ECE 510 Lecture 2
Plotting and Fitting 1

Histogram, CDF Plot, T&T 1.1-4,7-8
Reliability Functions, T&T 2.1-6, 9

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Looking At Data

Looking at Data

Bag #1

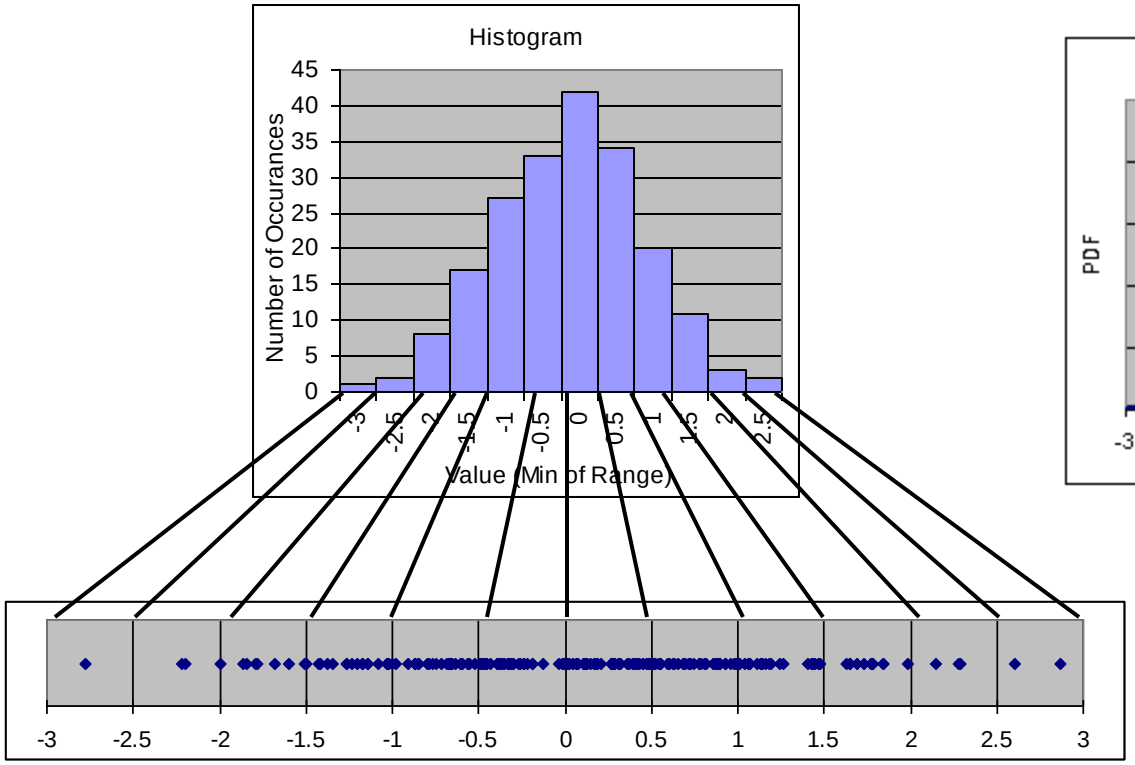
-1.26755	1.778466	-1.37188	-1.14666	1.437807	-0.60299	-1.02321	2.284605
2.145411	0.692451	-1.17339	0.364737	0.724378	-1.50313	0.190458	0.40733
1.650385	0.630984	-0.12599	1.264115	-1.84423	-0.48658	-0.66664	0.320823
0.316924	-0.33161	0.067807	0.481851	1.18916	0.933333	1.446249	0.373354
0.480242	-1.78896	0.485449	-0.74937	0.688161	-0.98282	-0.71612	-0.33363
-0.36264	-0.7888	0.269517	1.988823	-0.43457	0.926149	-0.48861	-0.6811
1.838188	-2.22009	0.772391	1.11014	0.01931	-1.34591	-0.01784	0.022294
-0.86969	1.461931	0.190981	-0.00919	0.077722	0.495746	1.00924	0.38849
-0.5533	-0.6787	0.819628	-0.30203	-0.44853	0.957826	-0.76691	0.873608
-0.32181	-1.99142	0.518891	-0.59561	-1.78149	-0.79414	1.0625	1.83861
0.626424	0.179701	-1.85872	0.269425	0.858583	0.419005	1.40497	-0.63827
0.976309	2.280774	2.866851	1.634329	0.990006	-0.23951	0.127575	-2.19514
0.44894	1.075119	1.689274	1.475581	-1.03203	-0.18468	0.866304	-1.19854
0.558334	-0.85079	0.067652	-0.21733	-0.27136	-1.08395	-0.47462	1.246703
-0.65523	-0.86594	1.650949	0.042898	0.893246	1.769013	-0.00528	0.505914
-1.26232	1.013604	1.147206	0.105458	0.590284	-1.02945	-0.65664	0.521887
0.902779	0.286925	-0.18876	0.272094	-0.39127	0.280675	-2.77599	1.424694
-1.17387	2.605709	-0.39121	0.122448	0.43523	0.314019	-0.37809	-0.66442
0.726144	-0.24025	-0.03335	0.791683	-1.231	-1.59685	0.149208	0.455159
1.18528	0.043876	1.777507	-0.30699	-0.29853	0.657965	0.601112	0.803147
1.138225	0.887483	-0.52012	1.734477	0.1218	-0.46349	1.165336	0.171781

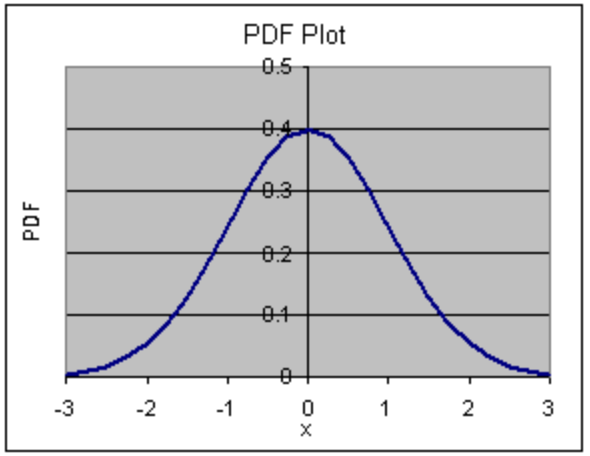
Bag #2

1.265675	0.848201	0.819197	0.189162
2.914639	0.067836	3.785975	1.267826
0.686888	0.098782	6.034544	0.912695
1.029218	4.281229	0.711612	0.958154
6.985271	1.921583	1.121907	0.799197
0.54227	1.326231	1.582003	0.999151
0.428173	4.567446	0.19616	4.988572
8.785572	3.877789	5.698939	1.455257
0.191375	0.721186	0.633513	3.18961
3.753661	8.632928	3.928738	1.61795
0.442747	0.78904	0.182824	1.007515
4.614461	6.452247	1.54774	1.167165
3.775211	2.233818	0.39789	0.779513
0.791782	1.422401	0.766199	0.372987
0.857405	0.095834	7.152579	0.319819
2.591271	0.677541	5.013876	5.268087
0.799215	3.002185	0.366671	7.439692
1.79157	0.902246	1.771052	5.918061
4.16152	0.35055	1.357161	2.058974
1.521754	0.841953	1.838735	1.537069

- What do you do with a bag of numbers?

