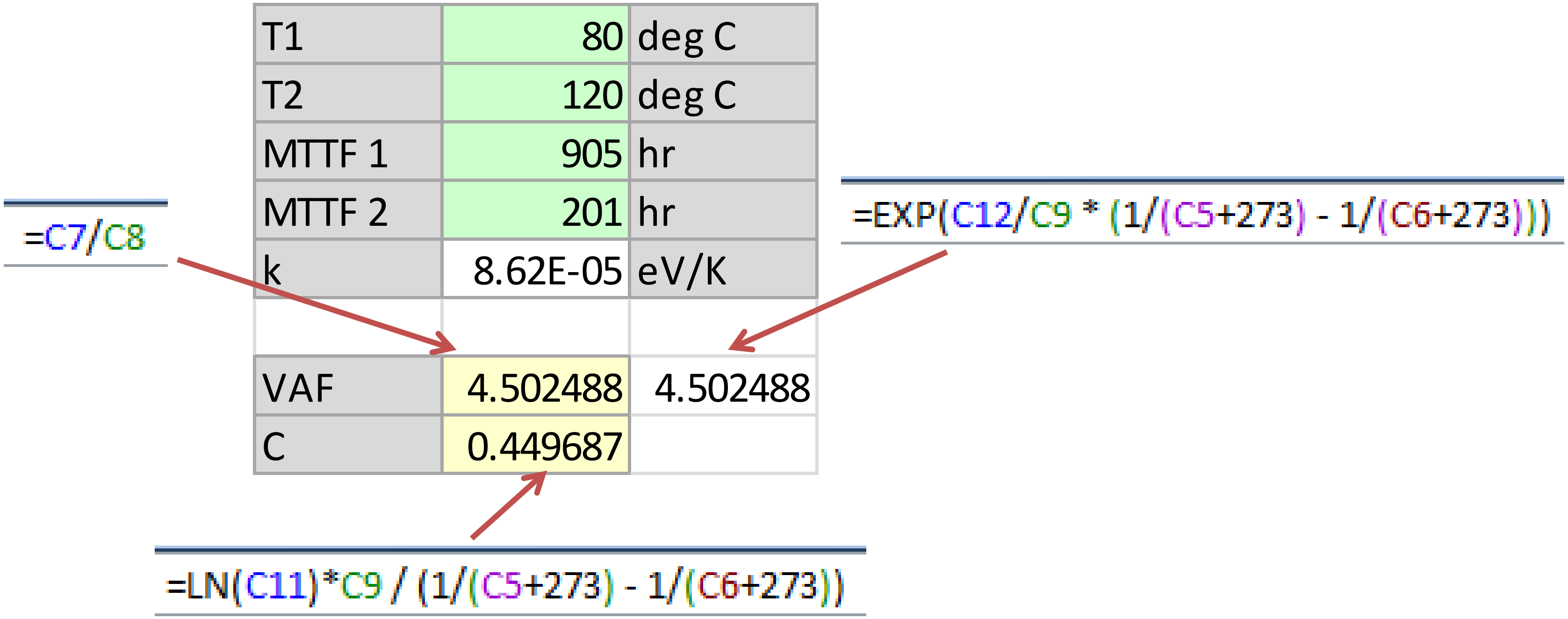
If two samples of devices give these MTTFs:

- 905 hours at 80 deg C
- 201 hours at 120 deg C

find the

- Temperature Acceleration Factor (TAF)
- Activation energy Ea in the an Arrhenius temperature acceleration model

Solution 8.2



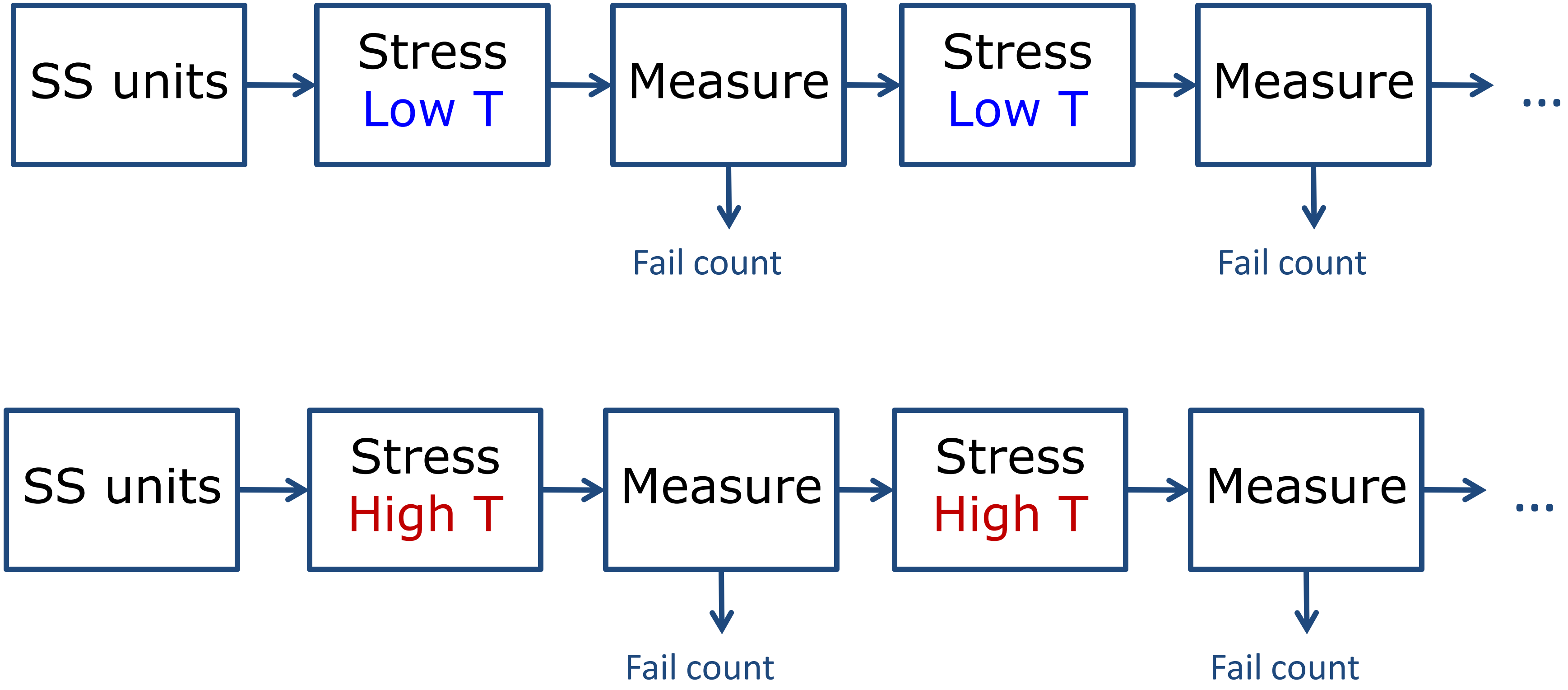
T1	80	deg C
T2	120	deg C
MTTF 1	905	hr
MTTF 2	201	hr
k	8.62E-05	eV/K
VAF	4.502488	4.502488
C	0.449687	

=C7/C8

=EXP(C12/C9 * (1/(C5+273) - 1/(C6+273)))

=LN(C11)*C9 / (1/(C5+273) - 1/(C6+273))

Acceleration Experiment



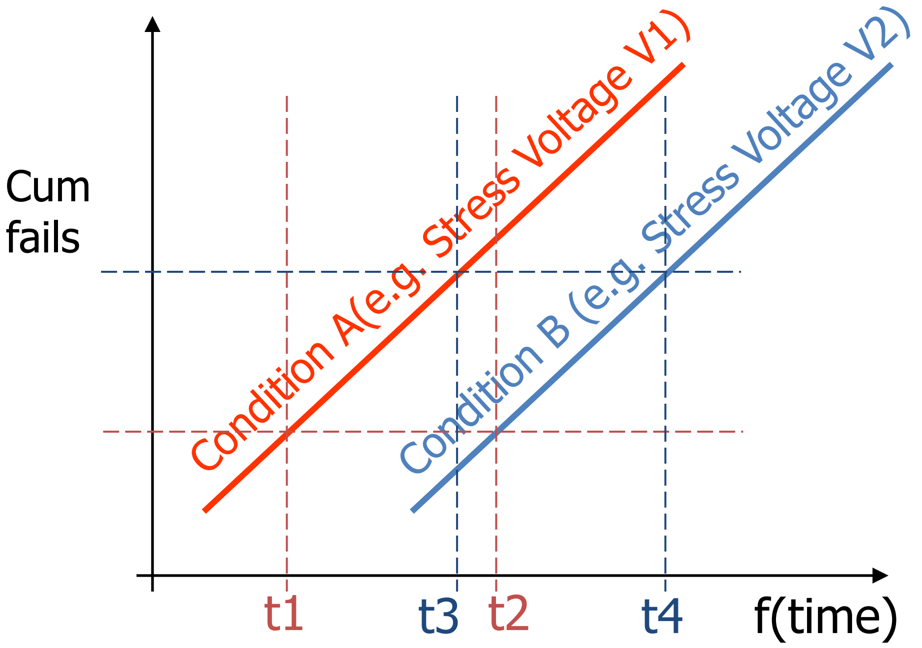
SS units → Stress Low T → Measure → Stress Low T → Measure → …

Measure → Fail count

SS units → Stress High T → Measure → Stress High T → Measure → …

Measure → Fail count

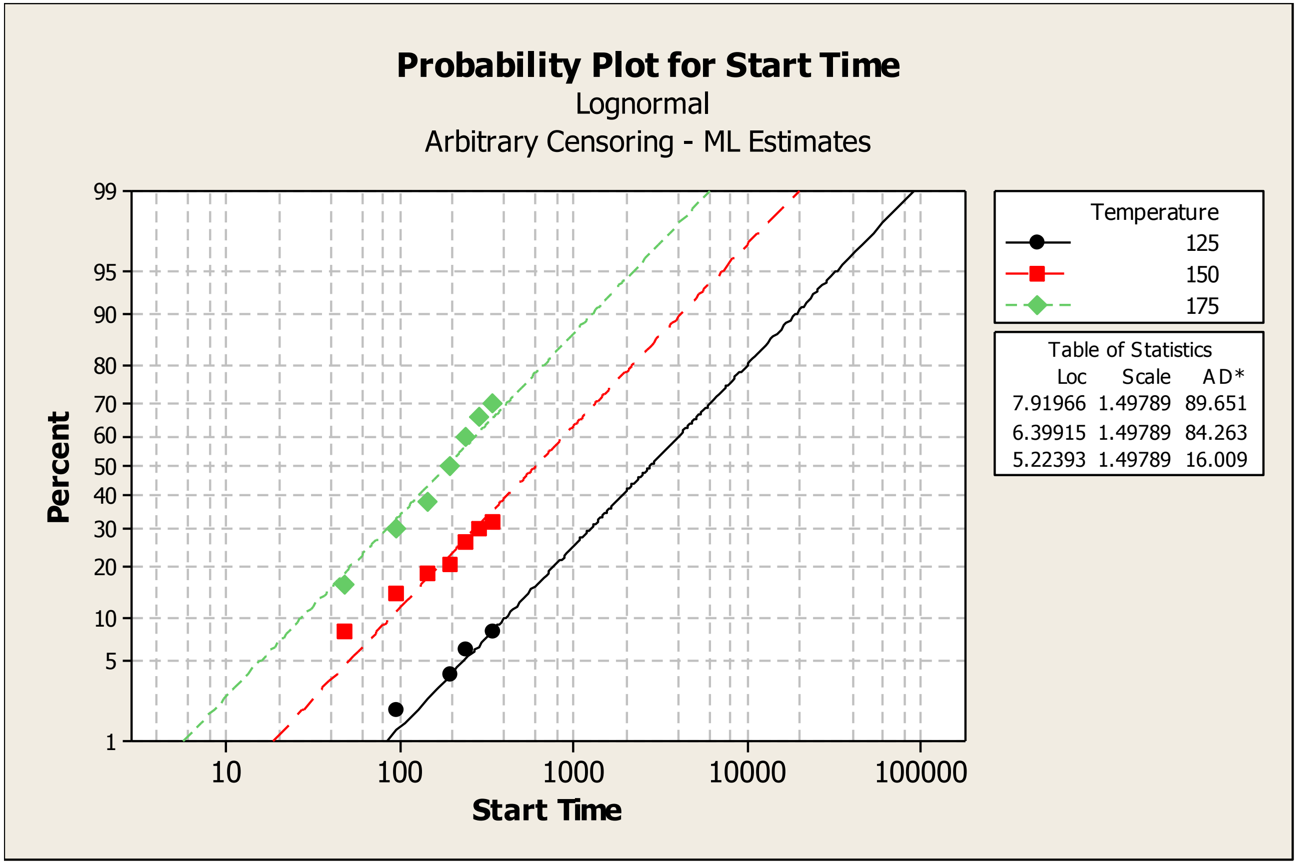
Acceleration Concept



$$AF = \frac{t2}{t1} = \frac{t4}{t3}$$

- Distributions at both conditions must match for acceleration concept to make sense

Acceleration Example



A temperature acceleration experiment showing the same distribution shape (slope) at each stress temp

Accelerated Stress Testing

- Special-purpose equipment accelerates various fail mechanisms

An LCBI burn-in system gives V and T stress to accelerate Si fail mechanisms

A HAST system gives pressure and humidity along with V and T to accelerate package fail mechanisms

Maximum Likelihood Method and the Exponential Distribution

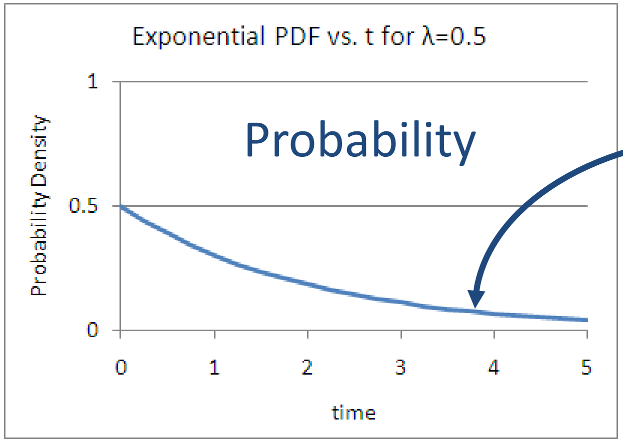
MLE

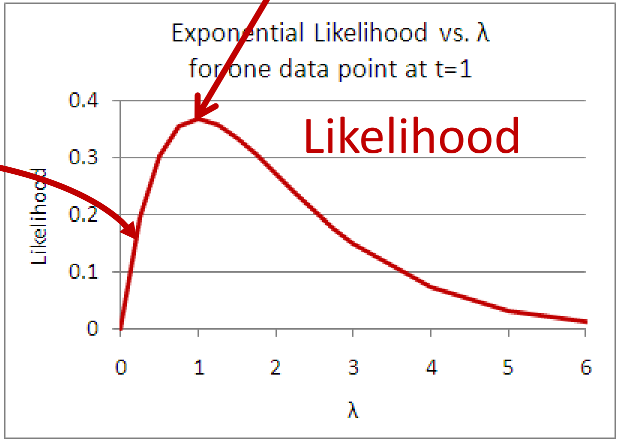
- Maximum Likelihood Estimation (MLE) is a fitting technique that is good for any model
- Principle
  - We can’t ask: What is the most likely model?
    - Because we don’t have some well-defined space of possible models
  - We can ask: Given this model, how likely is this data set?
  - (This is a fairly Bayesian approach. We are usually frequentists.)

