RAT Selection Games in HetNets

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***Abstract*—We study the dynamics of network selection in heterogeneous wireless networks (HetNets). Users in such networks selfishly select the best radio access technology (RAT) with the objective of maximizing their own throughputs. We propose two general classes of throughput models that capture the basic properties of random access (*e.g.*, Wi-Fi) and scheduled access (*e.g.*, WiMAX, LTE, 3G) networks. Next, we formulate the problem as a non-cooperative game, and study its convergence, efficiency, and practicality. Our results reveal that: (i) Single-class RAT selection games converge to Nash equilibria, while an improvement path can be repeated infinitely with a mixture of classes. We next introduce a hysteresis mechanism in RAT selection games, and prove that with appropriate hysteresis policies, convergence can still be guaranteed; (ii) We analyze the Pareto-efficiency of the Nash equilibria of these games. We derive the conditions under which Nash equilibria are Pareto-optimal, and we quantify the distance of Nash equilibria with respect to the set of Pareto-dominant points when the conditions are not satisfied; (iii) Finally, with extensive measurement-driven simulations we show that RAT selection games converge to Nash equilibria in a small number of steps, and hence are amenable to practical implementation. We also investigate the impact of noisy throughput measurements, and propose solutions to handle them.**

I. Introduction

Heterogeneity of wireless network architectures (*e.g.*, the coexistence of 2.5G, 3G, 4G, Wi-Fi, femto, etc) is increasingly becoming an important feature of the current and next generation of wireless networks. At the same time, mobile devices are increasingly equipped with multiple radio access technologies (RATs) that can connect to and choose among the different access networks. In such heterogeneous wireless environments, an important question that arises is *how should a user select the best access network at any given time?* We can think about cars autonomously switching lanes on a rush-hour highway, which often leads to oscillations, chaos, an reduction in everyone's speed, and realize that the answers may not be straight-forward.

Network selection has been extensively studied in heterogeneous networks (for a survey refer to [1]), particularly in cases when there is assistance from the network ([2], [3]), or when a central controller is able to distribute users across networks in order to optimize some notion of system performance ([4]). In this paper we instead focus on a user-centric approach, in which users make decisions to select the appropriate network, without requiring any signaling overhead or coordination among the different access networks. Users in such networks only strive to maximize their own throughputs without regard for other users. The multi-user RAT selection problem is then essentially a non-cooperative game, named as *RAT selection games*, in which users are the players of the game and the strategies correspond to the selection of RATs.

The main challenge in analyzing the behavior of these games is to incorporate realistic models that (i) capture the multi-rate property of heterogeneous networks (*i.e.*, each user has a distinct transmission rate for each access technology), and (ii) accurately model the impact of each user's decision on other users' received throughputs. We divide the throughput models of different access networks into two general classes. In class-1 throughput models, users on the same base station (BS) achieve the same throughput, however, different user combinations result in distinct throughput values. This class of throughput models is especially suitable to model *throughput-fair* access networks such as Wi-Fi [5]. In class-2 throughput models, each user receives a user-specific throughput value that depends on the number of other users sharing the same BS. This class of throughput models is especially suitable to model *time/bandwidth/proportional-fair* access networks in 3/4G networks. We next analyze some of the most important properties of the equilibria in these games, such as convergence and Pareto-efficiency. We further perform extensive measurement-driven simulations to investigate the performance of distributed RAT selection in practice. The key results are summarized as follows:

- *Convergence:* We prove that in single-class RAT selection games, convergence to Nash equilibria is guaranteed [Theorems 1, 2]. When a mixture of classes is considered, we provide an example 2-user game in which an improvement path can be repeated infinitely without reaching an equilibrium. Thus motivated, we introduce a hysteresis mechanism to RAT selection games, and prove that by applying appropriate hysteresis policies, convergence to equilibria can still be guaranteed [Theorem 3].
- *Efficiency:* We investigate the optimality of Nash equilibria with respect to the set of Pareto-dominant points. We show conditions under which the Nash equilibria are also Pareto-optimal [Theorems 4, 5]. When the conditions are not met, we introduce a metric termed *average Pareto-efficiency gain* to quantify the distance between the Nash points and the set of Pareto-dominant points. We show that in class-1 games, the distance between a Nash point and Pareto-dominant points can become unbounded [Theorem 4]. However, we provide tight constant approximation bounds for class-2 games [Theorems 6, 7].
- *Practicality:* We perform hundreds of measurements across multiple access technologies (*e.g.*, HSPA, HSPA+, Wi-Fi) to obtain information on the availability and quality of access networks in an indoor environment. With extensive measurement-driven simulations, we show that RAT selec-