Histograms





- One way to look at data is a histogram
  - Counts number of data points per bin
  - Bin range is adjustable, depends on data
  - Lumpy approx. to the PDF (Probability Density Function)
- Useful for seeing the overall shape of the distribution

Making a Histogram in Excel

Say data is in C2:C201

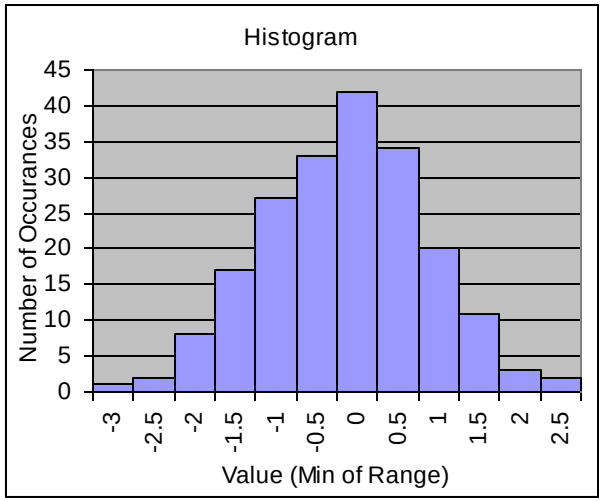
User must specify bins

=COUNTIF(C2:C201, ">="&G2) - COUNTIF(C2:C201, ">="&H2)

	C
1	Data
2	-1.26755
3	2.145411
4	1.650385
5	0.316924
6	0.480242
7	-0.36264
8	1.838188
9	-0.86969
10	-0.5533
11	-0.32181
12	0.626424
13	0.976309
14	0.44894
15	0.558334

G	H	I
Min	Max	Number
-3	-2.5	1
-2.5	-2	2
-2	-1.5	8
-1.5	-1	17
-1	-0.5	27
-0.5	0	33
0	0.5	42
0.5	1	34
1	1.5	20
1.5	2	11
2	2.5	3
2.5	3	2
		200

Sum of counts to verify that all data points have been counted

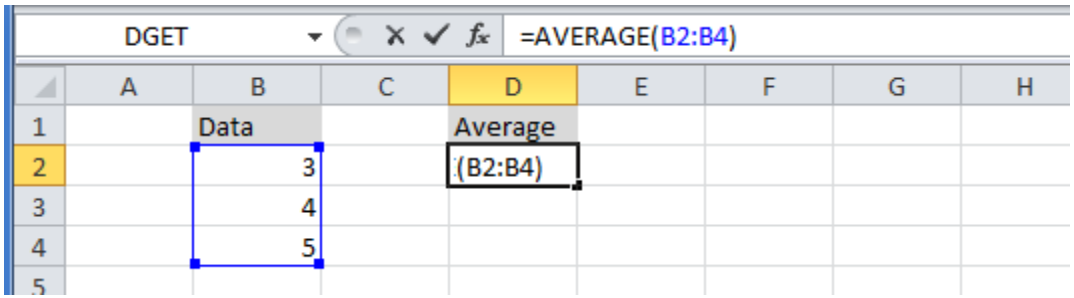


- Instructive – you must create your own bins
  - Note, "FREQUENCY" function is another method

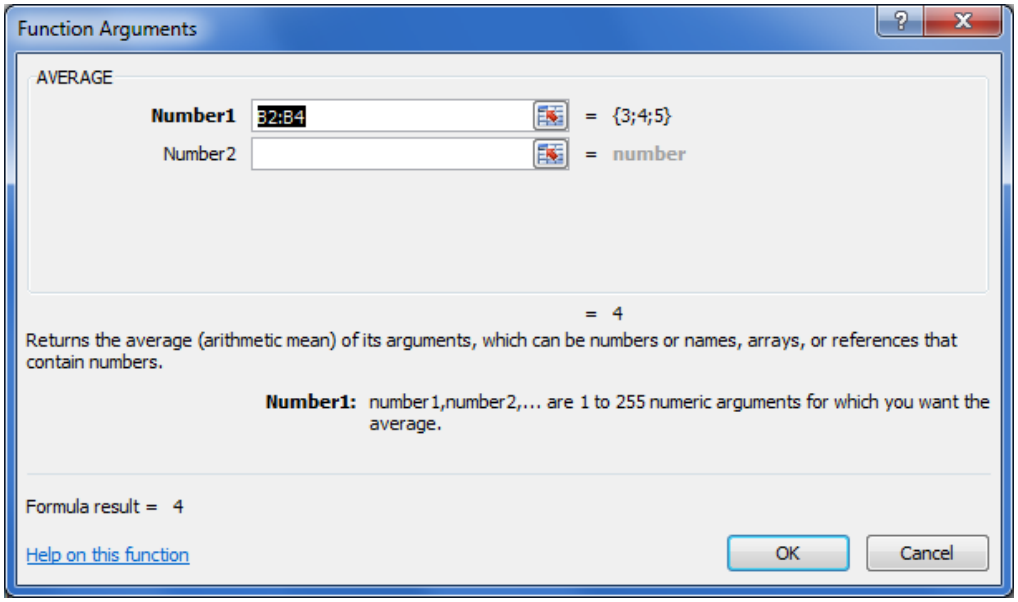
Using Excel

Cell Functions

Excel's greatest strength is cell functions (in my opinion)