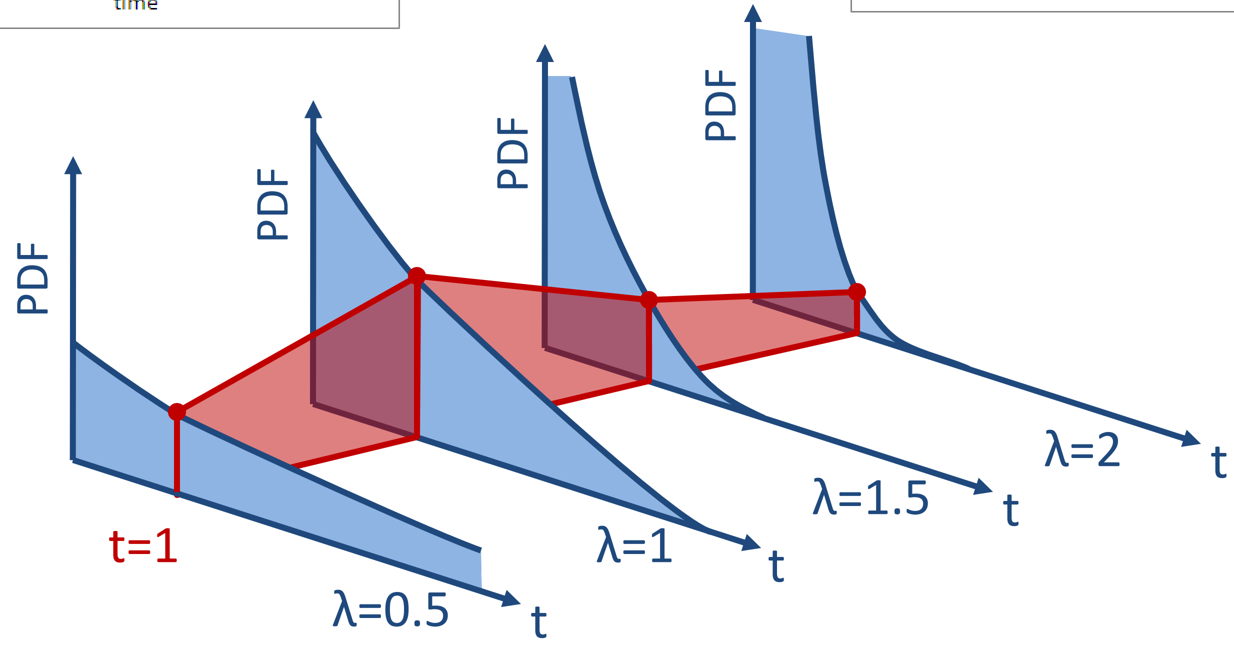
Probability vs. Likelihood

Maximum likelihood

$\lambda e^{-\lambda t}$







MLE

- Likelihood for each point
  - For exact values (exact times to fail), use the PDF
  - For ranges (failed between two readout times), use CDF delta
  - Multiply all together (or add logs)
- Use
  - Choose a model functional form with adjustable parameters
  - Adjust the parameters to maximize the likelihood

MLE for Exponential Data

- For a complete set of times to fail, likelihood is the PDF:

$$PDF_i = \lambda\, e^{-\lambda t_i}$$

- Take log of PDF:

$$\ln PDF_i = \ln \lambda - \lambda t_i$$

- Add up likelihood for each data point:

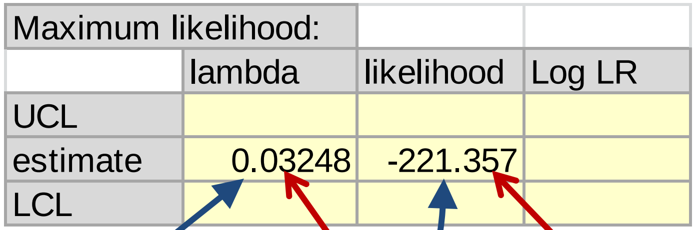
$$\text{Device hours} = \sum_i t_i$$

$$L = \sum_i \ln PDF_i = \sum_i \left(\ln \lambda - \lambda t_i\right) = N \ln \lambda - \lambda \sum_i t_i$$

$$\text{Sample Size} = N$$

- Then choose λ to maximize L

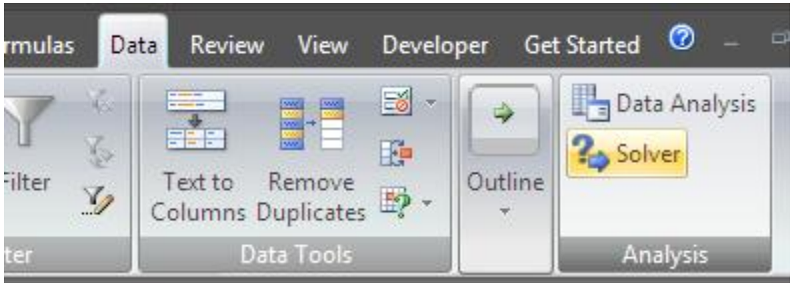
Ex 8.3a – MLE for Exponential

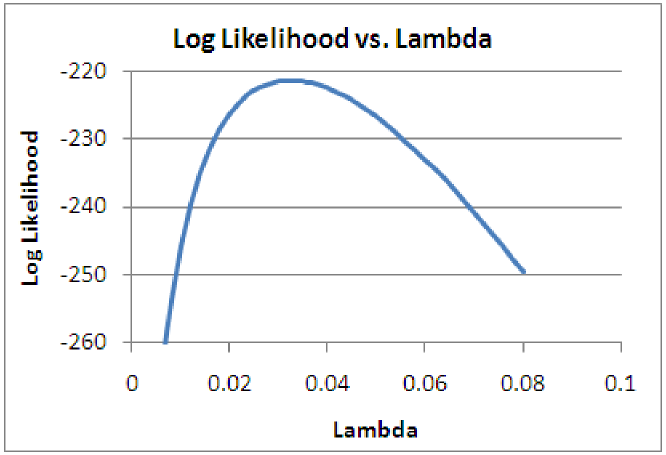


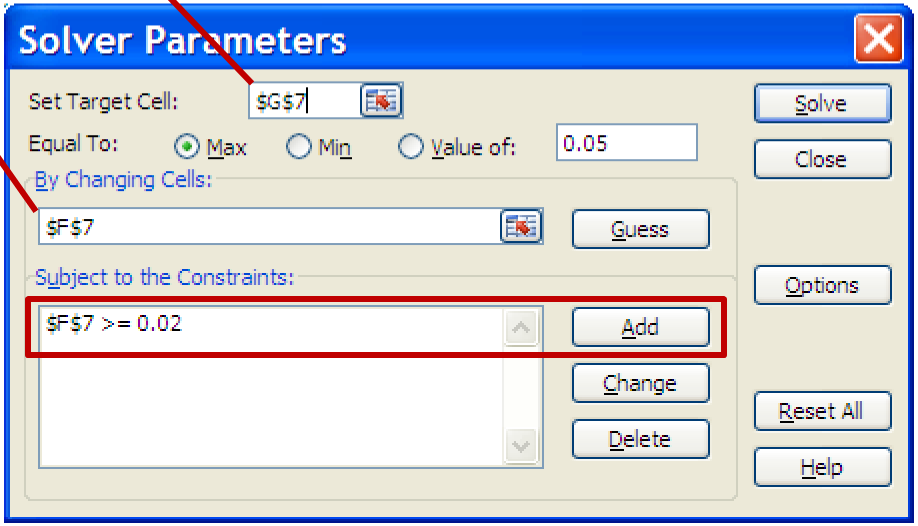
Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			

guess

```
=$C$3*LN(F8) - F8*$C$4
```







Set Target Cell: G7

Equal To: ● Max ○ Min ○ Value of: 0.05

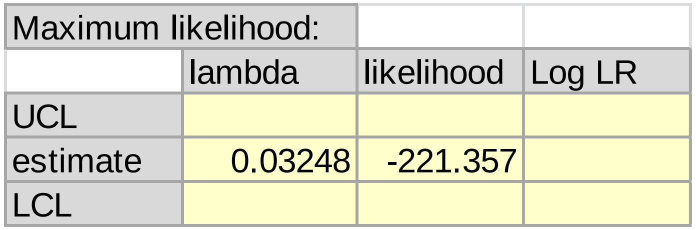
By Changing Cells:

F7

Subject to the Constraints:

F7 >= 0.02

Solution 8.3a



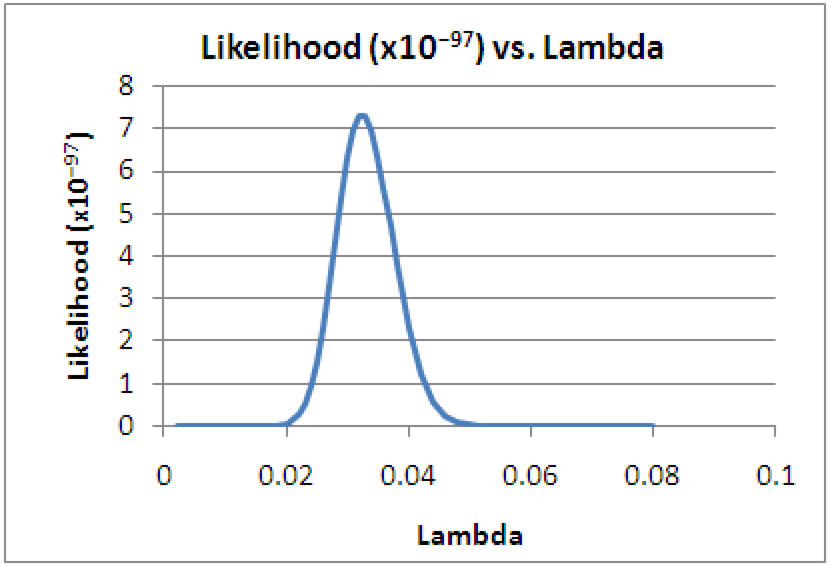
Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			

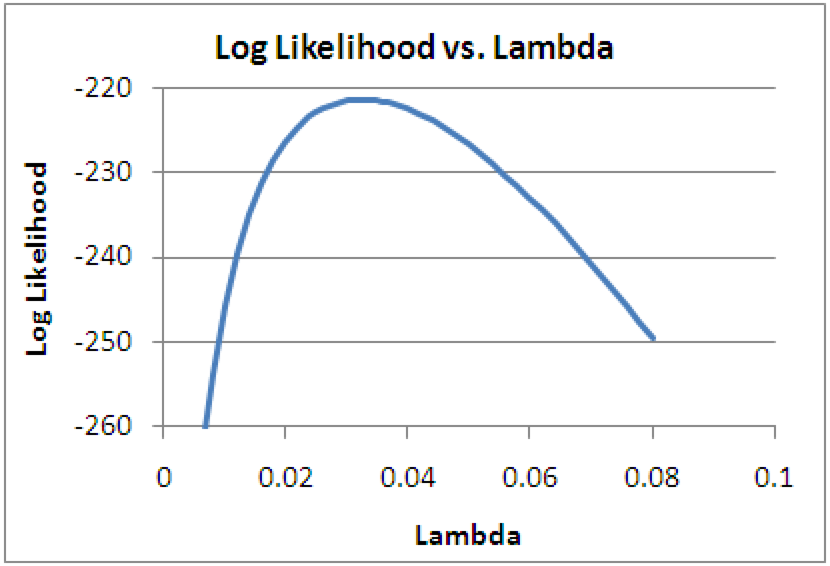
λ = 0.032 per hour = 3.2% per hour

MTTF = 1/λ = 30.8 hours

Graphs of Likelihood vs. Lambda

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			





Analytic λ

- For exponential, can maximize analytically:

$$L = N\ln\lambda - \lambda\sum_i t_i$$

$$\frac{dL}{d\lambda} = \frac{N}{\lambda} - \sum_i t_i = 0$$

$$\lambda = \frac{N}{\sum_i t_i} = \frac{\text{Number of fails}}{\text{Total device hours}}$$

Even works for type I censored data

Exercise 8.3b

- Calculate λ for the Ex 8.3 data set using the analytic expression and compare it to what you got from the MLE technique

Solution 8.3b

- Same as MLE technique

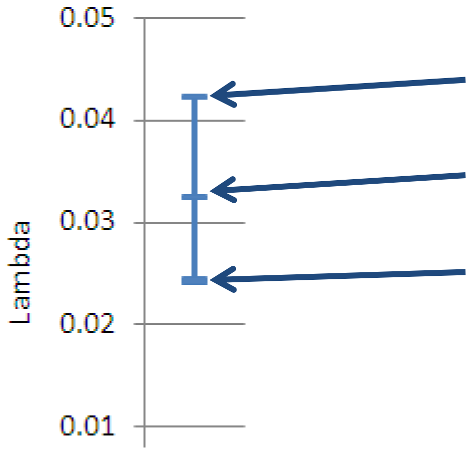
Analytic

fail count	50
device hours	1539.413
lambda (fails / dev hrs)	0.03248

MLE

Maximum likelihood:			
	lambda	likelihood	Log LR
UCL			
estimate	0.03248	-221.357	
LCL			

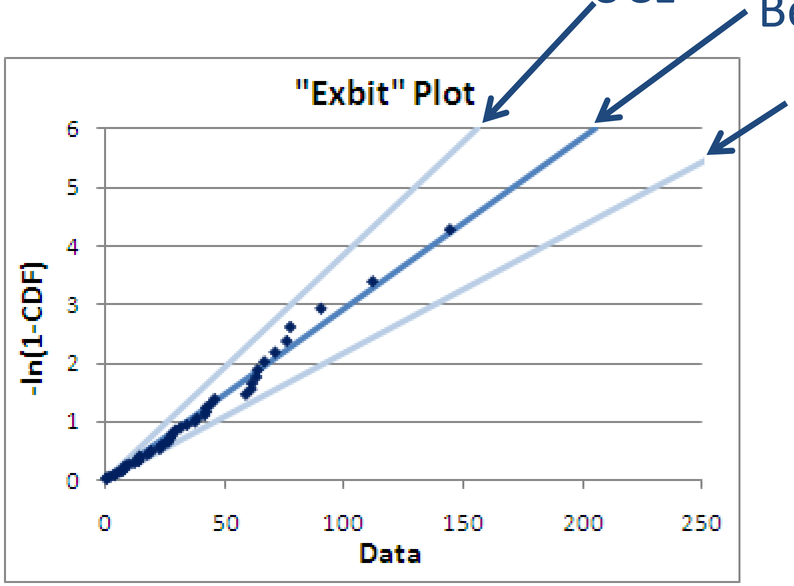
Uncertainty Range of Lambda



Upper Confidence Limit (UCL)

Best estimate

Lower Confidence Limit (LCL)

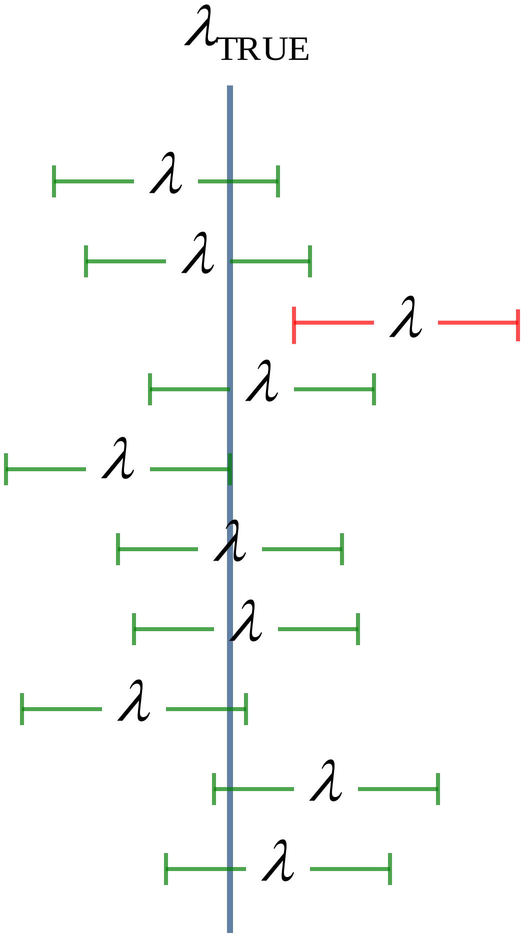


UCL

Best estimate

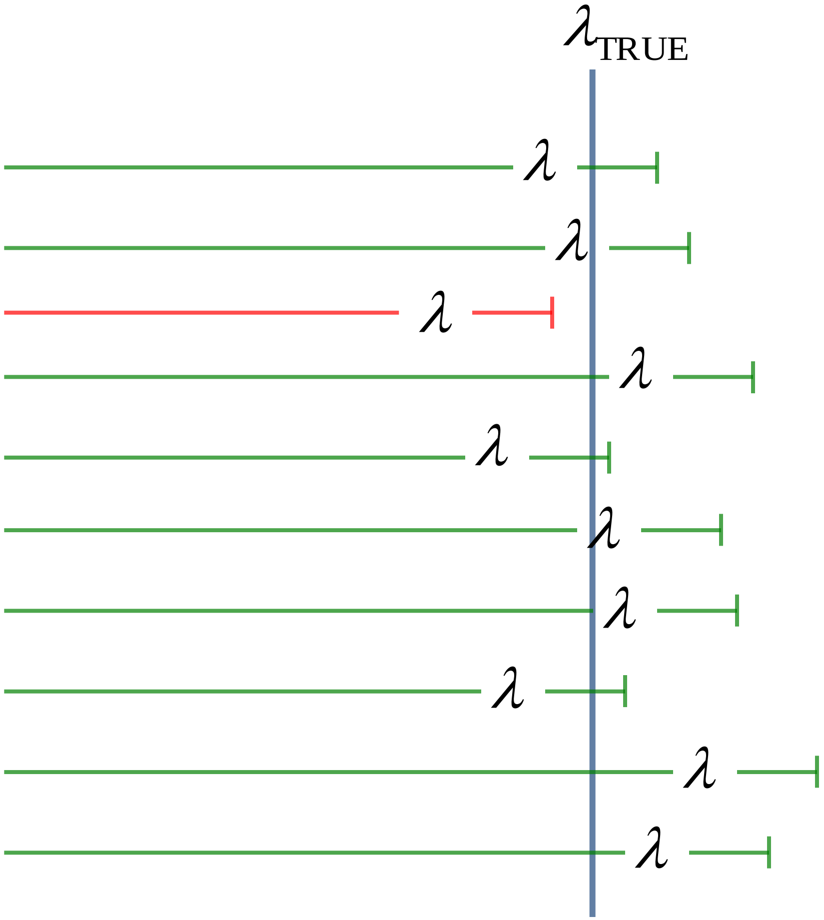
LCL

Confidence Interval (2-Sided)