978-1-4673-5946-7/13/$31.00 ©2013 IEEE

tion games converge to equilibria with a small number of switchings. We also show that the appropriate selection of switching threshold provides a balance between convergence time and the efficiency of equilibria. Finally, we investigate the impact of noisy throughput estimates and propose solutions to handle them.

This paper is organized as follows. We discuss the related work in Section II. We present our system model in Section III. In sections IV and V we investigate the convergence and Pareto-efficiency properties of RAT selection games, respectively. We present the results of our measurement-driven simulations in Section VI. Finally, we conclude in Section VII.

II. Related Work

There exist a large number of studies on network selection in HetNets. We highlight the crucial differences in the models and analysis between this paper and the most relevant samples.

Congestion Games and RAT Selection. Congestion games [6], [7] model the congestion externalities when users compete for limited resources. The idea here is that each user i pays a user-specific cost $c^i_r(x)$ when it uses resource r, which depends on the congestion level (x) and the specific preference of the user for r. The congestion impact of a user on a resource r is denoted by a weight. The congestion level (x) of resource r, is then the sum of the weights of the users that select r. Each user in these games aims to minimize its own cost. Over the last few decades several papers have studied the convergence properties of different classes of these games. Majority of these proofs is based on giving *potential functions* [8] (functions in which the gain (loss) observed by any user's unilateral move, is the same as the gain (loss) in the potential function). The convergence properties of a subclass of these games with separable preferences and player-independent costs was studied in [9]. Our proof in Theorem 1 is an application of [9] to the class-1 RAT selection games. The convergence properties of congestion games with separable preferences and player-independent weights was studied in [10]. Our class-2 throughput models have similarities to the games studied in [10]. However, unlike [10] (and the majority of convergence proofs in related work such as [6], [8], [9], [11]), we present a new proof methodology [Theorem 2] that does not rely on potential functions. More importantly, a key issue we must face in RAT selection games is that different technologies have different classes of throughput models. None of the prior work in game theory has studied the equilibria properties when a mixture of classes in considered.

Fairness and Pareto-Efficiency. Pareto-efficiency is a desirable outcome for non-cooperative games. Over the last few decades several fairness concepts that achieve Pareto-optimality have been introduced ([12], [13]). Our metric of average *Pareto-efficiency gain* quantifies the distance of Nash equilibria with respect to Pareto-dominant points. Similar concepts have been recently introduced in [14], [15] for load balancing, but do not apply to RAT selection games. Other work introduced the concepts of price of anarchy (PoA) [16] and price of stability (PoS) [17]. PoA bounds the distance of any Nash point with respect to an optimum defined by a social welfare function (*e.g.*, sum of throughputs). PoS bounds the distance of the best Nash from the social optimum. In contrast, the Pareto-efficiency metric is more general and fits the questions about RAT selection better. However, one can still derive upper bounds on PoA and PoS in RAT selection games based on our proposed techniques on average Pareto-efficiency gains.

Game Theory Applications in Network Selection. Congestion game based network selection was considered in [18], [19]. However, the model in [18] does not capture the multi-rate property of HetNets, while the model in [19] assumes only a single BS in each class of throughput models. As we will show later, in general multi-rate, multi-BS RAT selection games, convergence cannot be always guaranteed. Other work considered evolutionary game models to study the problem of network selection [20], [21]. In evolutionary games, a group of players form a population, and players from one population may choose strategies against users from other populations. These games assume a large number of users in which each of them has a negligible impact on others. This is not the case with RAT selection games in which an individual user has a major impact on the performance of all other users.