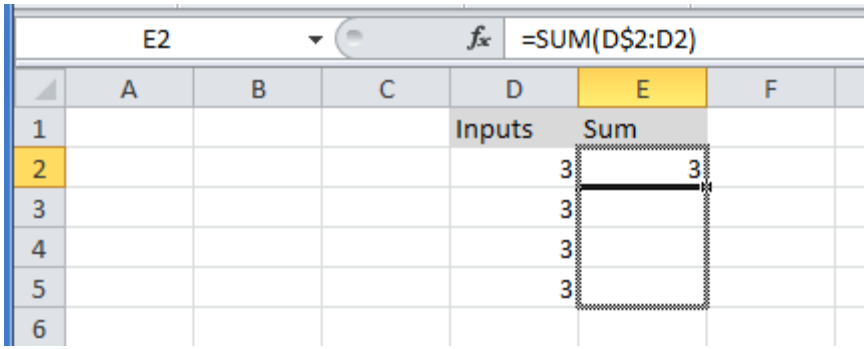


Clicking the fx button

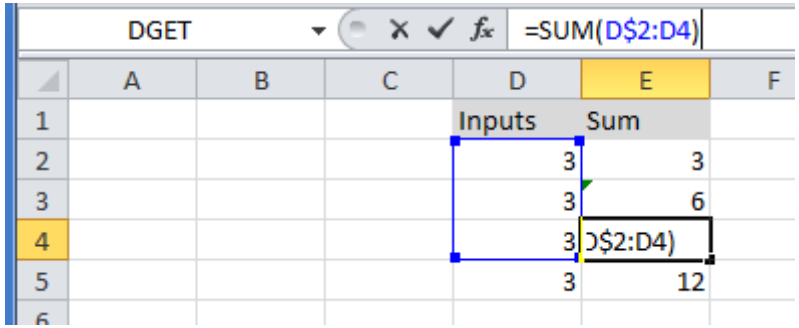


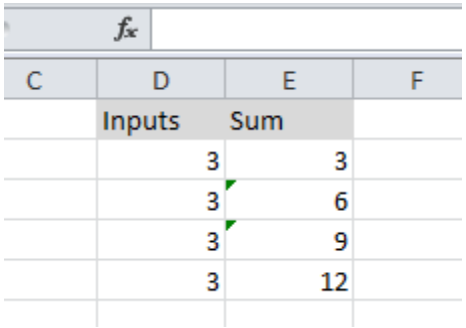
Relative Addressing, Copying Functions

Copy functions by dragging the black square



$ means absolute address, which doesn't change while copying

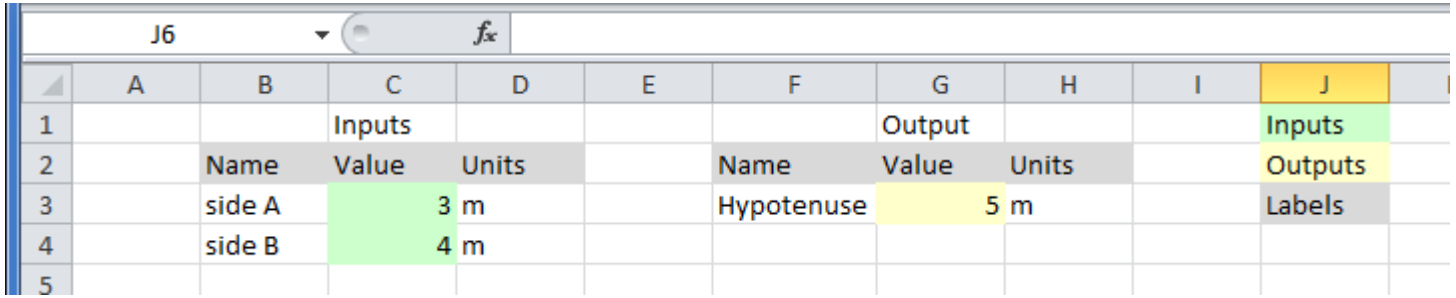




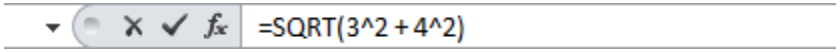
Style Suggestions

Strive to make your spreadsheets understandable to someone else (or to you next year)

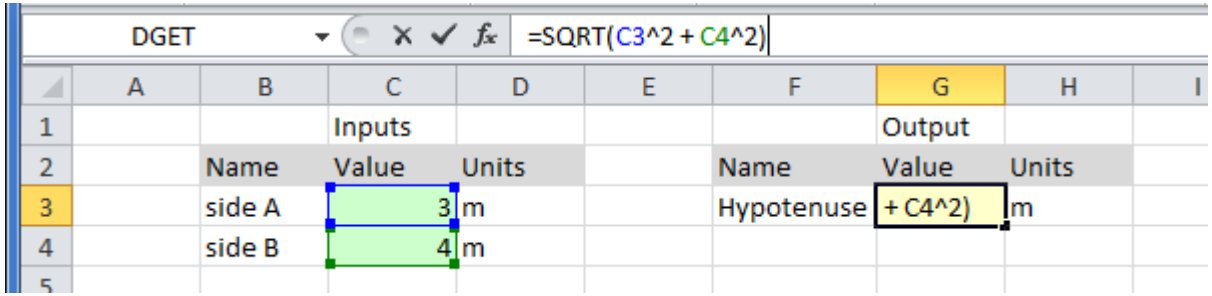
Put inputs and outputs in tables with labels; color coding *sometimes* helps



Don't put input values as numbers in cells

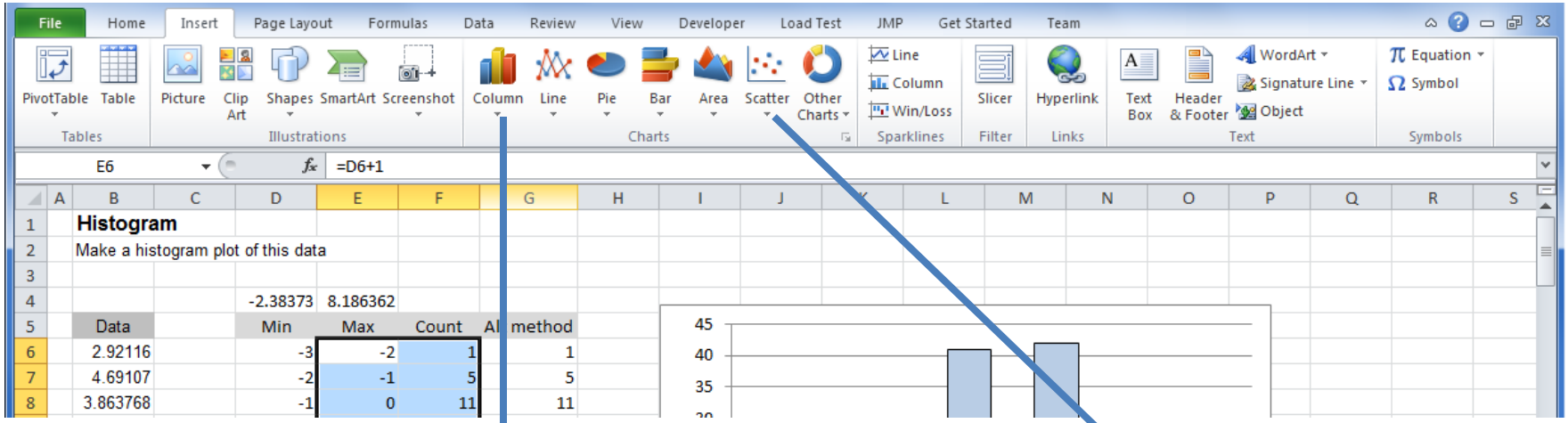


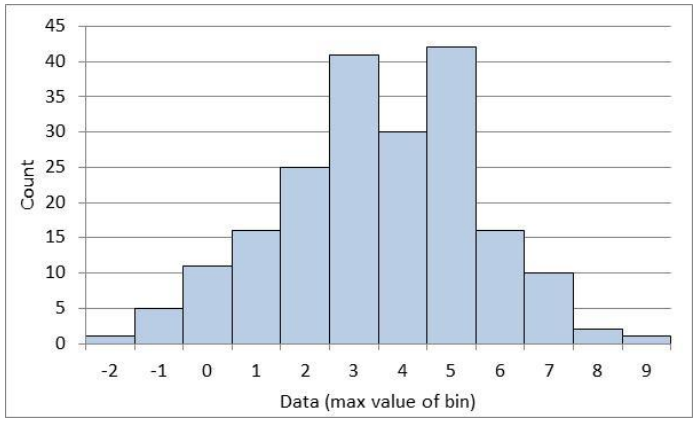
Put values in other cells and reference them