- 90% of random sample λ's with this confidence interval include the true population λ

Confidence Interval (1-Sided)

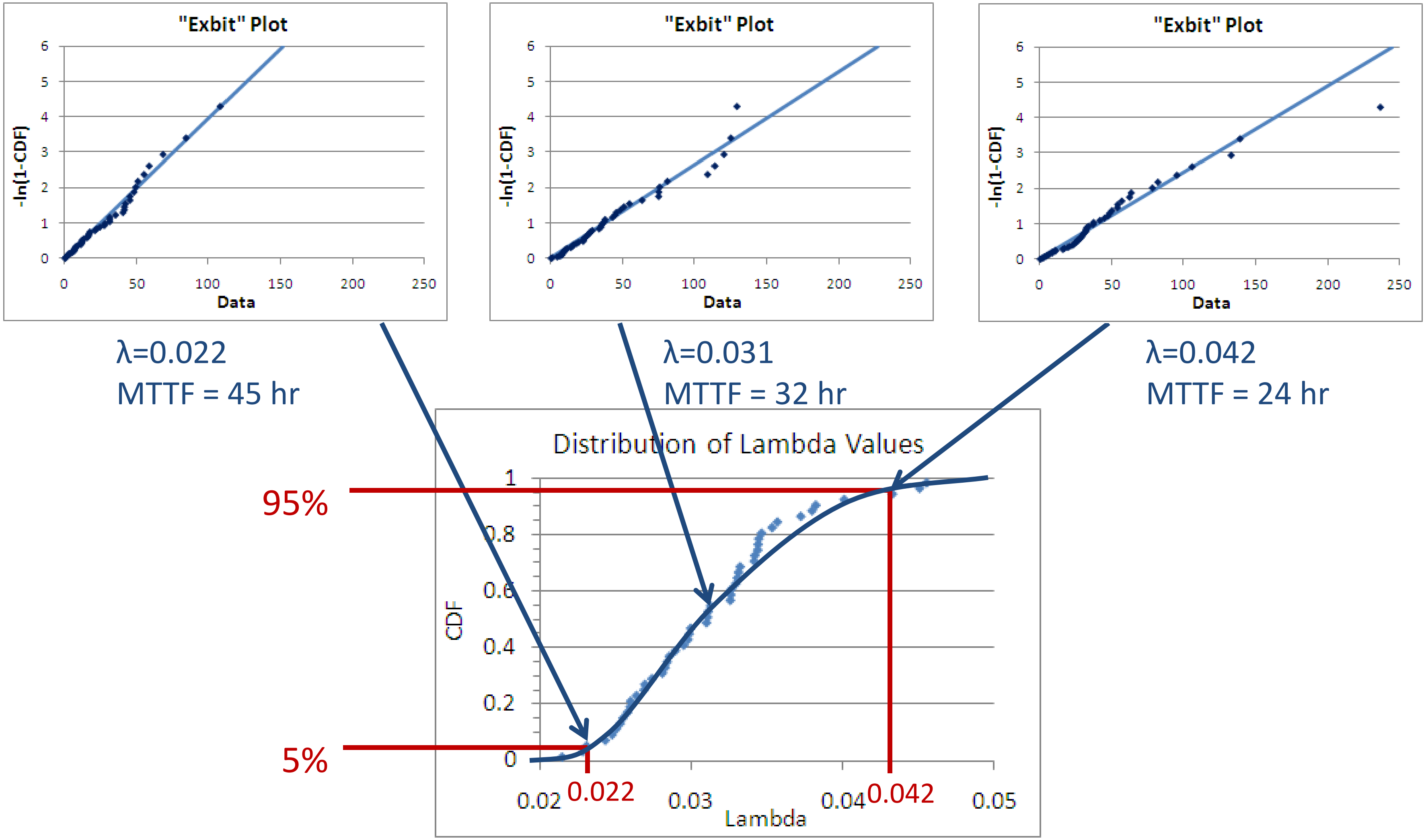


Uncertainties on Parameters

To calculate:

- Monte Carlo
- Likelihood ratio
- Analytic

Recall Monte Carlo Lambda Uncertainty

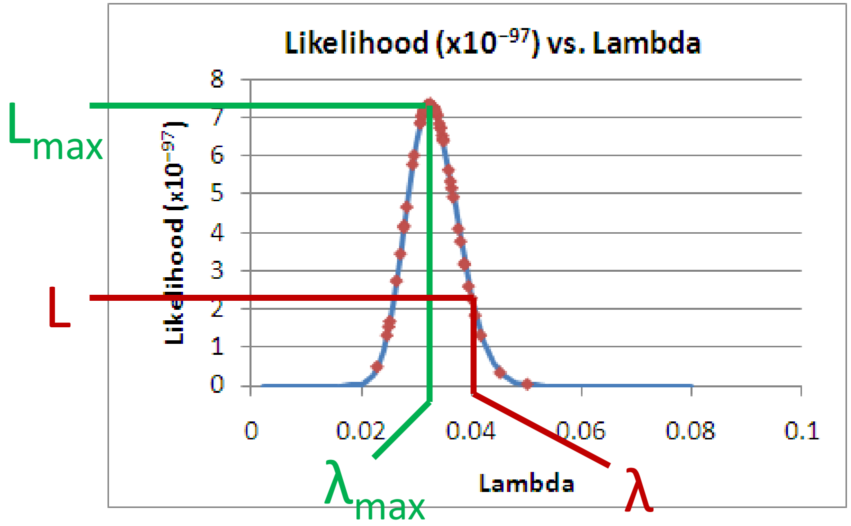


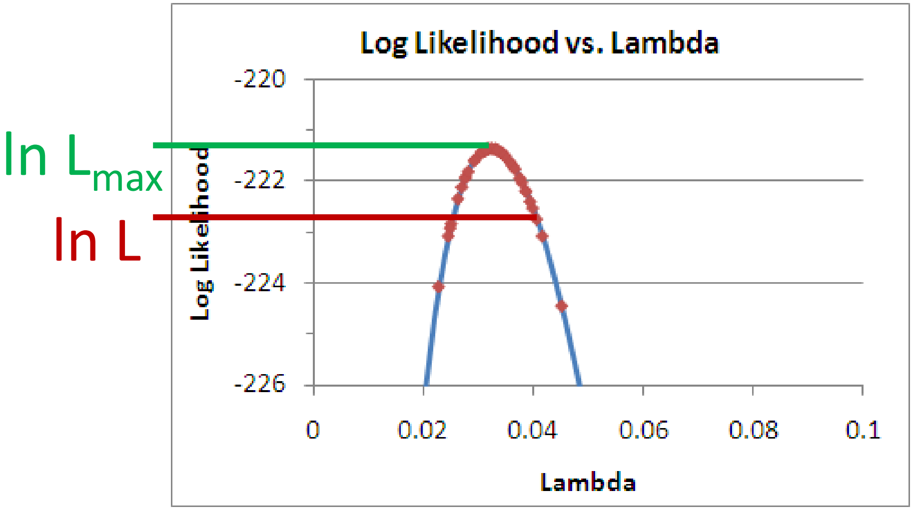
λ=0.022
MTTF = 45 hr

λ=0.031
MTTF = 32 hr

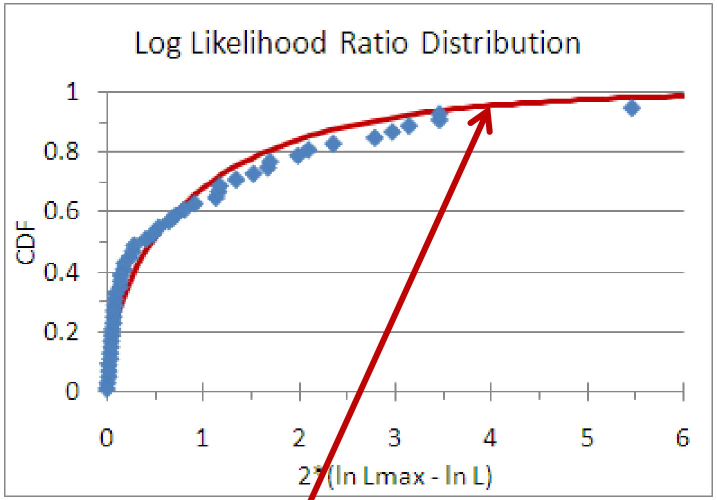
λ=0.042
MTTF = 24 hr

Likelihood Ratio Lambda Uncertainty


Likelihood (x10^{-97}) vs. Lambda


Log Likelihood vs. Lambda

$$\ln\left(\frac{L_{max}}{L}\right)^2 = 2\times\left(\ln L_{\max} - \ln L\right)$$

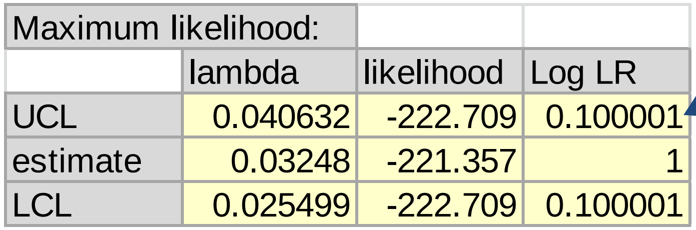

Log Likelihood Ratio Distribution

1–CHIDIST(Log LR, 1)

Number of parameters in model (=1 for exponential)

Exercise 8.3c

- Calculate UCL and LCL for lambda:
  - Calculate Log LR for each (below)
  - Choose lambda for each to set Log LR = 0.1
    - Do by hand first, then use Solver to fine-tune



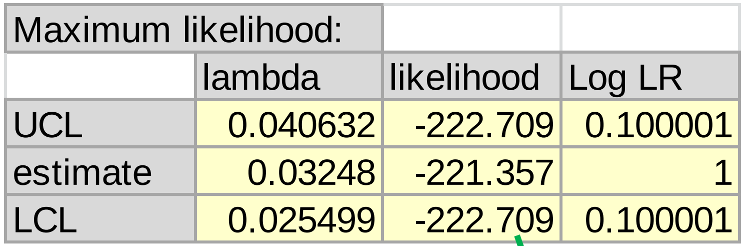
Maximum likelihood:			
	lambda	likelihood	Log LR
UCL	0.040632	-222.709	0.100001
estimate	0.03248	-221.357	1
LCL	0.025499	-222.709	0.100001

Likelihood of best estimate

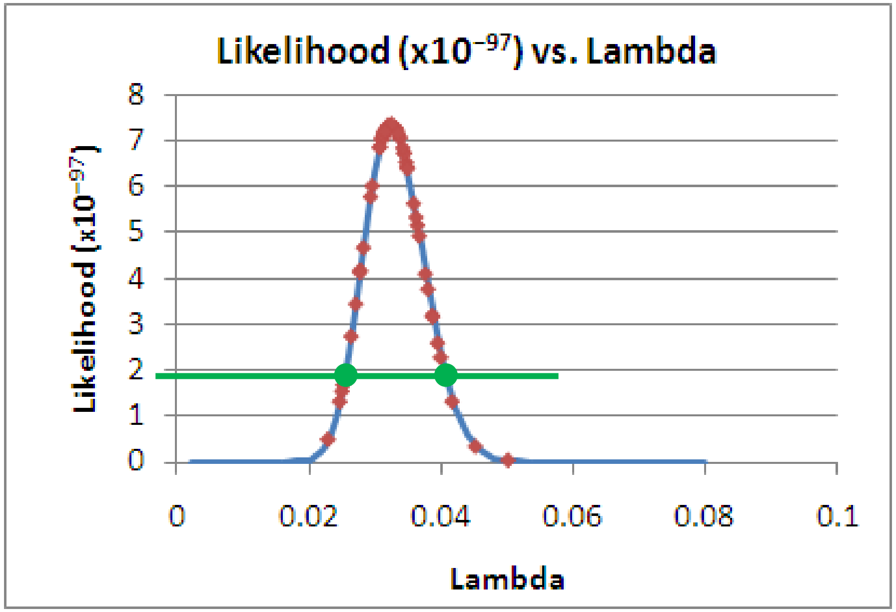
Likelihood of UCL

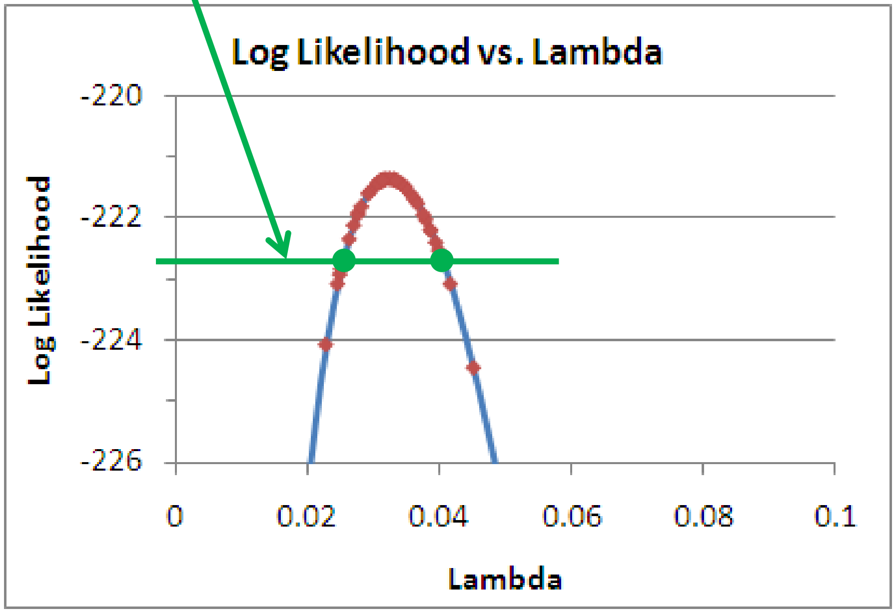
=CHIDIST(2*(G8-G7), 1)

Solution 8.3c



Maximum likelihood:			
	lambda	likelihood	Log LR
UCL	0.040632	-222.709	0.100001
estimate	0.03248	-221.357	1
LCL	0.025499	-222.709	0.100001





Analytic Lambda Uncertainty

for 90% CL

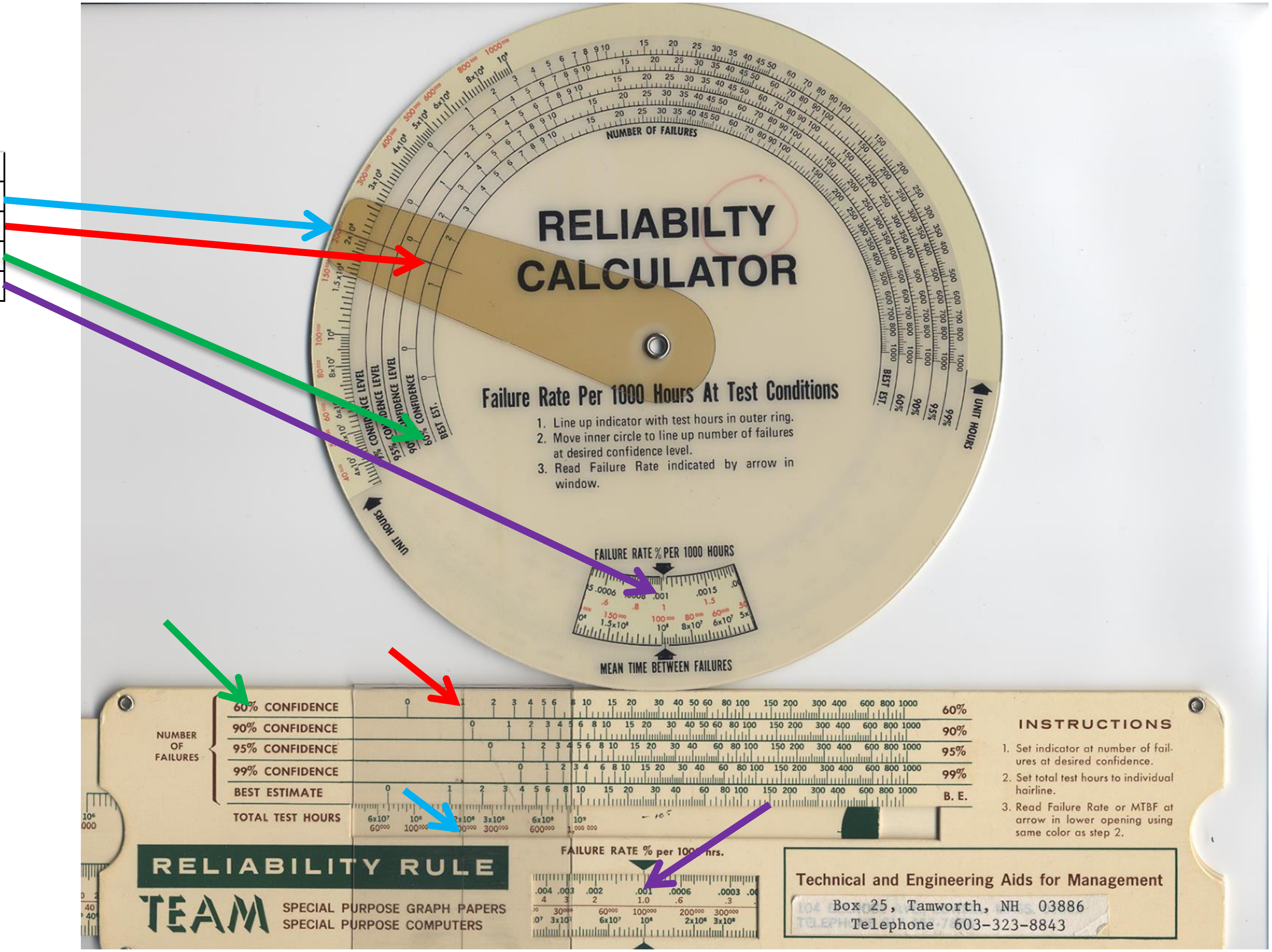
Note N+1 vs. N

$$\lambda_{UCL} = \lambda_{BE} \frac{\mathrm{CHIINV}\left(5\%,\, 2(N+1)\right)}{2N}$$

$$\lambda_{LCL} = \lambda_{BE} \frac{\mathrm{CHIINV}\left(95\%,\, 2N\right)}{2N}$$

Venerable Calculation

Reliability Calculator	
Device hours	200
Fails	1
Confidence	60%
Fail rate (%/hr)	1.01%



Failure Rate Per 1000 Hours At Test Conditions

1. Line up indicator with test hours in outer ring.
2. Move inner circle to line up number of failures at desired confidence level.
3. Read Failure Rate indicated by arrow in window.