III. System Model

In this section, we present the system model and propose a generic, distributed RAT selection algorithm with autonomous actions by each user.

A. Network Model

We consider a heterogeneous wireless environment which consists of M base stations (BSs) and N users. Here, BS is simply a generic term to collectively represent NB in 3G, eNB in 4G, AP in Wi-Fi, femtoBS in femto-cells, etc. The set of BSs and users are denoted by $\mathbf{M} = \{1, ..., M\}$ and $\mathbf{N} = \{1, ..., N\}$, respectively. We denote the set of users connected to BS k by $\mathbf{N}_k$. Fig. 1 shows an example of such a heterogeneous network in which BSs consists of multiple access networks (LTE, 3G, and Wi-Fi). We assume that all BSs are interference-free by means of spectrum separation between BSs that belong to different access networks, and frequency reuse among same kind BSs. Each user has a specific number of RATs, and therefore has access to a subset of BSs. Note that due to the frequency separation between BSs, each RAT can receive beacon signals from *at most one BS*. If a user's wireless interface is able to receive beacon signals from multiple BSs, we model this functionality by assuming multiple RATs for such an interface. For example, an 802.11b wireless card that is able to receive signals from channels 1, 3, and 11 (in 2.4 GHz band), is denoted as a 3-RAT interface. Different access networks in heterogeneous networks have many different characteristics such as packet sizes, physical layer technology, modulation and coding scheme (MCS), etc. Given today's consumer device capability, while a user can switch its selected RAT (based on its expected performance on other RATs), we assume that *each user uses only a single RAT at any given time*.

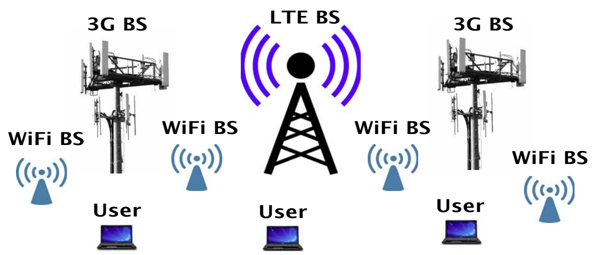

Fig. 1. An example heterogeneous network.

B. Throughput Model

The throughput achieved by a user i on a BS k, denoted as $\omega_{i,k}$, depends on the user's selected access network, the user-specific parameters (*e.g.*, transmission rate) and the other users that are connected to the same BS. The instantaneous PHY rate $R_{i,k}(t)$ of user i on BS k depends on its selected MCS and the channel conditions at time t. We assume stationary channel conditions without considering mobility.

The different access networks in heterogeneous networks have different medium access (MAC) protocols to share the bandwidth among the users. We divide the medium access protocols into two classes:

Class-1 Throughput Models: In this class, the throughput of a user i on BS k depends on the specific users that are connected to k. However, all users that share the same BS achieve the same throughput, *i.e.*, with abuse of notation

$$\omega_{i,k} = f_k(R_{1,k}, R_{2,k}, ..., R_{n_k,k}) \qquad \forall i \in \mathbf{N}_k \tag{1}$$

Here, n_k is the number of users that are connected to BS k. An example of such MAC protocols is the distributed coordination function (DCF) implemented in 802.11, in which a Wi-Fi BS provides fair access opportunity to uplink users [5], [22]. The throughput of the users on the downlink depends on the queuing technique implemented on the BS. The most common technique uses a round-robin scheme. Thus, the downlink throughput of a Wi-Fi user can be expressed as

$$\omega_{i,k} = \frac{L}{\sum\limits_{j \in \mathbf{N}_k} \frac{L}{R_{j,k}}} \qquad \forall i \in \mathbf{N}_k \tag{2}$$

Here, L is the packet size. Throughput models similar to Eq. (1) can also be derived for the uplink [22].