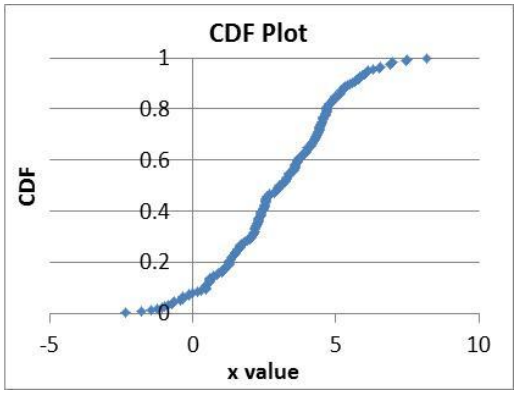


Graphs

Select data and then Insert the type of graph







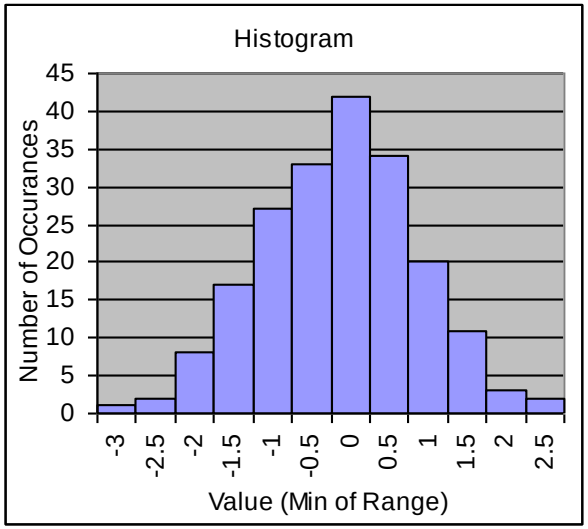
Back to data plotting

Exercise 2.1

- Make a histogram of the data in tab “Ex 2.1”.

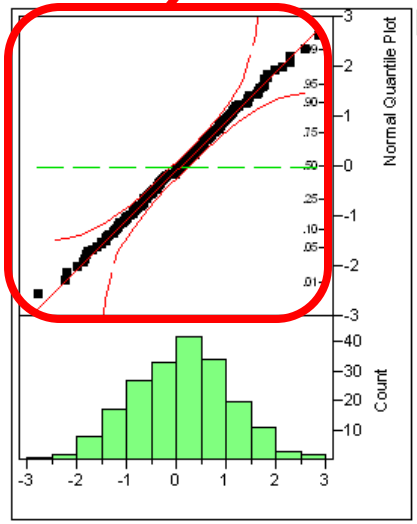
Histograms in JMP

Our Excel histogram:



CDF plot

JMP makes histograms automatically:



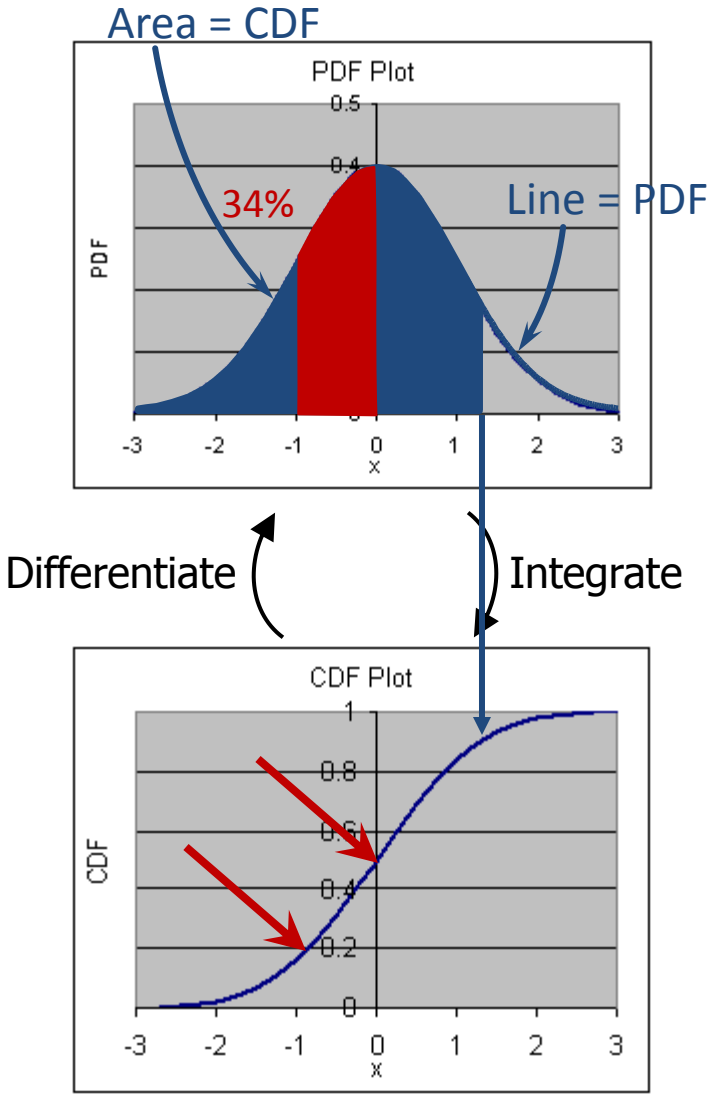
Quantiles			Moments	
100.0%	maximum	2.867	Mean	0.1003821
99.5%		2.866	Std Dev	1.0077467
97.5%		2.141	Std Err Mean	0.0712585
90.0%		1.436	upper 95% Mean	0.2409006
75.0%	quartile	0.787	lower 95% Mean	-0.040137
50.0%	median	0.125	N	200
25.0%	quartile	-0.604	Sum Wgt	200
10.0%		-1.196	Sum	20.076412
2.5%		-1.858	Variance	1.0155535
0.5%		-2.773	Skewness	-0.01569
0.0%	minimum	-2.776	Kurtosis	-0.107046
			CV	1003.9112
			N Missing	0

CDF Plot

- PDF (Probability Density Function)
  - Area under PDF = 1
- CDF (Cumulative Distribution Function)
  - Range of values is 0 to 1
- Related to each other:

$$CDF(x) = \int_{-\infty}^{x} PDF(x')\,dx'$$

$$PDF(x) = \frac{d}{dx} CDF(x)$$

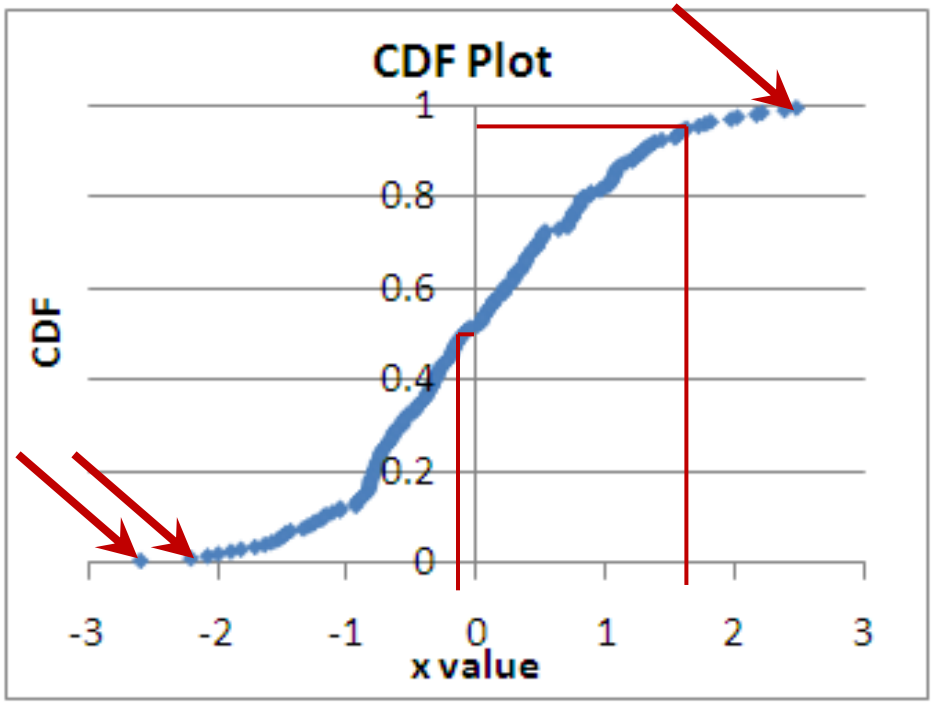


CDF Plot

Rank=1 for *lowest* data point

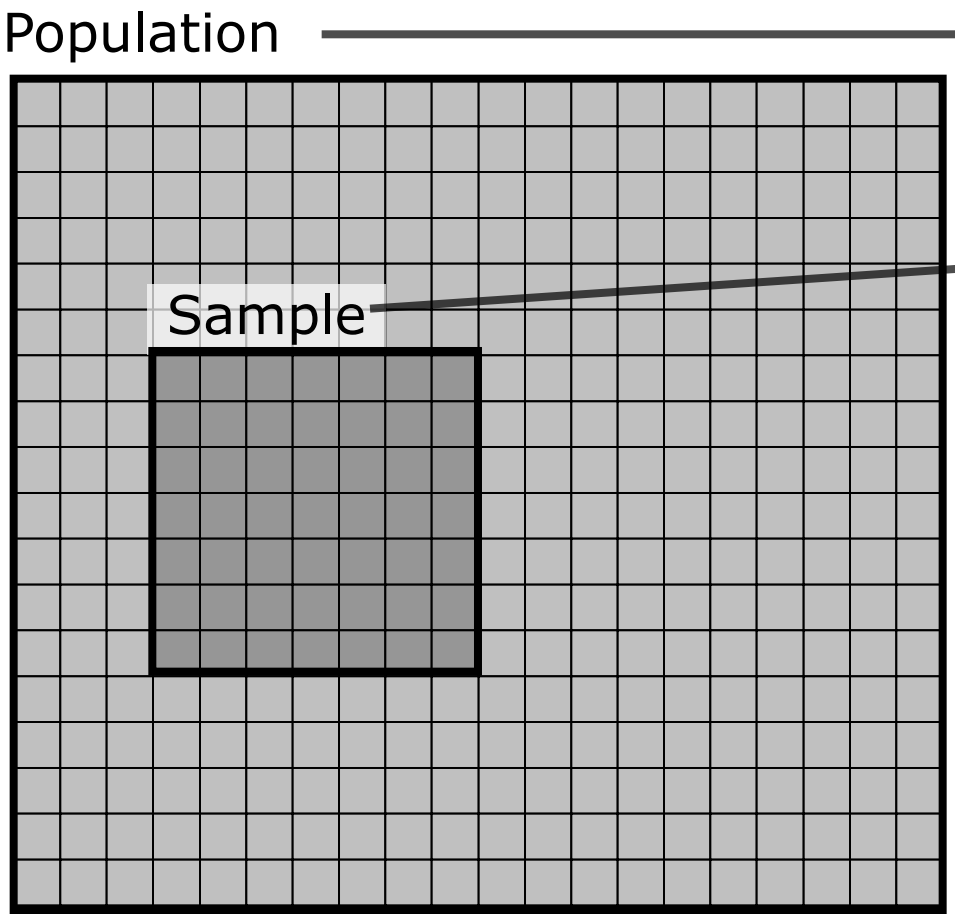
$$\frac{Rank - 0.3}{Count + 0.4}$$

2	Data	CDF
3	2.476147	0.996507
4	-0.93374	0.133234
5	0.126027	0.567365
6	-1.71652	0.038423
7	-0.14318	0.487525
8	-1.20213	0.098303
9	-0.75337	0.233034
10	0.057801	0.542415
11	-0.43195	0.352794
12	-0.15637	0.482535
13	0.35763	0.652196
14	-0.2927	0.422655
15	-0.30083	0.417665
16	-0.38647	0.372754
17	-1.26719	0.088323
18	1.812076	0.966567
19	-0.53628	0.327844
20	1.553529	0.936627