INSTRUCTIONS

1. Set indicator at number of failures at desired confidence.
2. Set total test hours to individual hairline.
3. Read Failure Rate or MTBF at arrow in lower opening using same color as step 2.

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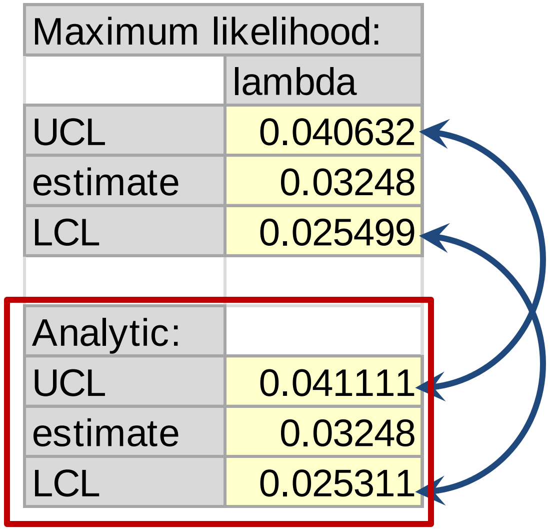
Exercise 8.3d

- Calculate lambda UCL and LCL analytically

Solution 8.3d

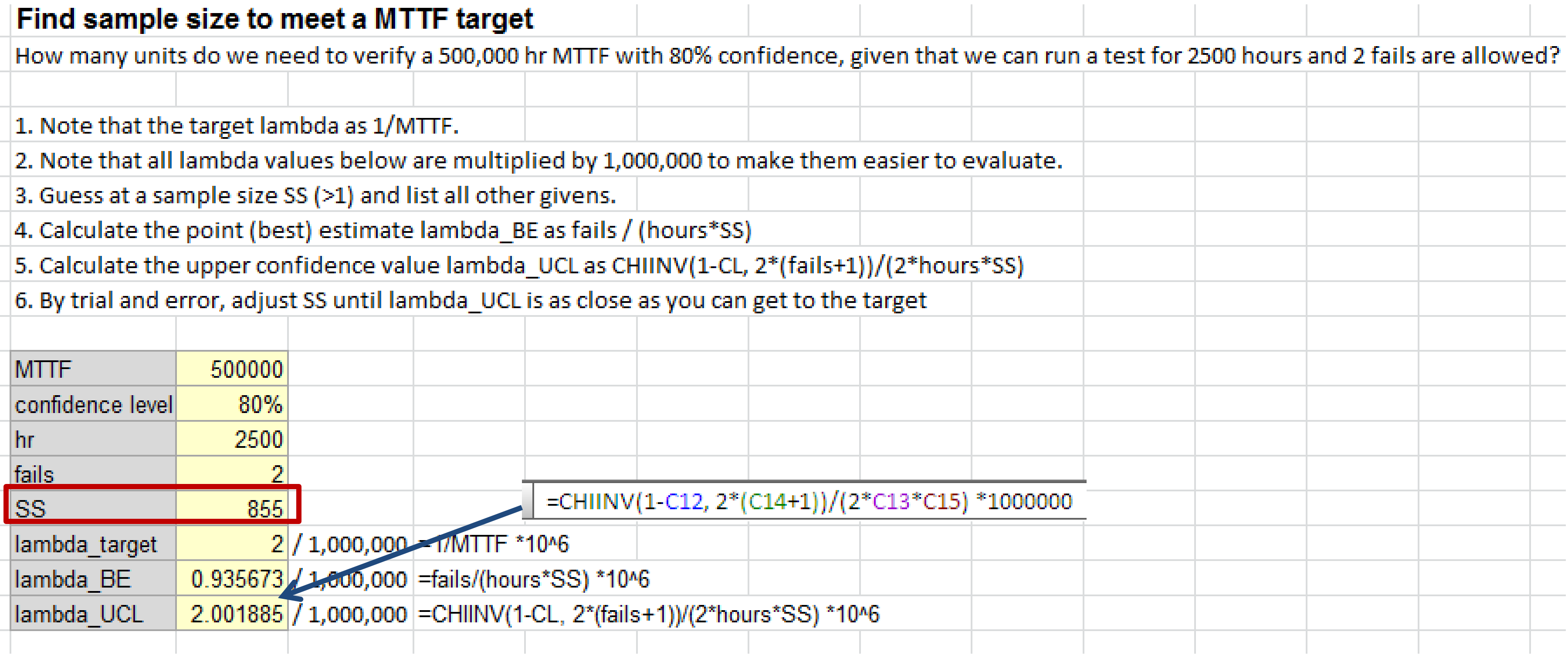


Maximum likelihood:	
	lambda
UCL	0.040632
estimate	0.03248
LCL	0.025499
Analytic:	
UCL	0.041111
estimate	0.03248
LCL	0.025311

Exercise 8.4

- This is Tobias & Trindade problem 3.1
- How many units do we need to verify a 500,000 hr MTTF with 80% confidence, given that we can run a test for 2500 hours and 2 fails are allowed?
- Hint: you can do this by trial and error. Calculate the UCL on λ as a function of sample size SS and then adjust SS until the UCL equals the target λ.

Solution 8.4



Find sample size to meet a MTTF target

How many units do we need to verify a 500,000 hr MTTF with 80% confidence, given that we can run a test for 2500 hours and 2 fails are allowed?

1. Note that the target lambda as 1/MTTF.
2. Note that all lambda values below are multiplied by 1,000,000 to make them easier to evaluate.
3. Guess at a sample size SS (>1) and list all other givens.
4. Calculate the point (best) estimate lambda_BE as fails / (hours*SS)
5. Calculate the upper confidence value lambda_UCL as CHIINV(1-CL, 2*(fails+1))/(2*hours*SS)
6. By trial and error, adjust SS until lambda_UCL is as close as you can get to the target

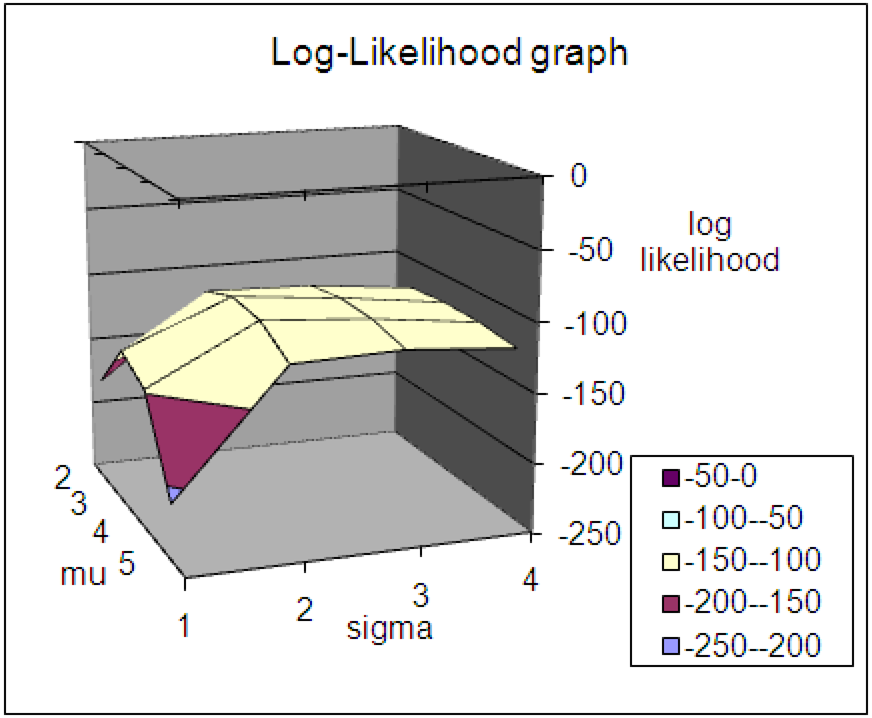
MTTF	500000		
confidence level	80%		
hr	2500		
fails	2		
SS	855		
lambda_target	2	/ 1,000,000	=1/MTTF *10^6
lambda_BE	0.935673	/ 1,000,000	=fails/(hours*SS) *10^6
lambda_UCL	2.001885	/ 1,000,000	=CHIINV(1-CL, 2*(fails+1))/(2*hours*SS) *10^6

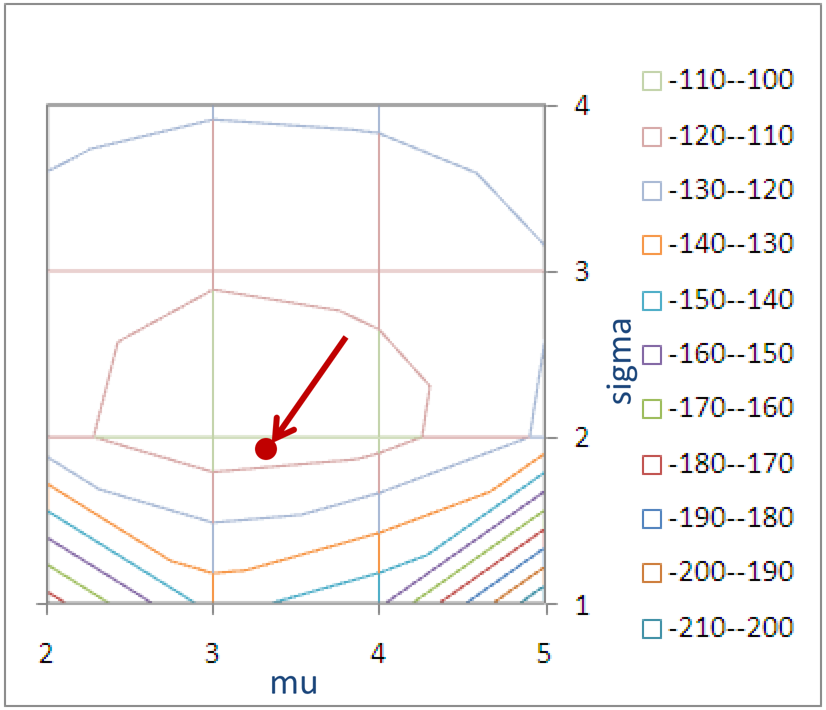
=CHIINV(1-C12, 2*(C14+1))/(2*C13*C15) *1000000

Normal Distribution MLE and Analytic

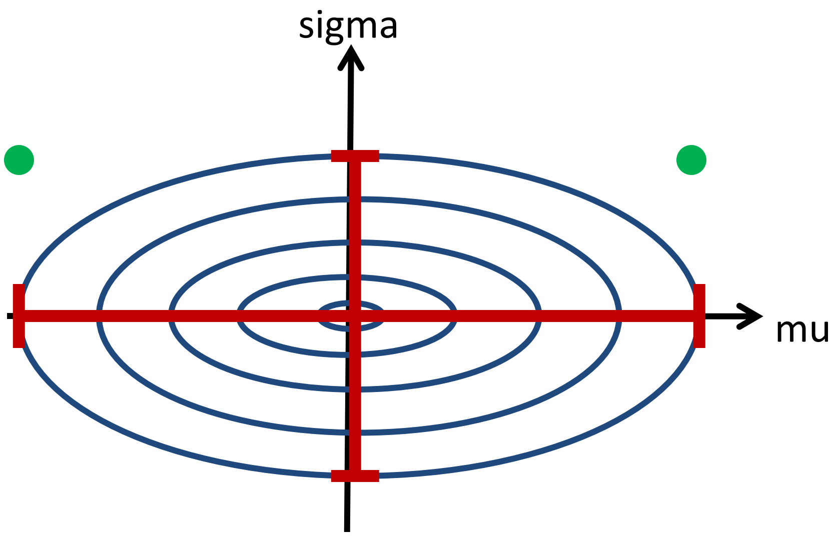
MLE for the Normal

$$L_i = \ln\left(\text{NORMDIST}\left(\text{data}_i, \overset{\downarrow}{\mu}, \overset{\downarrow}{\sigma}, \text{false}\right)\right)$$

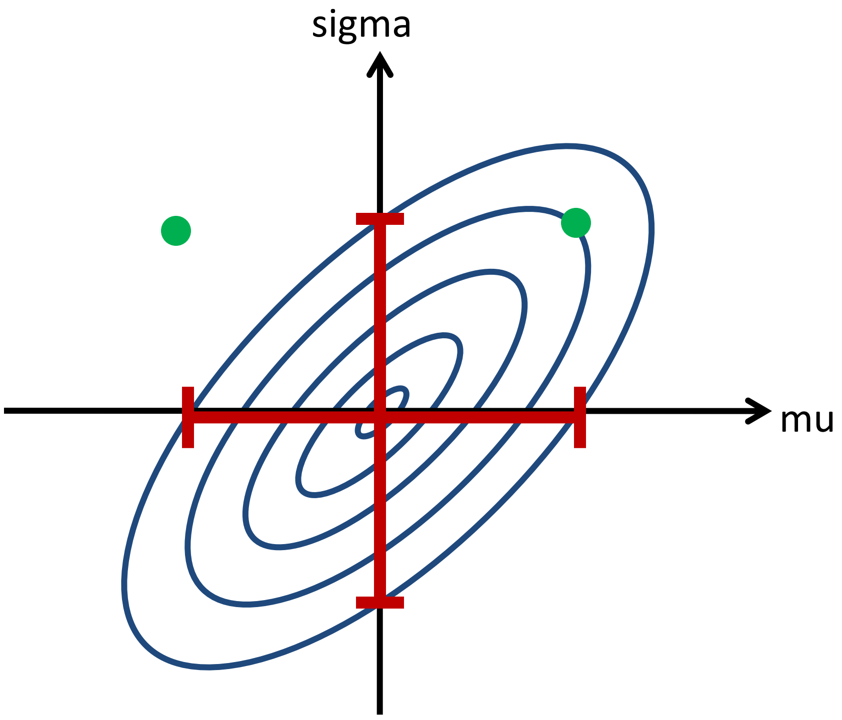




2D MLE



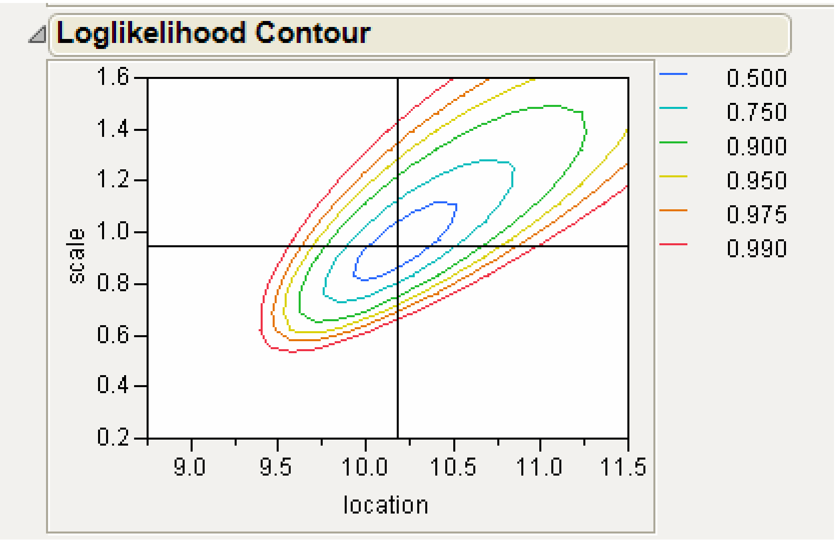
Uncorrelated



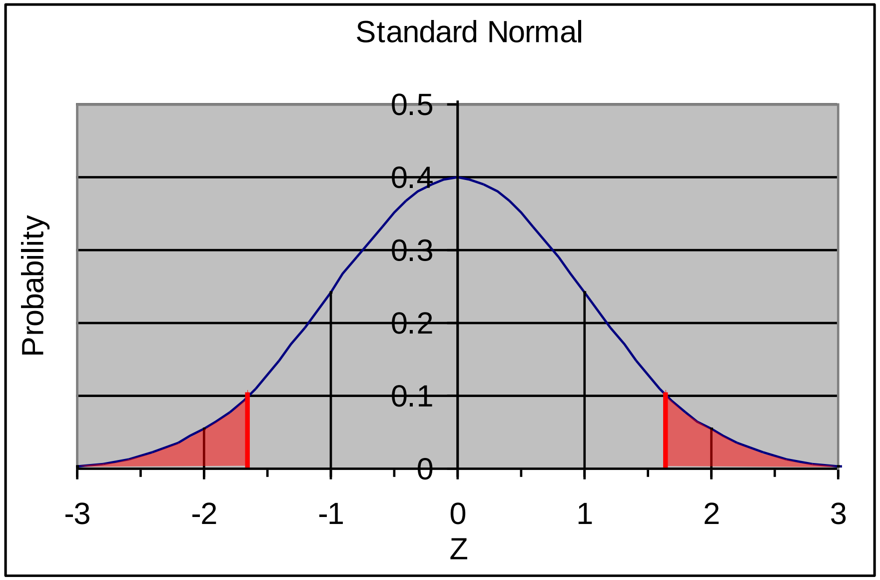
Correlated

Described by covariance matrix

JMP MLE Correlations



Analytic Uncertainty of Mean



Z-statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$\bar{X}$ = sample mean