Class-2 Throughput Models: In this class, the throughput of a user i on BS k depends only on the total number of users that share the same BS (*i.e.*, n_k), instead of the specific user combination. However, the throughput of each user can be different from other users , *i.e.*,

$$\omega_{i,k} = R_{i,k} \times f_k(n_k) \qquad \forall i \in \mathbf{N}_k \tag{3}$$

Time-fair TDMA MAC protocols are an example of class-2 throughput models. Here the wireless medium is time-shared among all the users such that each user has the same time duration to access the medium. Therefore, the throughput of a user i connected to a time-fair BS k is given by

$$\omega_{i,k} = \frac{R_{i,k}}{n_k} \qquad \forall i \in \mathbf{N}_k \tag{4}$$

OFDMA based MAC protocols with fair subcarrier sharing (*e.g.*, WiMAX) are another example of class-2 throughput models. With fair spectrum sharing, users receive a similar number of sub-carriers. Hence, the throughput of a user i is roughly dependent only on the total number of users sharing the same BS, and would be similar to Eq. (4).

Another example of Class-2 models is proportional-fair scheduling (PFS) in 3G networks. Here, the PFS algorithm schedules at the next slot, the user that has the highest instantaneous rate relative to its average throughput (for details refer to [23]). With PFS, the closed-form expression of the average throughput of a user with Rayleigh fading is

$$\omega_{i,k} = \frac{R_{i,k}}{n_k} \times \sum_{j=1}^{n_k} \frac{1}{j} \qquad \forall i \in \mathbf{N}_k \tag{5}$$

Here, $\sum_{j=1}^{n_k} \frac{1}{j}$ appears due to channel fading [23].

C. RAT Selection Games

We model the RAT selection problem in heterogeneous networks as a non-cooperative game, in which users select RATs in a distributed manner to increase their own individual throughputs. Thus, the player set is the set of users, *i.e.*, $\mathbf{N}$. Player strategies are the choice of the RATs (or the corresponding BSs). We denote player i's strategy by σ_i. The strategy profile of all users is denoted by $\sigma = (\sigma_1, \sigma_2, ..., \sigma_N)$.

A strategy profile σ is said to be at Nash equilibrium if each player considers its chosen strategy to be the best under the given choices of other players. Therefore, at Nash equilibrium, no user will profit from deviating its strategy unilaterally.

We define a *path* as a sequence of strategy profiles in which each strategy profile differs from the preceding one in only one coordinate. If the unique deviator in each step strictly increases its throughput, the path is called an *improvement path*.

D. Distributed RAT Selection Algorithm

We propose a generic distributed RAT selection algorithm. While the algorithm is simple as we wished, its performance analysis is actually consequently more difficult. Consider synchronized slotted time for now. In RAT selection games, each user uses only one RAT at any given time for communication. However, a user is able to decode the traffic on its other RATs. For example, if RAT j of user i is tuned to BS k, then user i is able to decode the packets transmitted by k, and therefore has the information on the number of users on k and their rates. Thus, each user can estimate its expected throughput if it decides to use another RAT for communication.

Algorithm 1 summarizes the RAT selection algorithm operated by each user. In order for user i to make a switch at time $t+1$ from BS k to BS k' (by changing its RAT), the expected gain defined as $\frac{\omega_{i,k'}[t+1]}{\omega_{i,k}[t]}$ should be higher than a given threshold (η) for the past T time slots (Line 2).

Here, T corresponds to the frequency of measurement prior to switching. Note that if multiple users switch to a BS concurrently, their expected throughputs would be different from their achieved throughputs. In order to minimize the number of concurrent switches to the same BS, we assume that users switch probabilistically with probability $p < 1$ (Line 4). The randomization parameter, p, depends on the congestion in the network and acts similarly to the 802.11 contention window mechanism. Similar to the binary exponential back-off in the 802.11 DCF, we assume that when concurrent migrations to a BS happen, a user sets its randomization parameter to p^{m_i+1} (Line 6), in which m_i is the number of past consecutive concurrent migrations observed by i.