- See all data points; no binning

Statistical Inference



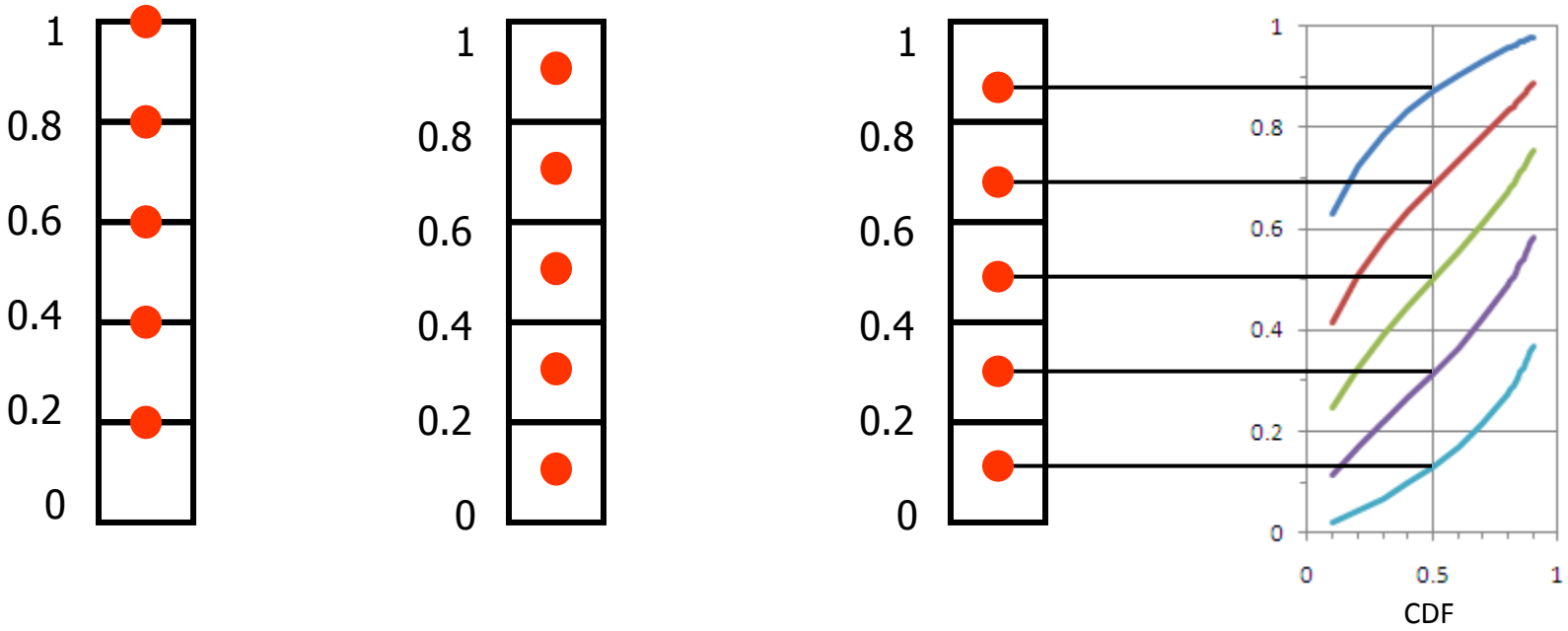
True ("population") value
= parameter

Sample value
= statistic

- Use a sample ***statistic*** to estimate a population ***parameter***

CDF Counting

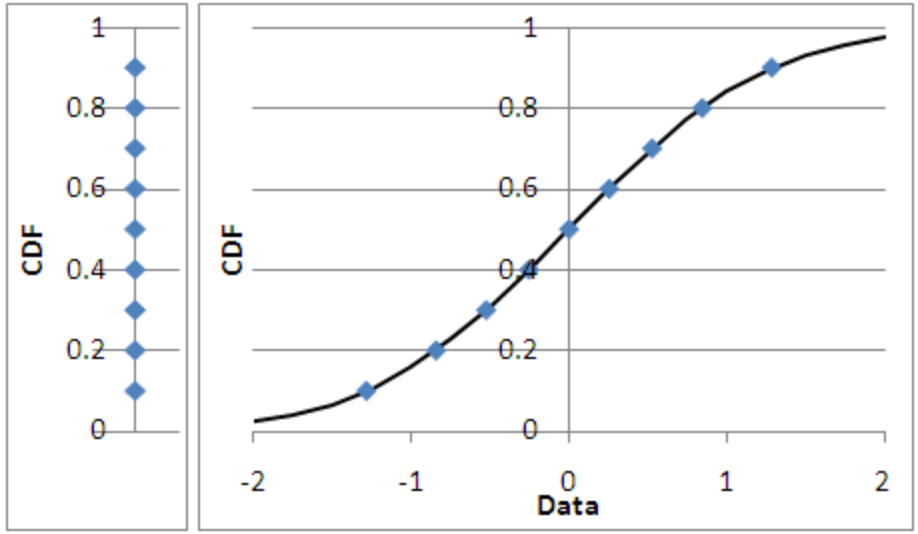
$$\frac{Rank}{Count} \qquad \frac{Rank-0.5}{Count} \qquad \frac{Rank-0.3}{Count+0.4} = \text{"Median Rank"}$$



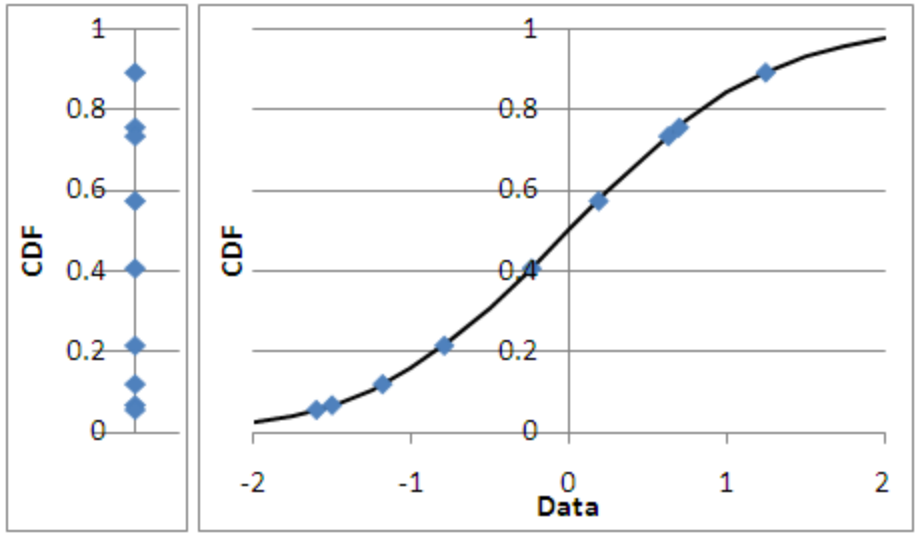
- Why CDF = (Rank–0.3)/(Count+0.4) ?
- Median rank gives the median location if experiment repeated many times