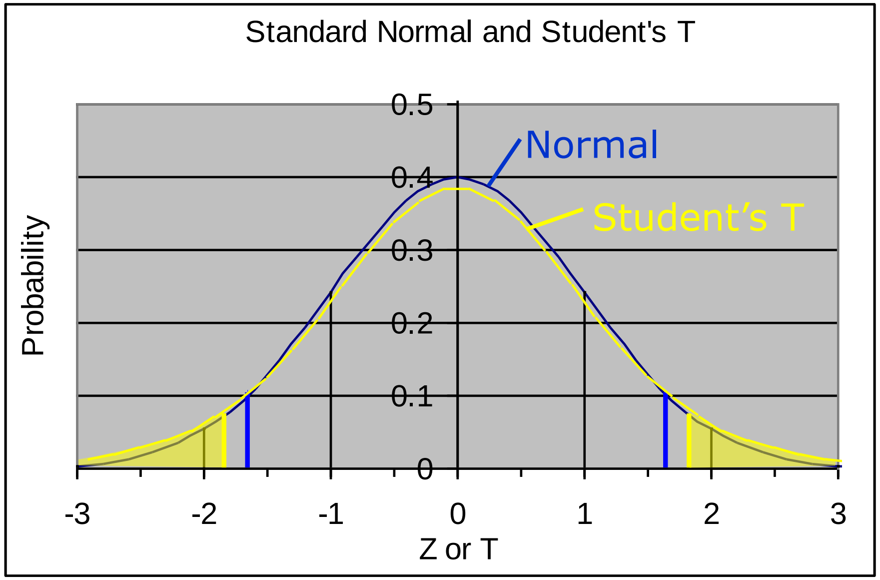
μ = population mean

σ = population stdev

n = sample size

- Using a *Z-statistic* is exactly equivalent to the previous slides
  - Gives error in $\bar{X}$ in units of $\sigma/\sqrt{n}$
- (Note that we need the true population standard deviation)

Analytic Uncertainty of Mean



T-statistic:

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$\bar{X}$ = sample mean

μ = population mean

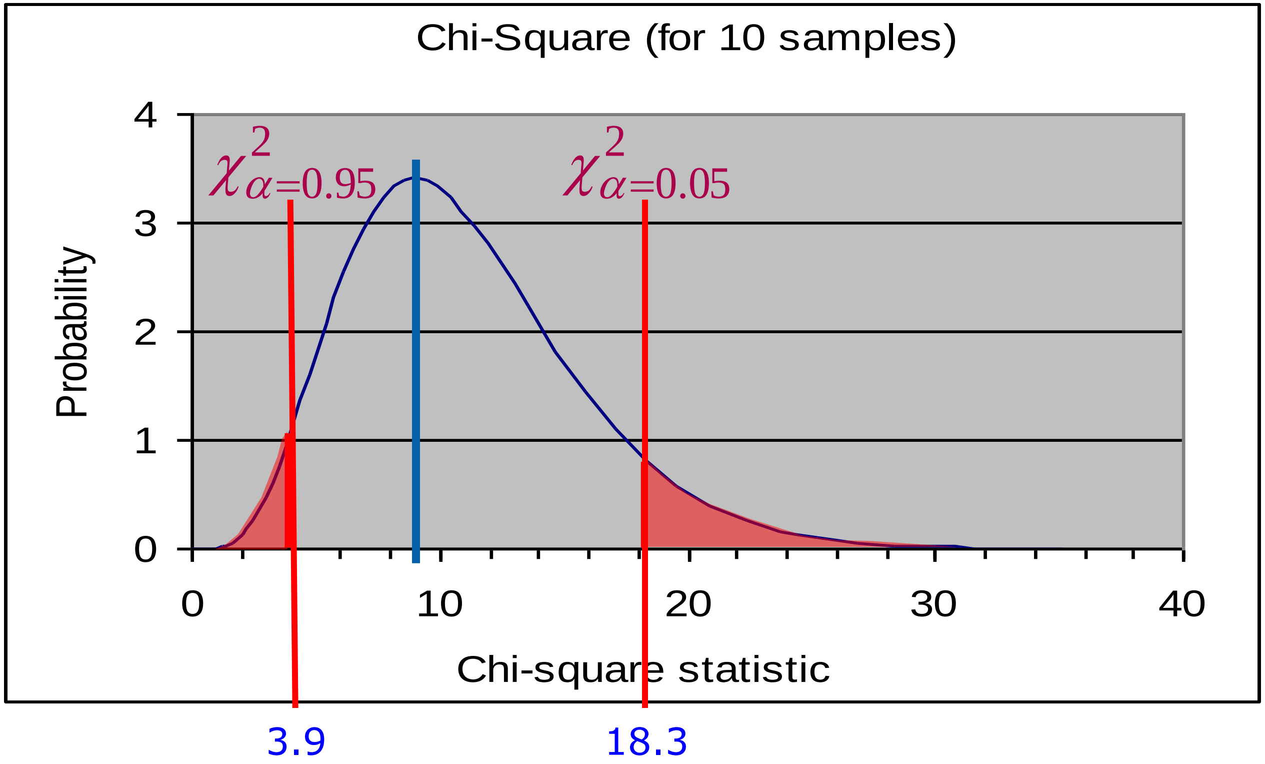
S = sample stdev

n = sample size

- Similar to *Z-statistic* but
  - Gives error in $\bar{X}$ in units of $S/\sqrt{n}$
- Preferred over *Z* because true σ is usually not known
- Calculate μ range from:

$$\bar{X} - \frac{t_{\alpha/2,n-1}\,S}{\sqrt{n}} \le \mu \le \bar{X} + \frac{t_{\alpha/2,n-1}\,S}{\sqrt{n}}$$

Analytic Uncertainty of Standard Deviation



Chi^2 statistic:

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

$S =$ sample stdev

$\sigma =$ population stdev

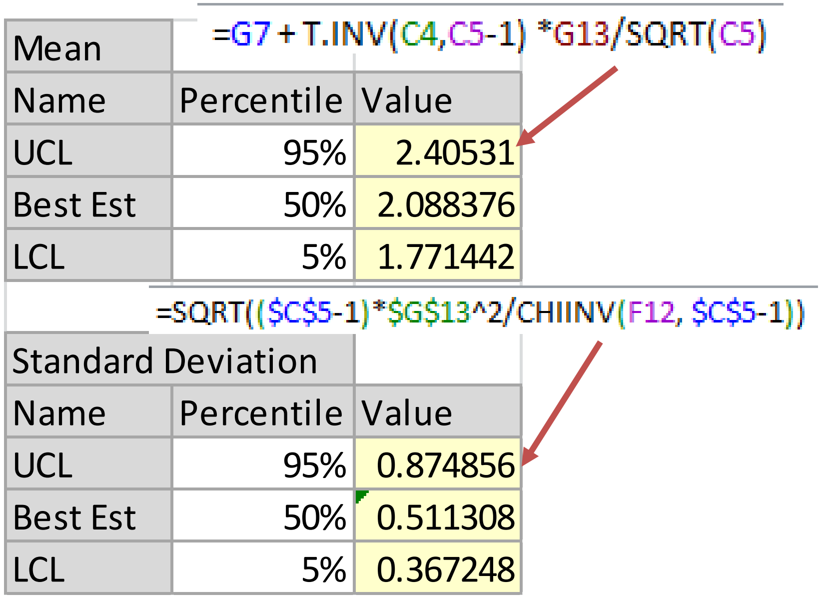
$n =$ sample size

- Gives error as ratio of sample variance / pop variance (times n-1)
- Distribution of chi^2 statistic follows a chi^2 distribution
- Calculate σ^2 range from: $$\frac{(n-1)\,S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

Exercise 8.5a

- For the 9 data points given, extract mu, sigma, and their 95% confidence intervals.

Solution 8.5a

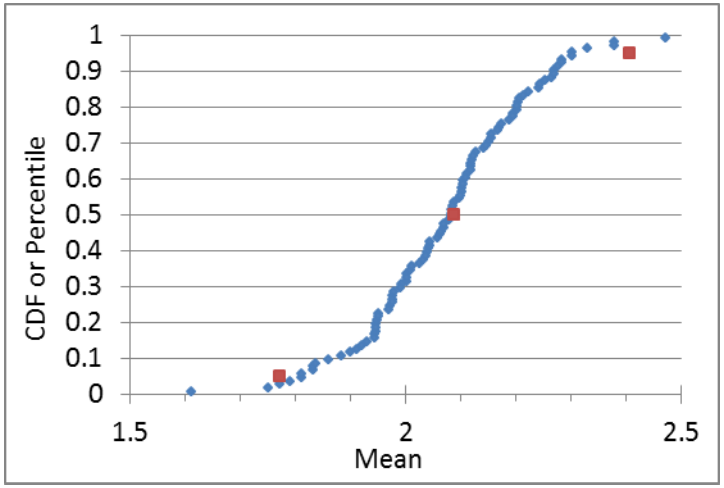


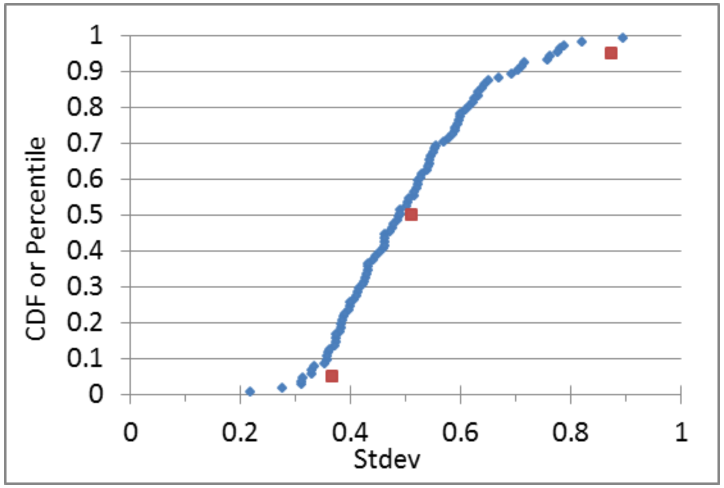
=G7 + T.INV(C4,C5-1) *G13/SQRT(C5)

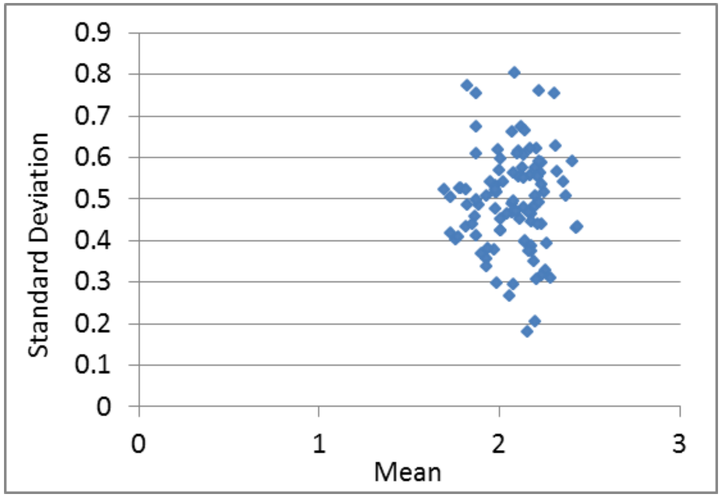
Mean		
Name	Percentile	Value
UCL	95%	2.40531
Best Est	50%	2.088376
LCL	5%	1.771442

=SQRT((C5-1)*G13^2/CHIINV(F12, C5-1))

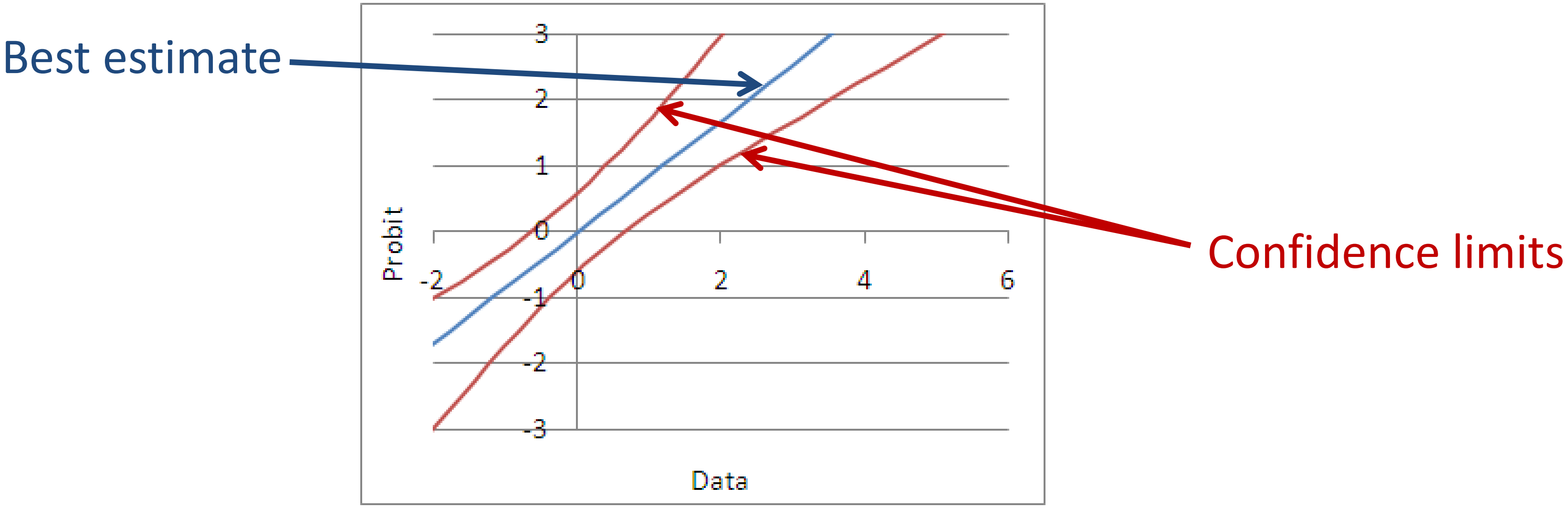
Standard Deviation		
Name	Percentile	Value
UCL	95%	0.874856
Best Est	50%	0.511308
LCL	5%	0.367248