Algorithm 1: RAT Selection Algorithm

Input: user i's parameters: η, T, p, h, Set of RATs
Output: Decision to switch, and the selected RAT
1 **for** *each RAT* k' **do**
2   **if** $\frac{\omega_{i,k'}[t+1]}{\omega_{i,k}[t]} > \eta, \forall\ t = t-T+1, ..., t$ **then**
3   **if** $class(k') = class(k)$ **then**
4   **if** *rand* $< p^{m_i+1}$ **then**
5   switch to k'
6   **if** *concurrent move* **then** increment m_i
7   **else** reset m_i to 0
8   **else**
9   **if** $\omega_{i,k'} > h$ **then**
10   **if** *rand* $< p^{m_i+1}$ **then**
11   switch to k', update h
12   **if** *concurrent move* **then** increment m_i
13   **else** reset m_i to 0

Since users in the RAT selection games selfishly switch their selected RAT to increase their own throughput, it is possible for some of the users to keep switching without reaching an equilibrium. A system design mechanism to dampen oscillations and guarantee convergence is to employ hysteresis. The hysteresis parameter in the RAT selection games, h, denotes the dependence of the RAT switching to the history of past switches that a user has made. Algorithm 1 shows a hysteresis policy in which a user that changes its class of BSs (*e.g.*, from class-1 to class-2) needs to have an expected throughput higher than its hysteresis value (Lines 8-9). In section IV we define our hysteresis policy in detail and demonstrate how it can guarantee convergence to equilibria in RAT selection games.

IV. Convergence

In this section, we investigate the convergence properties of the RAT selection games. We first consider the case in which all BSs belong to the same class of throughput models. Next, we consider the case when a mixture of the two classes exists. The behaviors qualitatively differ, as we will show.

In RAT selection games, different users can occasionally join and/or leave a *single BS* concurrently. However, due to the presence of the randomization parameter p, such events happen infrequently and diminish rapidly when there is network congestion (due to the exponential decrease of p with congestion). For the rest of this section, we ignore these events.

TABLE I
MAIN NOTATION

N	Number of users
M	Number of BSs
n_k	Number of users on BS k
$\mathbf{N}_k$	Set of users on BS k
$R_{i,k}$	PHY rate of user i to BS k
$\omega_{i,k}$	Throughput of user i to BS k
σ_i	Strategy profile of user i
η	Switching threshold
p	Randomization parameter
T	Frequency of measurement prior to switching
h	Hysteresis parameter in the switching algorithm

A. Single-Class RAT Selection Games

We first consider the case in which all BSs belong to the same class of throughput models. Note that in our model each user has a different rate for each BS. In addition, each BS has a BS-specific model to share the throughput among users.

Theorem 1. *Class-1 RAT selection games converge to a Nash equilibrium.*

Proof: Our proof is in essence similar to the proof given in [9]. Here we apply it to the RAT selection problem and present it for completeness. Denote the throughput of user i by ω_i. For simplicity, assume the following ranking of user throughputs

$$\omega_1 \leq \omega_2 \leq ... \leq \omega_N \tag{6}$$

Define a function g on the *ordered throughput values* as

$$g = \omega_1 \times S^{N-1} + \omega_2 \times S^{N-2} + ... + \omega_N \tag{7}$$

Here, S is a very large number (*i.e.*, $S \ggg \omega_i$, $\forall$ possible $\omega_i : i \in \mathbf{N}$). Now assume that user i migrates from BS a to BS b. Note that in class-1 throughput models, all same-BS users achieve the same throughput. Thus, due to i's migration, the throughput of all users on BSs a and b would be affected. The throughput of users on BS a would increase, since a user has left a. The throughput of users on BS b would decrease, since a user has joined. However, the throughput of users on b would be higher than $\omega_{i,a}$, or else user i would not have migrated. Thus, in the new ranking of user throughputs, the value of g in Eq. (7) strictly increases. As the number of users and BSs is finite, function g cannot increase indefinitely and would terminate at a point, *i.e.*, the Nash equilibrium. ∎

We next focus on class-2 RAT selection games.

Theorem 2. *Class-2 RAT selection games converge to a Nash equilibrium.*

Proof: Our proof is based on contradiction. Define the system state of the network as the set of BSs and their connected users. Now assume that there is a loop in the system, *i.e.* there exists a system-state sequence with identical start and end states, as shown in Fig. 2.