Sampling a CDF

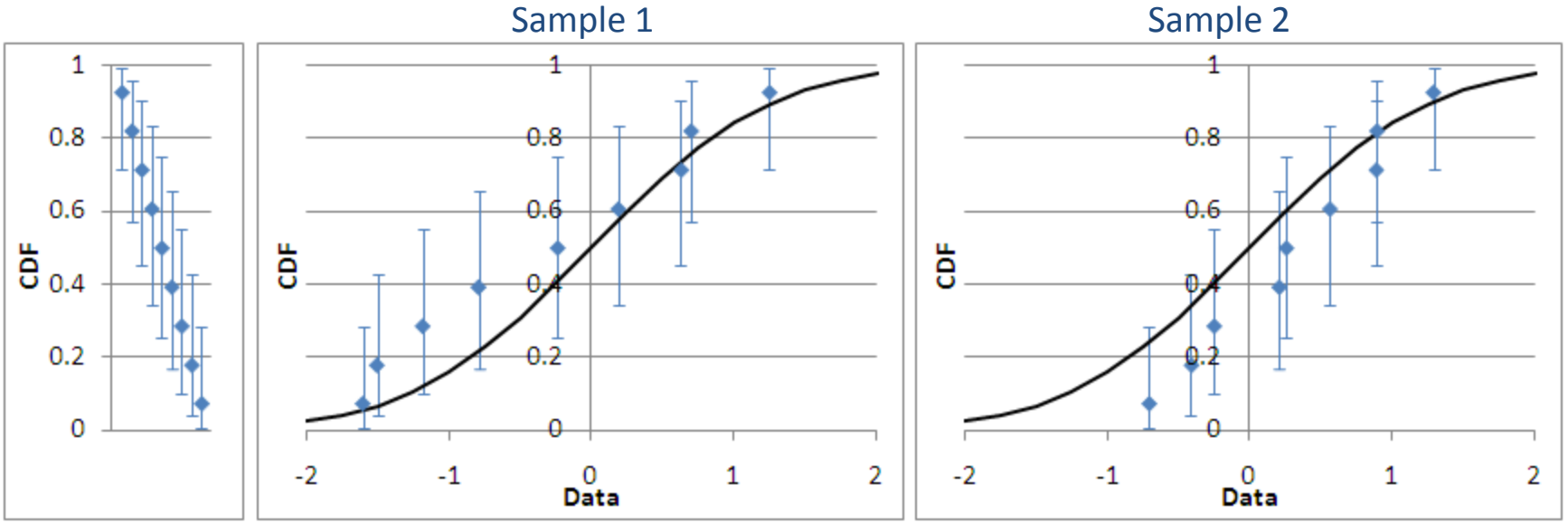
Want to sample uniformly



Actually sample randomly



Sampling a CDF



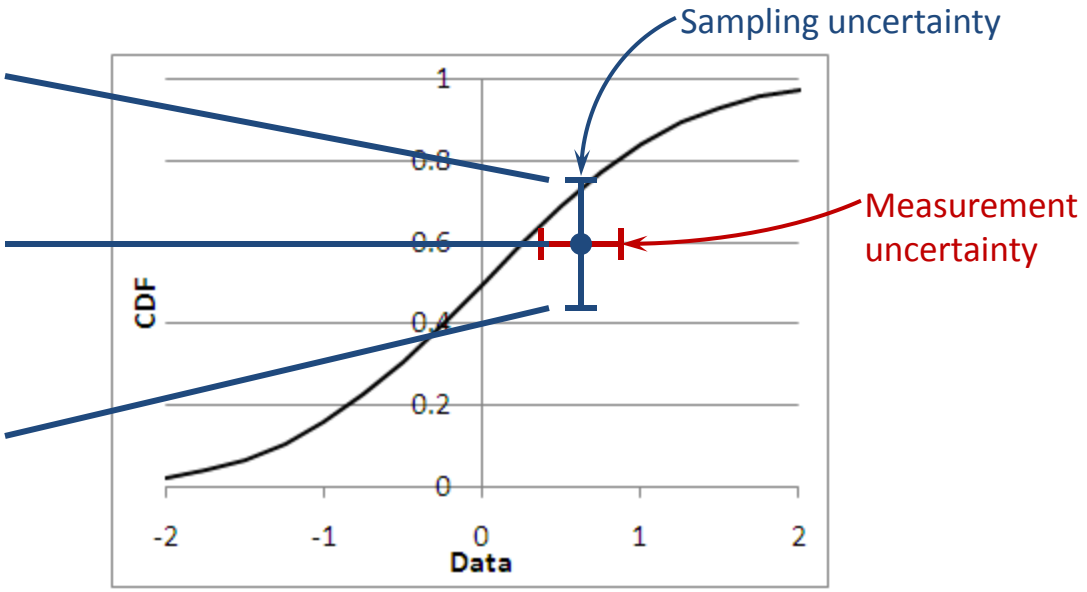
- Range of possible CDF locations for each sample
- Median rank is median of this range

Sampling Uncertainty

BETAINV(0.95, Rank, Count-Rank+1)

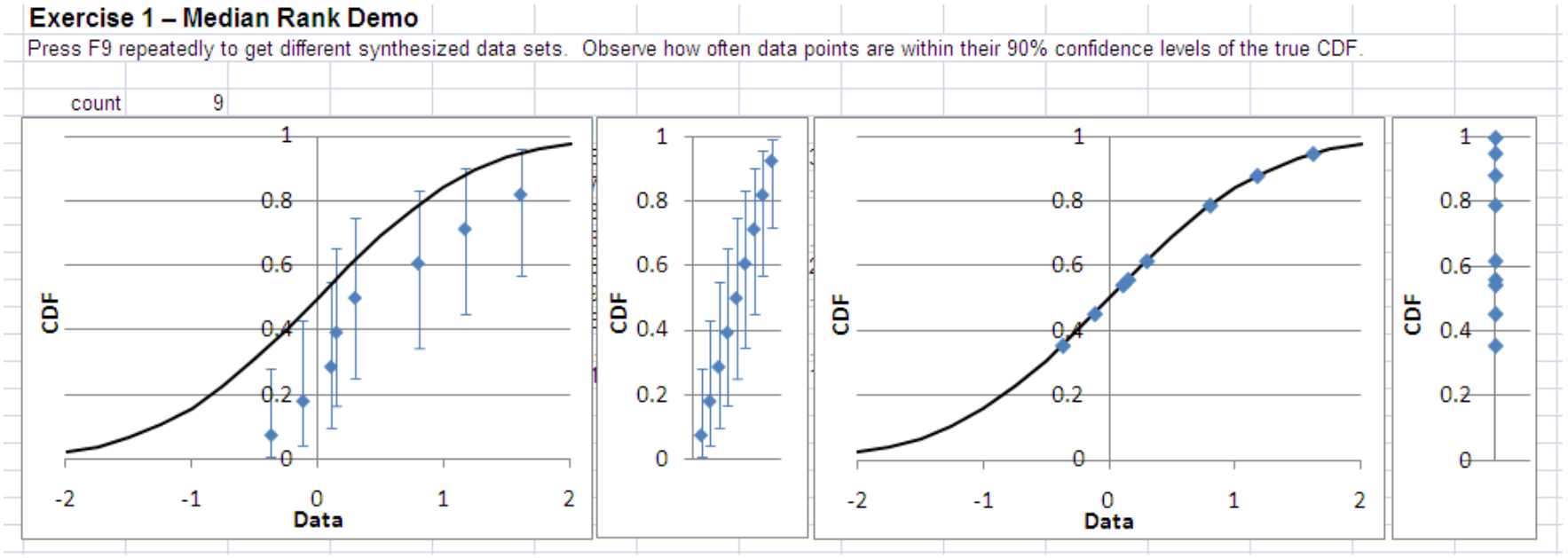
BETAINV(0.50, Rank, Count-Rank+1)
≈ (Rank – 0.3) / (Count + 0.4)

BETAINV(0.05, Rank, Count-Rank+1)