Normal Distribution Uncertainties



Best estimate

Confidence limits

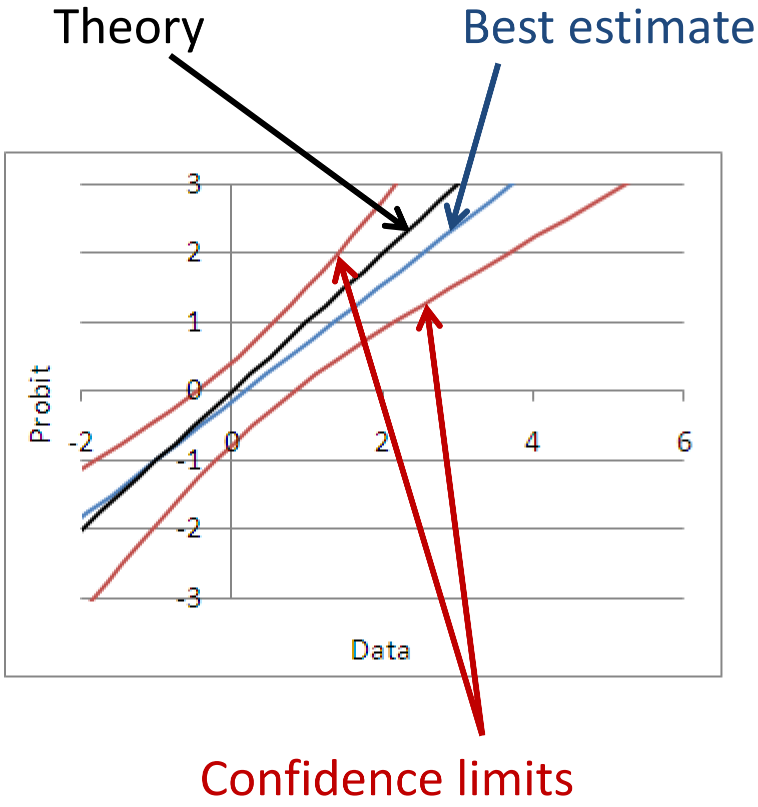
	Best estimate	Uncertainty
Mean μ	μ = AVERAGE(data)	m = NORMSINV(CL) * σ / SQRT(N)
Stdev σ	σ = STDEV(data)	s = σ * (SQRT(CHIINV(1-CL, N-1) / (N-1)) - 1)
Percentile	μ + z*σ	UCL = μ + z*σ + SQRT(m^2 + z^2*s^2) LCL = μ + z*σ – SQRT(m^2 + z^2*s^2)

CL = confidence level (e.g., 95%)

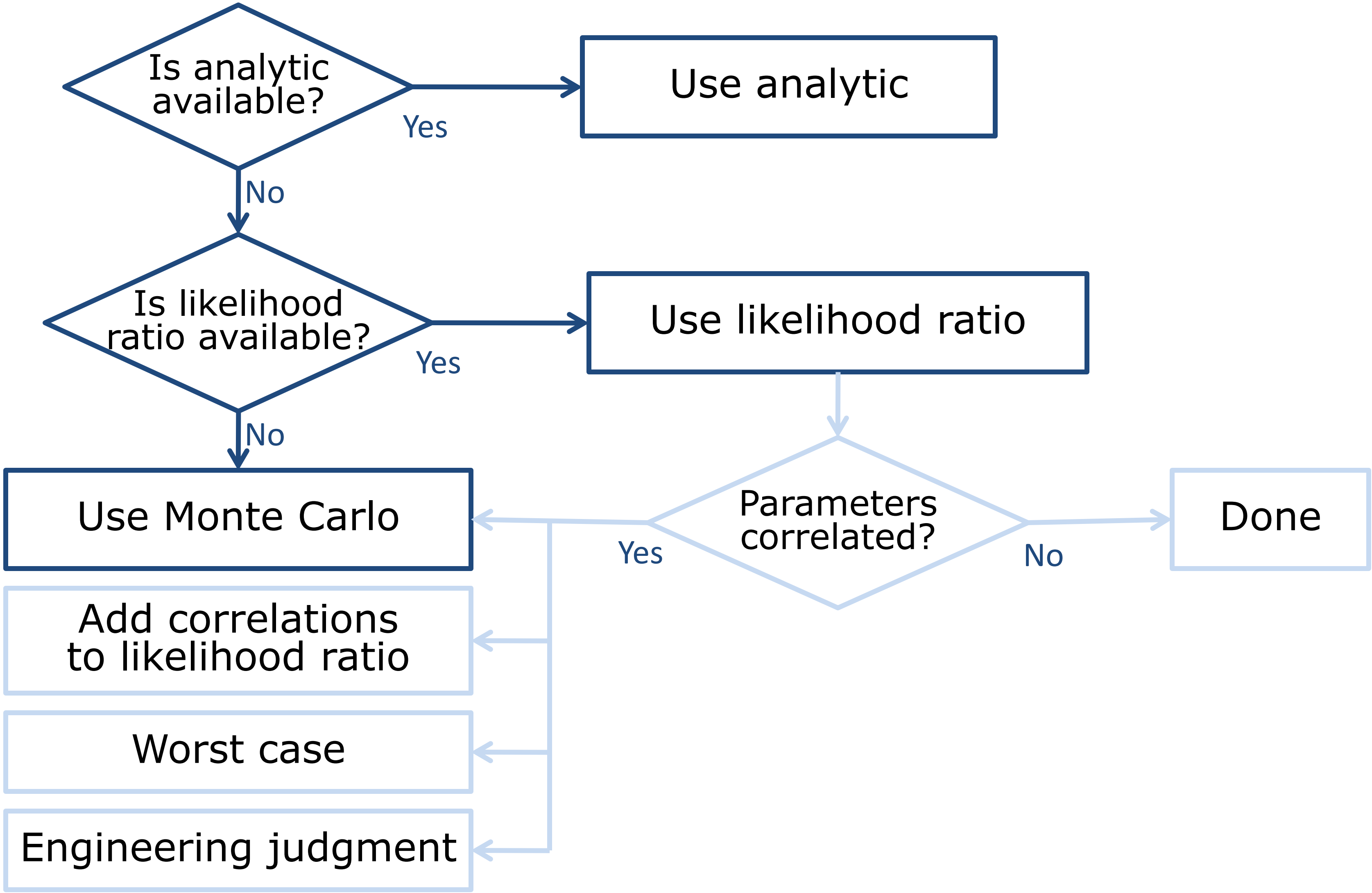
z = probit value at which to evaluate distribution (e.g., -2)

N = number of samples in data set

Exercise 8.5b



Calculation Method Flowchart



Is analytic available?

Yes

Use analytic

No

Is likelihood ratio available?

Yes

Use likelihood ratio

No

Use Monte Carlo

Parameters correlated?

Yes

No

Done

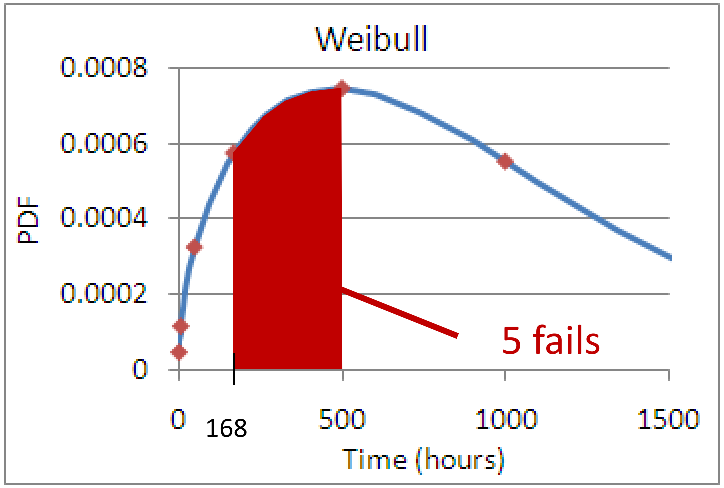
Add correlations to likelihood ratio

Worst case

Engineering judgment

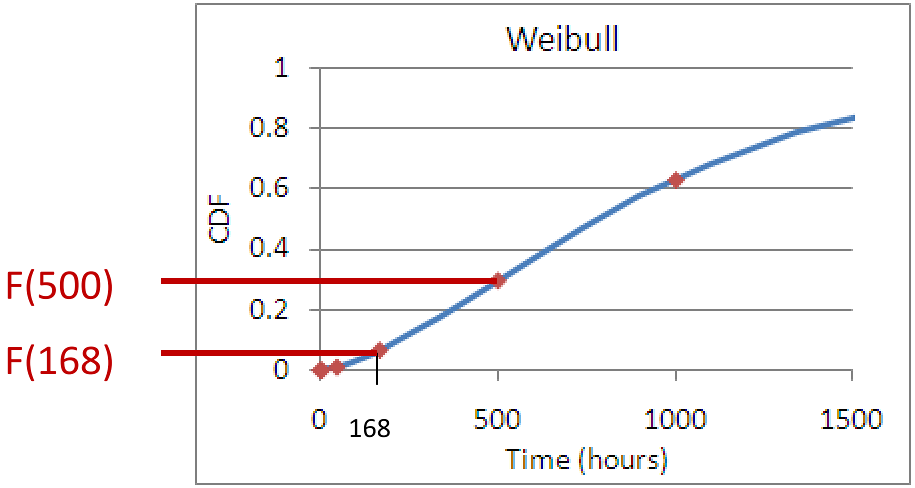
Weibull MLE with Readout Data

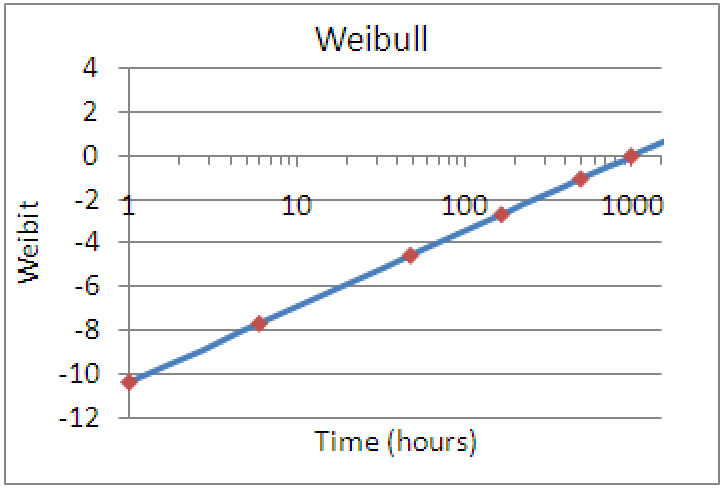
Weibull Readout Data



$$LIK = \left[F(500) - F(168)\right]^5$$

$$L = 5 \cdot \ln\left[F(500) - F(168)\right]$$



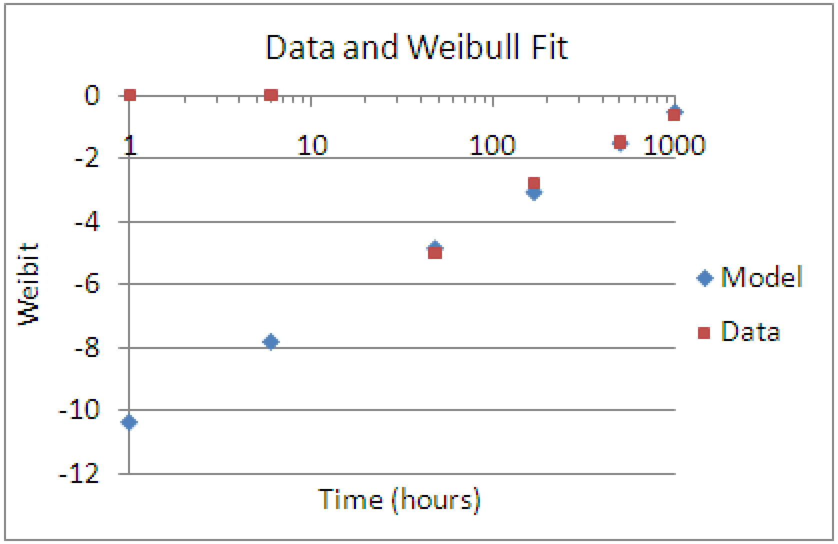


MLE for Weibull

Vary these to maximize this

shape	1.2
lifetime	1500
SS	300

time	fails	model F	L
0		0	
1	0	0.000154	0
6	0	0.001325	0
48	2	0.015948	-8.45037
168	16	0.069736	-46.763
500	43	0.234771	-77.4687
1000	63	0.459218	-94.1294
survivors	176	0.540782	-108.194
		Ltotal	-335.006



$$L = \sum_{r=1}^{R} \left[\boxed{n_r} \cdot \ln \boxed{\left\{ F(t_r) - F(t_{r-1}) \right\}} + \underbrace{d_r \cdot \ln S(t_r)} \right] + \boxed{s_R}\, \boxed{\ln S(t_R)}$$

$$\boxed{F(t) = 1 - \exp\left\{ -\left(\frac{t}{\alpha} \right)^{\beta} \right\}}$$

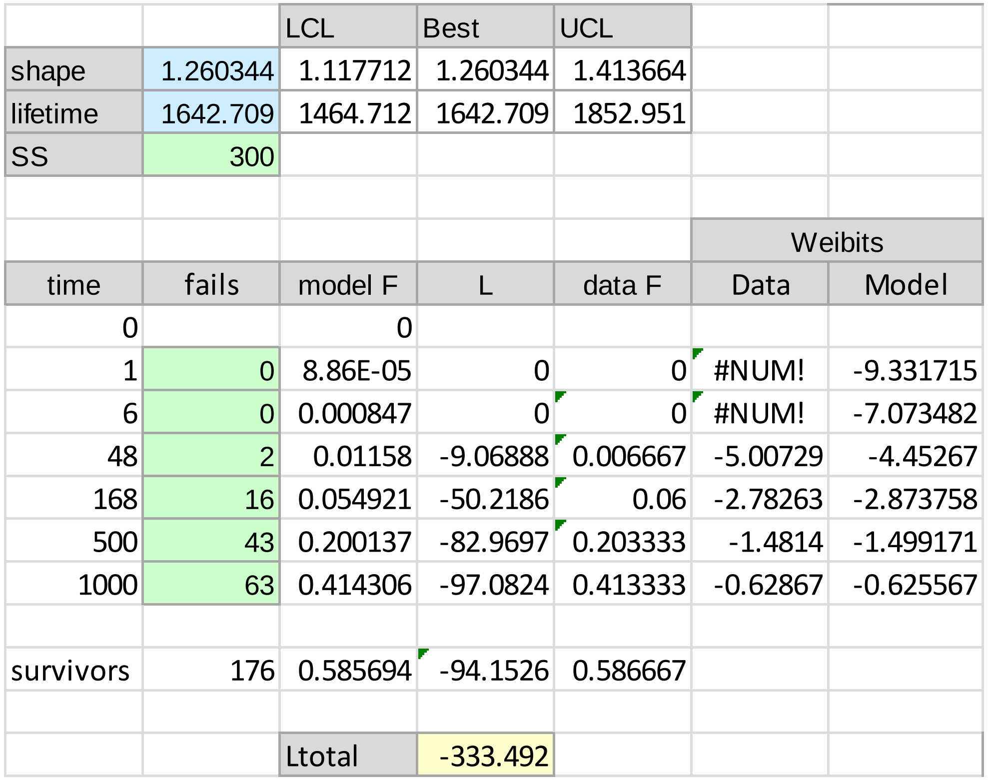
$$\boxed{S(t) = \exp\left\{ -\left(\frac{t}{\alpha} \right)^{\beta} \right\}}$$

Exercise 8.6a

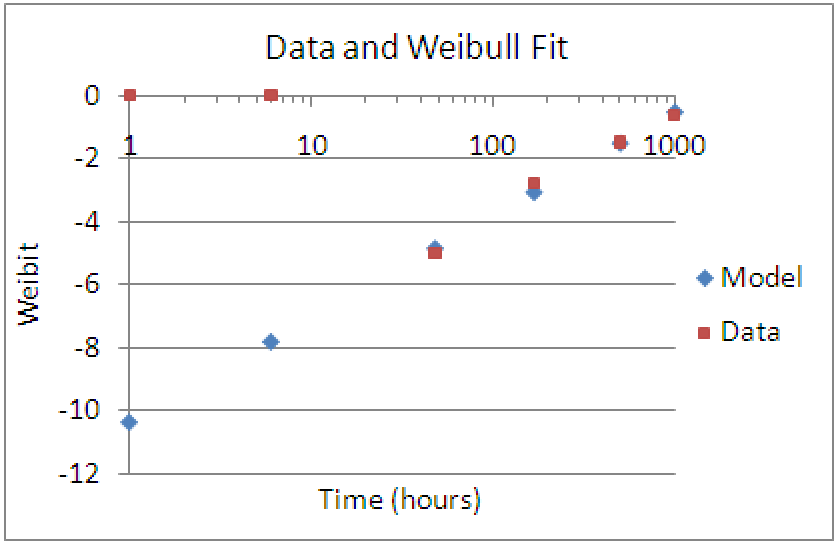
- Use MLE to determine Weibull fit parameters for the readout data given below and on the Ex16 tab.
- Also, find (separate) 90% confidence intervals for each parameter using the likelihood ratio technique. (That is the confidence where the LR=0.1.)