Next, consider the throughput inequalities of the migrating users between any two consecutive states. As in the definitions of the models in Section III, the throughput of a user i on BS k is equal to $R_{i,k} \times f_k(n_k)$, in which n_k is the number of users on BS k. For the example depicted in Fig. 2, we have the following inequalities for the intermediate states

$$R_{i,k} \times f_k(n'_k) > \dots \tag{8}$$

$$\dots \tag{9}$$

$$\dots > R_{i,k} \times f_k(n''_k) \tag{10}$$

Next, we multiply all the terms on the right hand sides, and all the terms on the left hand sides of the inequalities. Note that when a user i migrates to BS k, we have an inequality similar to Eq. (8). Similarly, when eventually user i migrates from BS k (in order to have a cycle), an inequality similar to Eq. (10) exists. Therefore, when we multiply the right and left hand sides, all the $R_{i,j}$ terms will cancel each other.

On the other hand, between any two consecutive states, the number of users on any given BS j goes up or down by 1, each time a user joins or leaves the BS j, respectively. Therefore, whenever the number of users on BS j becomes equal to n_j by a joining user (*i.e.*, there exists an $f_j(n_j)$ term on the left hand side), an $f_j(n_j)$ term would later appear on the right hand side when a user leaves BS j. Therefore, after multiplying the right and left hand sides of the inequalities, the $f_j(n_j)$ terms will also cancel each other. After all the cancellations we have $1 > 1$, which is a contradiction. Since a cyclic system state sequence can not exist, every class-2 RAT selection games terminates at an equilibrium, *i.e.*, the Nash equilibrium. ■

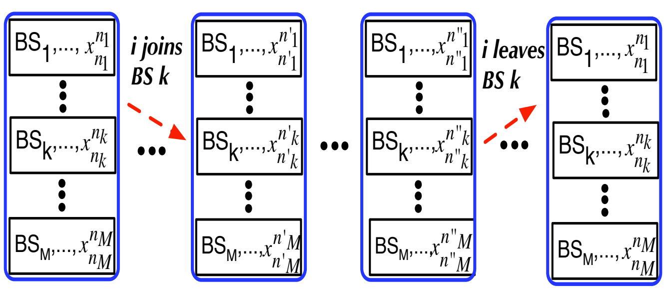

Fig. 2. System state evolution in class-2 RAT selection games. Here $x^i_{n_k}$ denotes the i'th user on BS k and n_k denotes the number of users on BS k. In the state evolution sequence shown above, the beginning and end states are the same, *i.e.* a cycle happens.

B. Mixed-Class RAT Selection Games

In this section, we investigate the convergence properties when there is a mixture of the two classes. We first provide an example 2-player 4-BS game, in which a cycle exists, and therefore convergence to an equilibrium cannot be guaranteed. Next, we show how adding appropriate hysteresis policies can guarantee convergence.

Fig. 3 shows an example 2-player RAT selection game in which an improvement path can be repeated infinitely. The BSs are shown as a, b, c, and d, and the players are displayed as 1 and 2. BSs b and d are throughput-fair and belong to class-1 (Eq. (2)), while BSs a and c are time-fair and belong to class-2 (Eq. (4)). The $R_{i,j}$ value of users on RATs/BSs is shown in Fig. 3.

Initially, users 1 and 2 are connected to BSs a and b, respectively. During each stage of the game, one of the users