- Different from measurement uncertainty

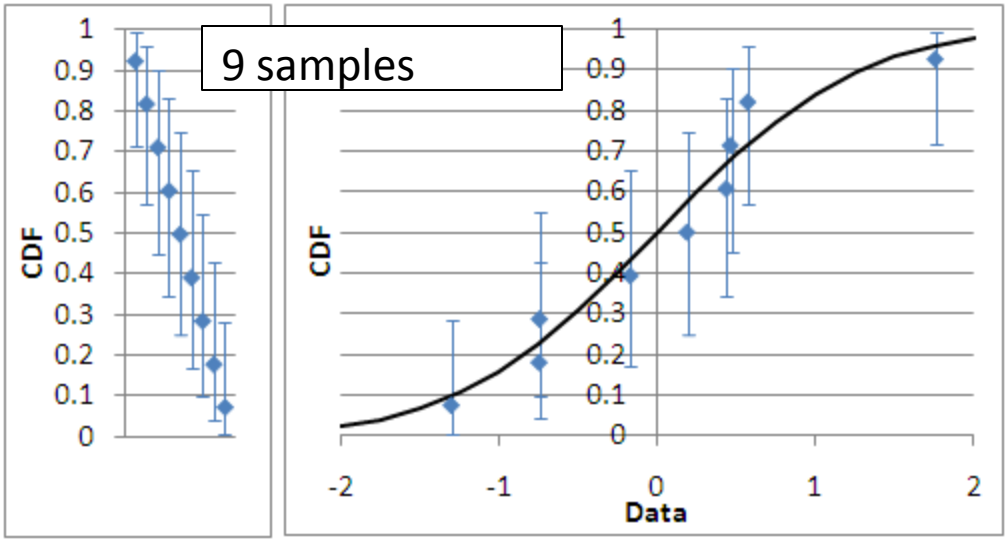
Exercise 2.2

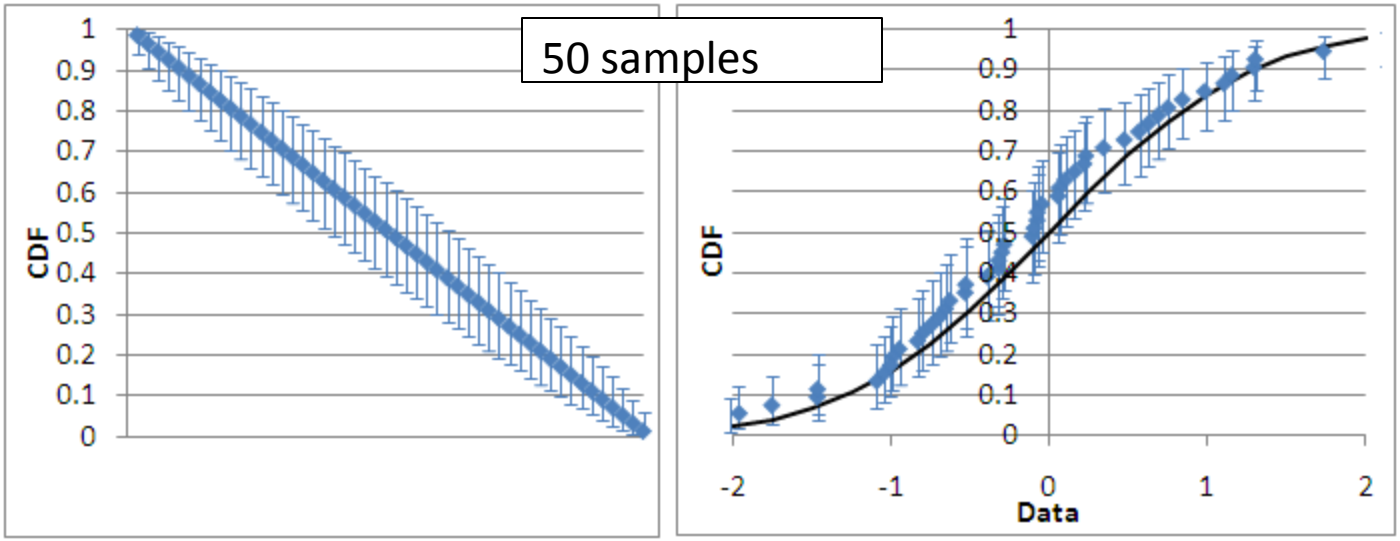


- Find the Median Rank Demo
- Press F9 several times to see different synthesized samples
- Observe the behavior

To Reduce Sampling Uncertainty…

…take more samples





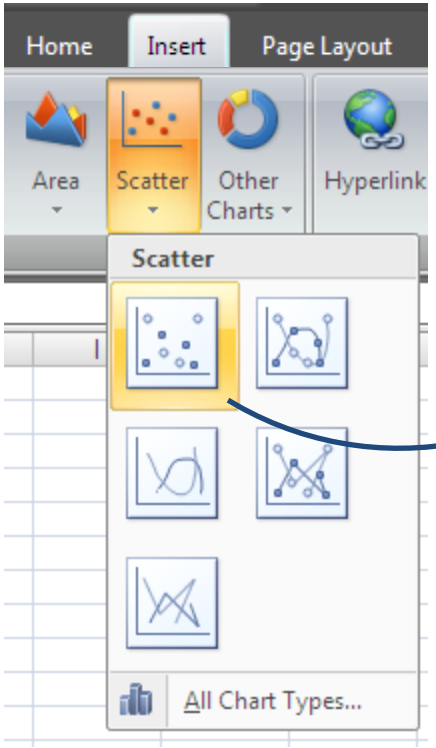
CDF Plot in Excel

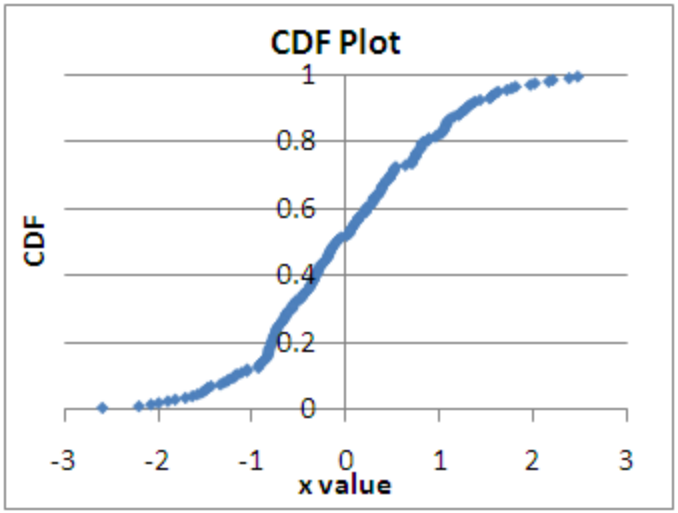
=COUNT(A3:A10000)

$$\frac{Rank - 0.3}{Count + 0.4}$$

=(RANK(A3, $A$3:$A$10000, 1) - 0.3) / ($B$1 + 0.4)

	A	B
1		200
2	Data	CDF
3	2.476147	0.996507
4	-0.93374	0.133234
5	0.126027	0.567365
6	-1.71652	0.038423
7	-0.14318	0.487525
8	-1.20213	0.098303
9	-0.75337	0.233034
10	0.057801	0.542415
11	-0.43195	0.352794
12	-0.15637	0.482535
13	0.35763	0.652196
14	-0.2927	0.422655
15	-0.30083	0.417665
16	-0.38647	0.372754
17	-1.26719	0.088323
18	1.812076	0.966567
19	-0.53628	0.327844
20	1.553529	0.936627





To remove "ties":

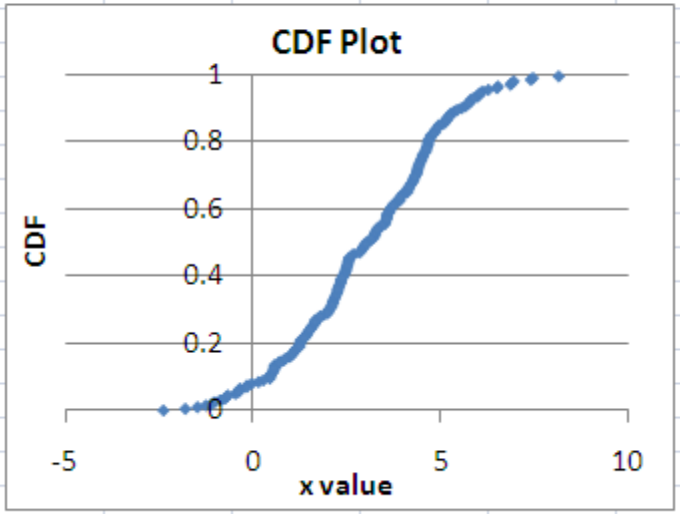
=(RANK(B6, $B$6:$B$10000, 1) + COUNTIF($B$6:B6, "="&B6)-1 - 0.3) / ($C$4 + 0.4)

Exercise 2.3

- Make a CDF plot of the data given in the Ex 2.3 tab

Exercise 2.3 Solution

	200
Data	CDF
2.92116	0.482535
4.69107	0.796906
3.863768	0.622255
0.556751	0.118263
4.273871	0.682136
2.150888	0.322854
4.165581	0.662176
2.175981	0.332834
0.584248	0.138224
4.663427	0.786926
1.139328	0.183134
5.582209	0.901697
4.966402	0.846806
2.507061	0.427645
4.636447	0.776946
2.979884	0.492515
0.565277	0.123253
5.83117	0.921657
3.933394	0.627246
2.024169	0.297904
3.316007	0.537425
5.833778	0.926647



The End