time	fails
0	
1	0
6	0
48	2
168	16
500	43
1000	63

Solution 8.6a



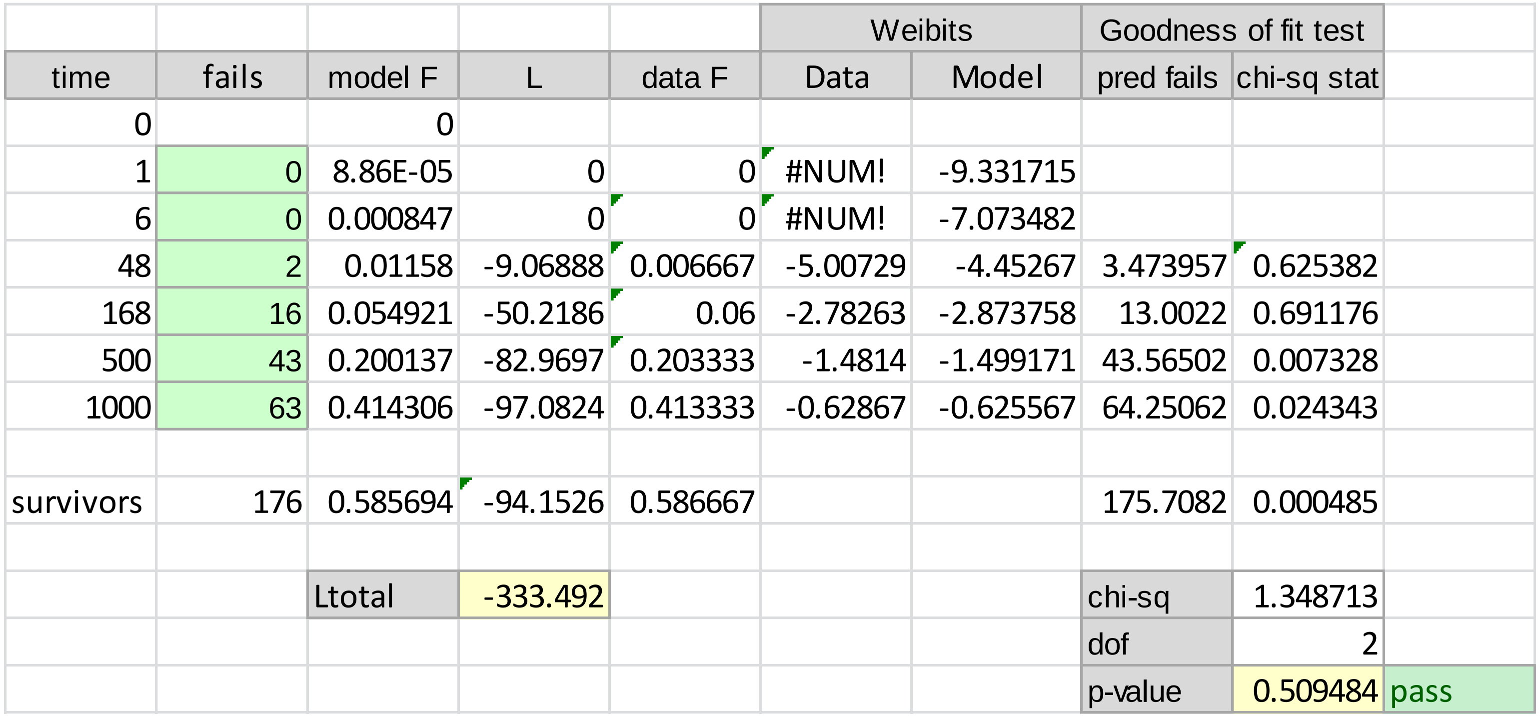
		LCL	Best	UCL		
shape	1.260344	1.117712	1.260344	1.413664		
lifetime	1642.709	1464.712	1642.709	1852.951		
SS	300					
					Weibits	
time	fails	model F	L	data F	Data	Model
0		0				
1	0	8.86E-05	0	0	#NUM!	-9.331715
6	0	0.000847	0	0	#NUM!	-7.073482
48	2	0.01158	-9.06888	0.006667	-5.00729	-4.45267
168	16	0.054921	-50.2186	0.06	-2.78263	-2.873758
500	43	0.200137	-82.9697	0.203333	-1.4814	-1.499171
1000	63	0.414306	-97.0824	0.413333	-0.62867	-0.625567
survivors	176	0.585694	-94.1526	0.586667		
		Ltotal	-333.492			



Exercise 8.6b

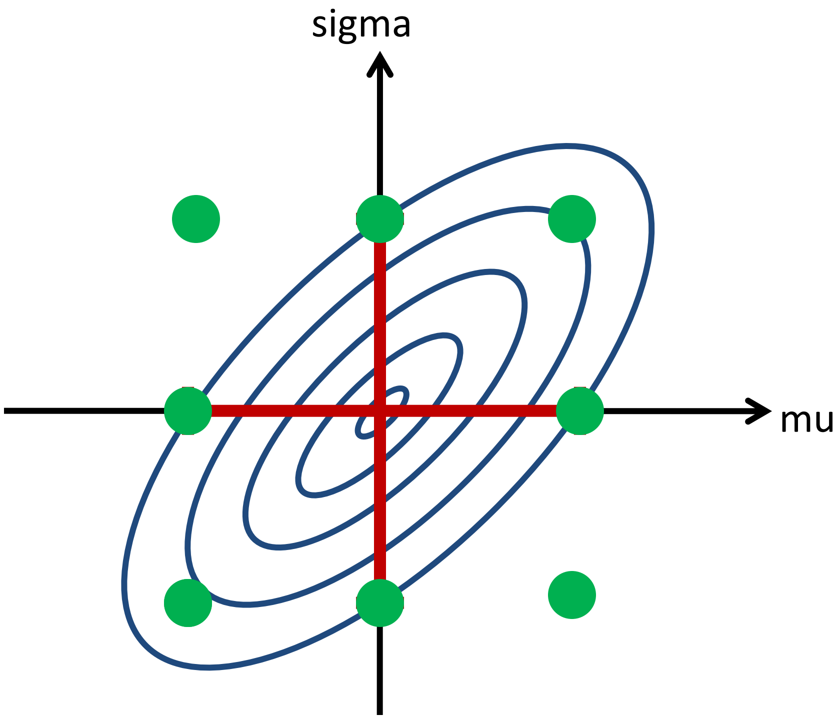
- Add a chi-square goodness-of-fit test to your fit from part (a). Recall that the bins should have more than about 5 fails each, so you will need to combine the first few readouts into 1 bin. So we do things the same way, combine the first 3 readouts into 1 bin, even though they only have 2 total fails.

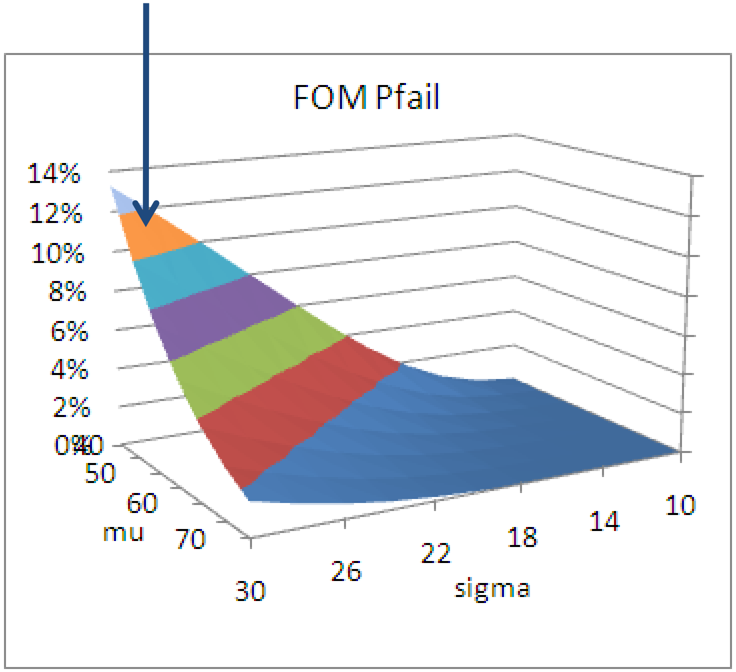
Solution 8.6b



					Weibits		Goodness of fit test		
time	fails	model F	L	data F	Data	Model	pred fails	chi-sq stat	
0		0							
1	0	8.86E-05	0	0	#NUM!	-9.331715			
6	0	0.000847	0	0	#NUM!	-7.073482			
48	2	0.01158	-9.06888	0.006667	-5.00729	-4.45267	3.473957	0.625382	
168	16	0.054921	-50.2186	0.06	-2.78263	-2.873758	13.0022	0.691176	
500	43	0.200137	-82.9697	0.203333	-1.4814	-1.499171	43.56502	0.007328	
1000	63	0.414306	-97.0824	0.413333	-0.62867	-0.625567	64.25062	0.024343	
survivors	176	0.585694	-94.1526	0.586667			175.7082	0.000485	
		Ltotal	-333.492				chi-sq	1.348713	
							dof	2	
							p-value	0.509484	pass

Confidence and Figures of Merit





Exercise 8.6c

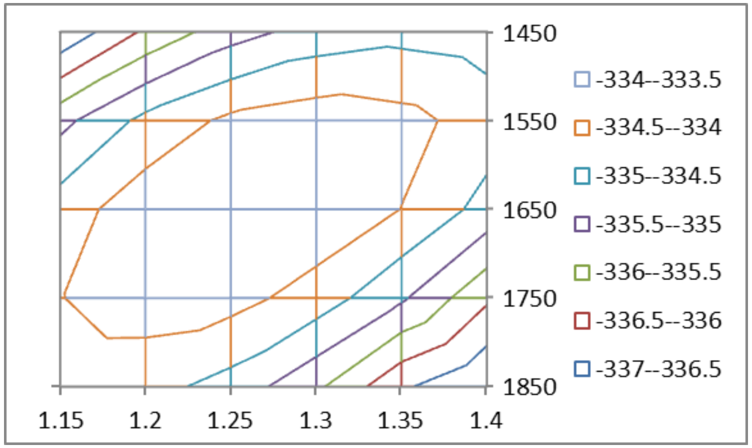
- Use the pfail at 2000 hours as the FOM for the Weibull model of 8.6. Evaluate this pfail as a function of the shape and lifetime parameters.
- Try various corners of the “space” of shape and lifetime values and find the worst case corner. Report these values as your worst case shape and lifetime values.

Solution

		LCL	Best	UCL		Worst case		FOM time	2000
shape	1.413664	1.117712	1.260344	1.413664		1.4136642		FOM pfail	0.788438
lifetime	1464.712	1464.712	1642.709	1852.951		1464.7121			0.722
SS	300								

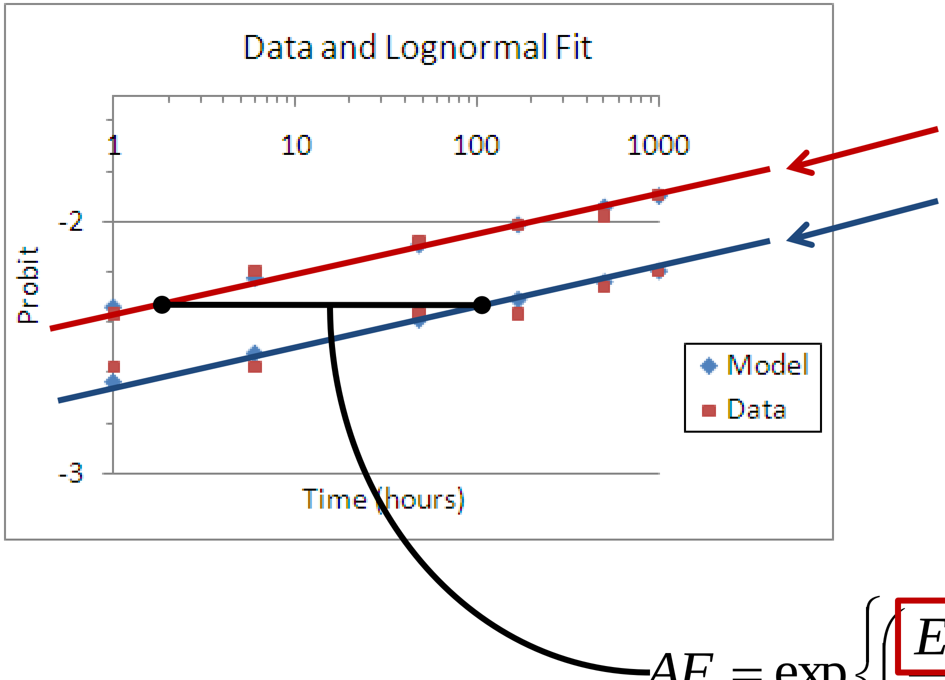
Ltotal	-334.69
best L	-333.492
LR p-value	0.121738

	1.15	1.2	1.25	1.3	1.35	1.4
1450	-336.92	-335.908	-335.208	-334.798	-334.656	-334.764
1550	-335.151	-334.357	-333.889	-333.724	-333.84	-334.218
1650	-334.247	-333.702	-333.497	-333.606	-334.008	-334.684
1750	-334.016	-333.741	-333.817	-334.22	-334.926	-335.915
1850	-334.314	-334.321	-334.691	-335.397	-336.417	-337.728



Acceleration Calculations

Acceleration



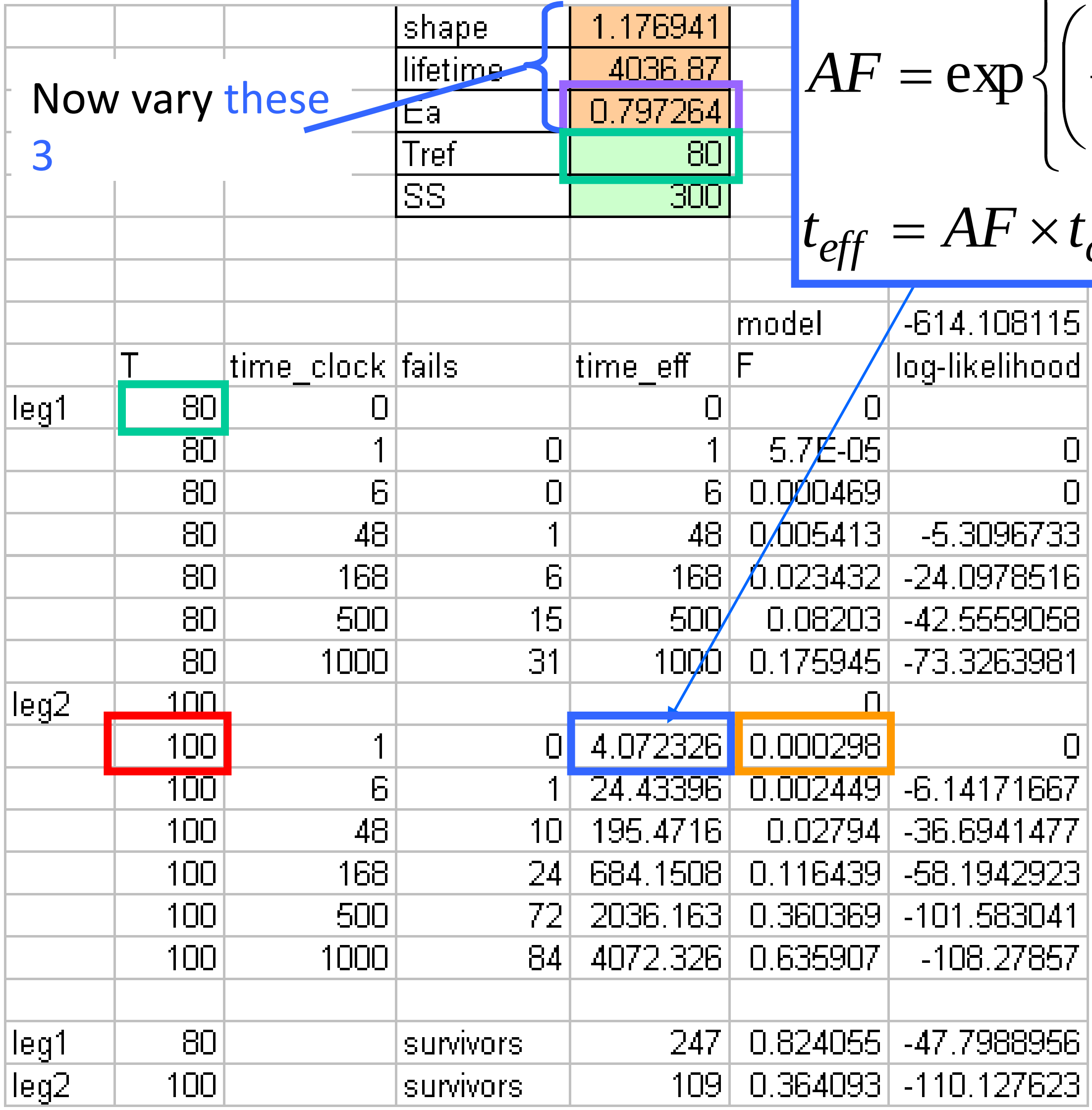
F(t) at high T (T_{stress})

F(t) at low T (T_{use})

$$AF = \exp\left\{\left(\boxed{\frac{E_a}{k}}\right) \times \left(\frac{1}{T_{use}} - \frac{1}{T_{stress}}\right)\right\}$$