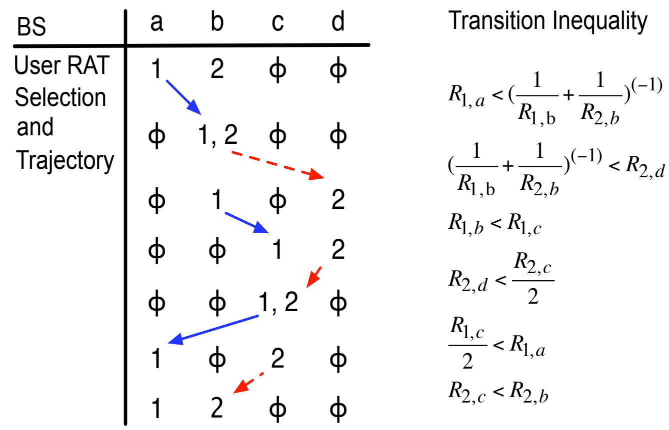

Fig. 3. An example infinite improvement path in a 2-player, 4-strategy RAT selection game with both class-1 and class-2 BSs. BSs a and c are class-2, whereas BSs b and d are class-1. The unique deviator user is shown through arrows in each step. This cyclic path is generated by the six strategy profiles shown, and it can be endlessly repeated. The inequality relevant to each step, *i.e.*, the one that guarantees the RAT switching user strictly increases its throughput is shown on the right. The six inequalities can all be validated for an infinite combination of $R_{i,j}$s. One such example is $R_{1,a}$ = 7.2, $R_{1,b}$ = 9, $R_{1,c}$ = 10.1, $R_{2,b}$ = 48, $R_{2,c}$ = 23.4, $R_{2,d}$ = 9. The selected rates are according to 802.11a for class-1, and 3G HSDPA for class-2.

migrates to another BS in order to increase its throughput. In the example depicted in Fig. 3, the improvement path starts from (a:1, b:2) strategy profile in which user 1 is connected to BS a, and user 2 is connected to BS b. The path continues as (b:1, b:2), (b:1, d:2), (c:1, d:2), (c:1, c:2), (a:1, c:2), and finally back to (a:1, b:2).

The transition inequalities for the migrating user is also depicted in Fig. 3. All these inequalities hold for the selected $R_{i,j}$ values (and can further hold for an infinite number of user-rate combinations). The existence of such a cyclic improvement path demonstrates that in generic mixed-class RAT selection games, an improvement path can be repeated infinitely.

The above example emphasizes the need to design system parameters that can stop infinite oscillations by users and guarantee convergence. We therefore introduce hysteresis, a mechanism that enforces the dependence of the system not only on its current selection, but also on its past selections.

In order to define hysteresis, we classify all the BSs according to their throughput class as depicted in Fig. 4. We next define the hysteresis value of a user i in a given class as its last achieved throughput in that class prior to switching to a different class of BSs. For example, if a user switches from a BS a in class-1 to a BS b in class-2, its hysteresis parameter in class-1 is defined as its throughput on BS a.

Definition 1. *Hysteresis Policy: Assume a user i that has moved from a class of BSs to another class of BSs. In order for i to return to a BS in the previous class, its expected throughput should be higher than the corresponding hysteresis value.*

Fig. 4 shows an example user i that has moved from BS a in class-1 to BS b in class-2. It next changes its selected BS in a series of selfish moves within class-2. Now if user i wants to go to BS d in class-1 from its position on BS c, not only

$\omega_{i,d}$ should be higher than $\omega_{i,c}$, but also $\omega_{i,d}$ should be higher than $\omega_{i,a}$ (*i.e.*, the hysteresis value).

Our next theorem demonstrates that such a hysteresis policy guarantees convergence to an equilibrium.

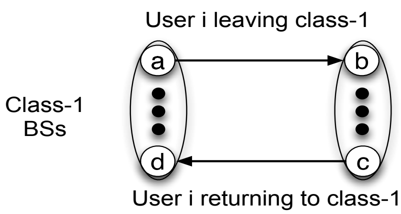

Fig. 4. Hysteresis value of user i in class-1 is its achieved throughput prior to leaving class-1, *i.e.* $\omega_{i,a}$ in the above example.

Theorem 3. *Mixed-class RAT selection games, with hysteresis policy, converge to an equilibrium.*

Proof: Our proof is based on contradiction. Define the system state of the network as the set of BSs and their connected users. Assume there is a loop in the system state evolution that can be repeated infinitely. Consider the second repetition of this loop. Note that in order to have a loop, every user that leaves its class has to return back to its class at a later time. Further, due to the loop repetition, such users have a history of being in both classes.

For simplicity, assume that the cycle starts when a user leaves a class-1 BS. Fig. 4 shows an example of such a user that leaves class-1, and returns back to class-1 (to form a cycle). Denote the throughput of user i prior to leaving class-1 as $\omega_{i,a}$, and immediately after returning to class-1 as $\omega_{i,d}$.