Acceleration MLE

Now vary these 3



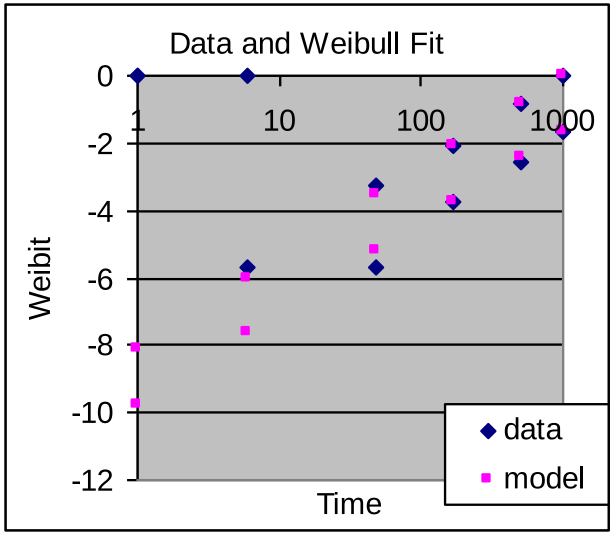
shape	1.176941
lifetime	4036.87
Ea	0.797264
Tref	80
SS	300

					model	-614.108115
	T	time_clock	fails	time_eff	F	log-likelihood
leg1	80	0		0	0	
	80	1	0	1	5.7E-05	0
	80	6	0	6	0.000469	0
	80	48	1	48	0.005413	-5.3096733
	80	168	6	168	0.023432	-24.0978516
	80	500	15	500	0.08203	-42.5559058
	80	1000	31	1000	0.175945	-73.3263981
leg2	100				0	
	100	1	0	4.072326	0.000298	0
	100	6	1	24.43396	0.002449	-6.14171667
	100	48	10	195.4716	0.02794	-36.6941477
	100	168	24	684.1508	0.116439	-58.1942923
	100	500	72	2036.163	0.360369	-101.583041
	100	1000	84	4072.326	0.635907	-108.27857
leg1	80		survivors	247	0.824055	-47.7988956
leg2	100		survivors	109	0.364093	-110.127623

$$AF = \exp\left\{\left(\frac{E_a}{k}\right)\times\left(\frac{1}{T_{ref}+273}-\frac{1}{T+273}\right)\right\}$$

$$t_{eff} = AF \times t_{clock}$$

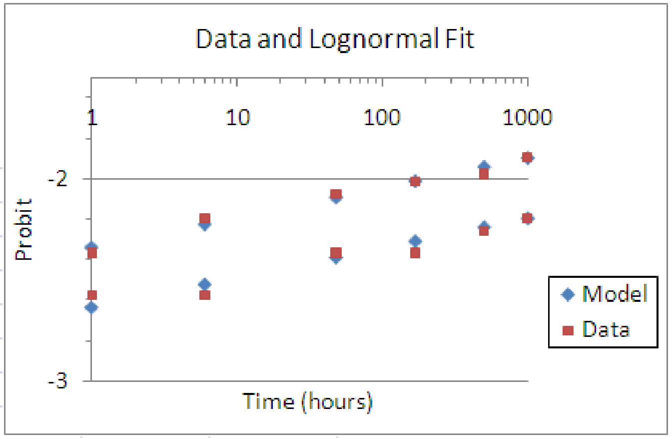
$$F(t) = 1-\exp\left\{-\left(\frac{t_{eff}}{\alpha}\right)^{\beta}\right\}$$



Exercise

- Do an MLE to get the 3 parameters (mu, sigma, and Ea) for a lognormal model of times to fail with an Arrhenius temperature acceleration for the data in tab Ex17.
- Also do a goodness-of-fit test to see if the lognormal is a good fit to the data.
- Also determine 90% confidence limits on all 3 parameters (separately).
- Use a FOM of 1 year at T=75 and find what the conservative combination of confidence limits is.

Solution

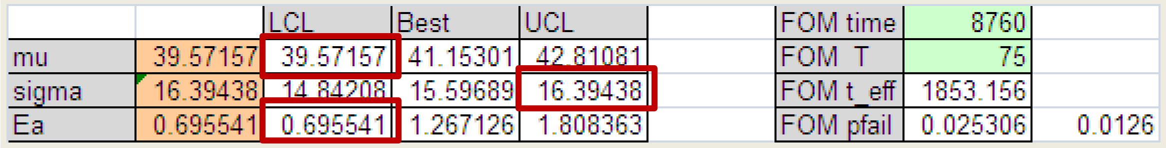


			-278.701	
		LCL	Best	UCL
mu	41.15301	39.57157	41.15301	42.81081
sigma	15.59689	14.84208	15.59689	16.39438
Ea	1.267126	0.695541	1.267126	1.808363
Tref	100			
SS	1000		LR p-value	0.999834

FOM time	8760
FOM T	75
FOM t_eff	517.0626
FOM pfail	0.012613

							Probits		Goodness of fit test	
					-278.701					
T	time_clock	fails	time_eff	model F	L	data F	Data	Model	pred fails	chi-sq stat
leg1	0		0	0						
100	1	5	1	0.004163	-27.4074	0.005	-2.57583	-2.63854	4.163201	0.168196
100	6	0	6	0.005807	0	0.005	-2.57583	-2.52366		
100	48	4	48	0.008416	-23.7944	0.009	-2.36562	-2.39034	4.25328	0.015083
100	168	0	168	0.010444	0	0.009	-2.36562	-2.31001		
100	500	3	500	0.012543	-18.499	0.012	-2.25713	-2.24009		
100	1000	2	1000	0.014059	-12.9833	0.014	-2.19729	-2.19565	5.642151	0.073085
leg2										
150	1	9	105.2628	0.009642	-41.7745	0.009	-2.36562	-2.33999	9.642155	0.042767
150	6	5	631.5765	0.013037	-28.4276	0.014	-2.19729	-2.22511	3.394783	0.759025
150	48	5	5052.612	0.018229	-26.3033	0.019	-2.07485	-2.09179	5.191912	0.007094
150	168	3	17684.14	0.022138	-16.6331	0.022	-2.01409	-2.01146	3.90937	0.211531
150	500	2	52631.38	0.026097	-11.0639	0.024	-1.97737	-1.94154	3.958352	0.968874
150	1000	5	105262.8	0.028908	-29.371	0.029	-1.8957	-1.8971	2.811069	1.704482
leg1	survivors	986		0.985941	-13.9602				985.9414	3.49E-06
leg2	survivors	971		0.971092	-28.483				971.0924	8.78E-06

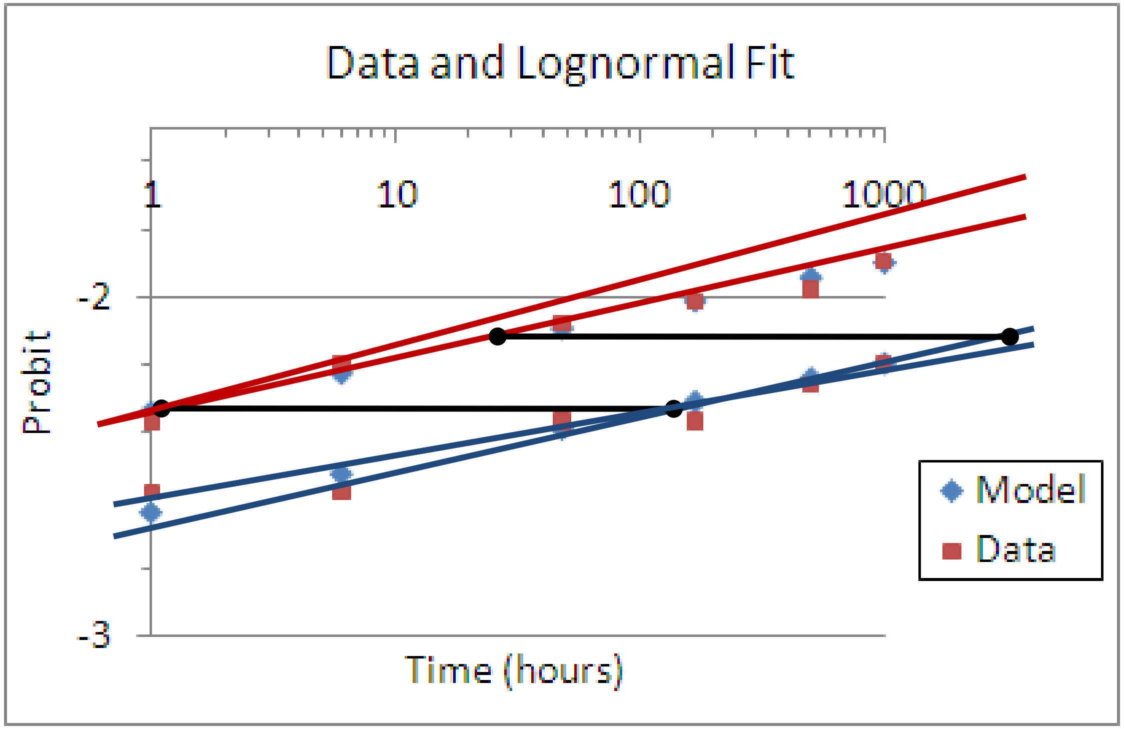
chi-sq	3.950149
dof	7
p-value	0.785501



		LCL	Best	UCL
mu	39.57157	39.57157	41.15301	42.81081
sigma	16.39438	14.84208	15.59689	16.39438
Ea	0.695541	0.695541	1.267126	1.808363

FOM time	8760	
FOM T	75	
FOM t_eff	1853.156	
FOM pfail	0.025306	0.0126

Is Acceleration Valid?



Likelihood Ratio Test

Likelihood ratio statistic:

$$LR = 2 \times (\ln L_1 - \ln L_2)$$

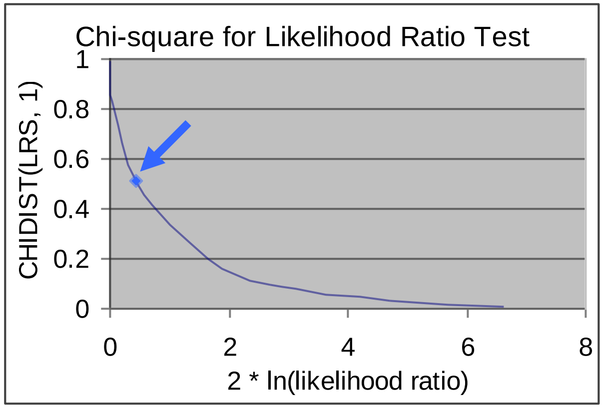
Chi-square distributed:

CHIDIST(LR, ΔDoF)

= CHIDIST(0.43, 1) = 0.51 (likely)

p-value

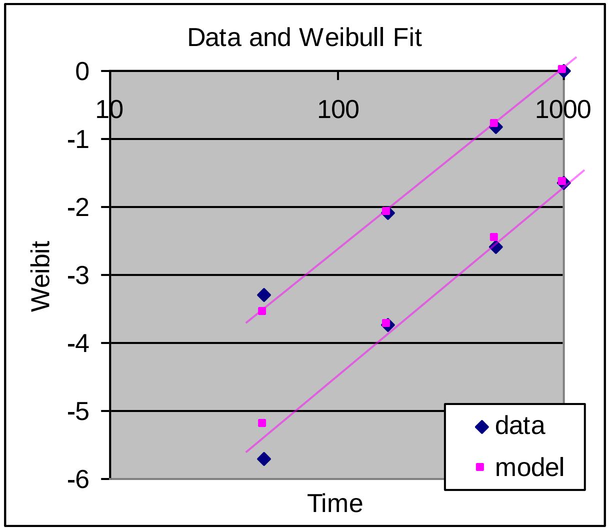
Acceleration is valid



Case 1: One Distribution

shape	1.176943
lifetime	4036.862
Ea	0.797262

$\log L_1 = -614.108$

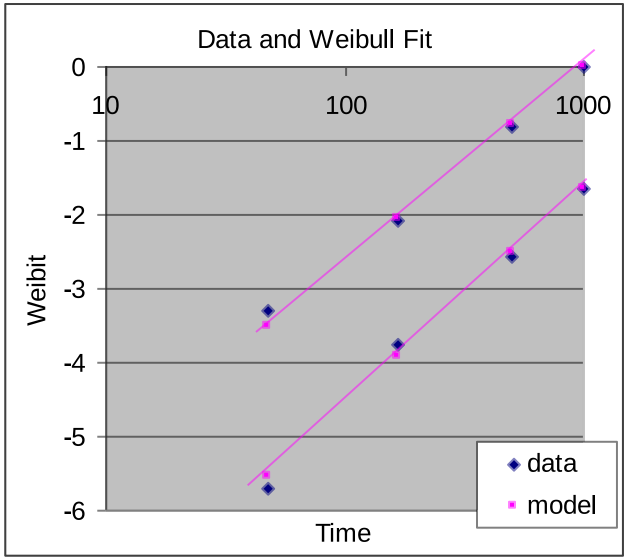


Case 2: Separate Distributions

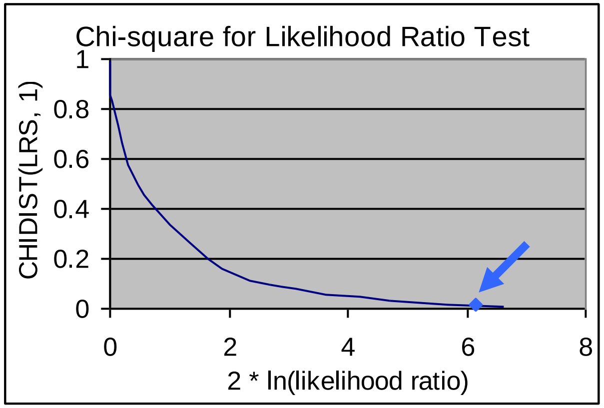
leg1	shape	1.281392
	lifetime	3592.847
leg2	shape	1.154944
	lifetime	4071.605

-192.912	Leg1
-420.981	Leg2

$\log L_2 = -613.893$



Likelihood Ratio Test



Likelihood ratio statistic:

$$LR = 2 \text{ x } (\ln L_1 - \ln L_2)$$

Chi-square distributed:

CHIDIST(LR, ΔDoF)

= CHIDIST(6.17, 1) = 0.013 (not likely)

p-value

Acceleration is ***not*** valid

Case 1: One Distribution

shape	1.198326
lifetime	3344.311
Ea	0.475108

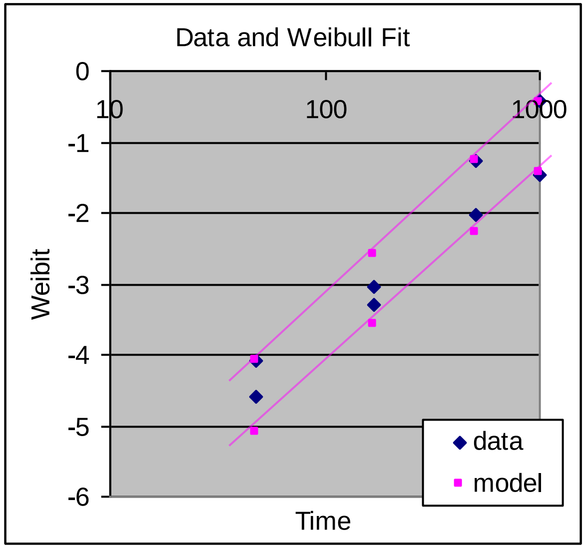
$\log \Gamma_1 =$ -580.91

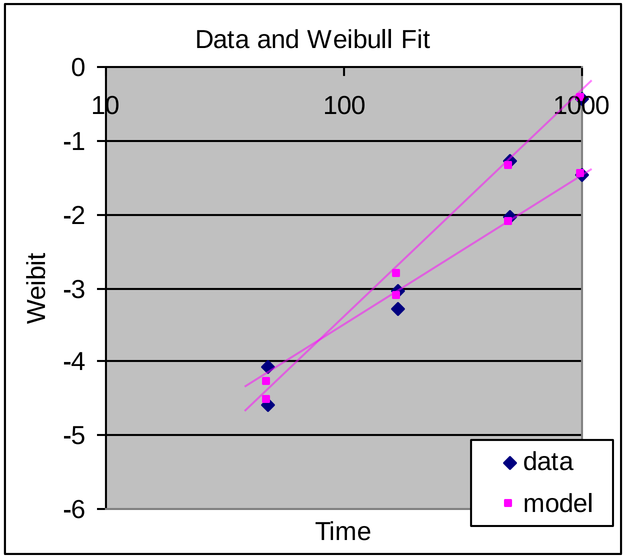
Case 2: Separate Distributions

leg1	shape	0.933166
	lifetime	4782.319
leg2	shape	1.350774
	lifetime	5607.301

-226.744	Leg1
-351.081	Leg2

$\log \Gamma_2 =$ -577.826





Exercise b

- Do a likelihood ratio test to see if the acceleration concept is valid for the Ex17 data set. You will have to use 2 separate likelihood sums and two sets of mu and sigma parameters with no Ea, and compare that to what you have for part (a).