Now assume that for *every* leave and return by *any* user in class-1, there exists a virtual BS. Each of these virtual BSs (*e.g.*, virtual BS v handling user i), handles only one specific user (e.g., by having zero rates for all other users), and offers a throughput equal to the average of the user's throughputs before leaving class-1 and immediately after returning to class-1. For example, the throughput of virtual BS v for user i is equal to $\omega_{i,v} = \frac{\omega_{i,a}+\omega_{i,d}}{2}$. Note that due to the hysteresis policy, $\omega_{i,d}$ is greater than $\omega_{i,a}$, and therefore we have the following inequality

$$\omega_{i,a} < \omega_{i,v} < \omega_{i,d} \tag{11}$$

Eq. (11) shows that user i gains by leaving BS a and joining virtual BS v, and also gains later by returning to BS d.

Now, consider the users that join class-1 from class-2. Such users would later return to class-2 due to the loop. Let j denote an example of such a user that joins BS e on class-1, from a BS in class-2. Note that j has a prior history of visiting a class-1 BS (*e.g.* BS g). Thus, we can assume that user j visits class-1 from a virtual BS v'. The throughput of virtual BS v' for user j is equal to $\omega_{j,v'} = \frac{\omega_{j,e}+\omega_{j,g}}{2}$. Note that due to the hysteresis policy, $\omega_{j,e}$ is greater than $\omega_{j,g}$, and hence $\omega_{j,e}$ is also greater than $\omega_{j,v'}$. Thus, we can correctly assume that user j visits class-1 from a virtual BS v'. Similarly, we can construct another virtual BS, that user j joins when leaving class-1. Thus, for any user that visits class-1 from class-2, we can construct the corresponding virtual BSs. Now, note that each virtual BS accommodates only one user, and therefore it can belong to both class-1 and class-2 throughput models (BSs). Thus, we can assume that all virtual BSs belong to class-1. Now by considering class-1 and all the virtual BSs, it follows that the loop is happening within class-1. However, in Section IV-A we proved that single-class RAT selection games do not have cyclic behavior, which is a contradiction. ■

V. Pareto-Efficiency

Beyond convergence properties, we analyze the Pareto-efficiency of Nash equilibria in RAT selection games. We show that in some cases the Nash equilibria are necessarily Pareto-optimal. When this is not the case, we quantify the improvement of the Pareto-optimal solutions, with respect to the Nash equilibria. In order to do this, we first present the formal definitions of some of the concepts used in this section.

Definition 2. *Let G be a game with a set* $\mathbf{N}$ *of players. We say that strategy profile σ' Pareto-dominates strategy profile σ if it holds that*

$$\forall i \in \mathbf{N}: \quad \omega_{i,\sigma'_i} \geq \omega_{i,\sigma_i} \tag{12}$$

Definition 3. *Let G be a game with N players. Let σ' denote a strategy profile that Pareto-dominates strategy profile σ. We define the average Pareto-efficiency gain of σ' to σ as*

$$\frac{\sum_{i=1}^{N} \frac{\omega_{i,\sigma'_i}}{\omega_{i,\sigma_i}}}{N} \tag{13}$$

For example, assume that strategy profile σ' has an average Pareto-efficiency gain of α with respect to σ. This means that users observe an average of α factor increase in their throughputs by changing from strategy profile σ to σ'. We next proceed to analyze the Pareto-efficiency of RAT selection games. We first do this for class-1 throughput models.

Theorem 4. *Let G be a class-1 RAT selection game with N users. Let σ^p denote a Pareto-optimal strategy profile, and σ^n denote a Nash profile. Let $\gamma = \frac{R_{\max}}{R_{\min}}$ denote the ratio between maximum and minimum rates across all the users. Then*
1) G has a Pareto-optimal Nash equilibrium,
2) The average Pareto-efficiency gain of σ^p to σ^n can become unbounded as $\gamma \to \infty$.

Proof: Part 1. Consider the user-BS profile, σ^1, that maximizes function g in Eq. (7). Since the value of g can not be further increased, σ^1 is a Nash equilibrium. Now assume σ^1 is not Pareto-optimal. Then, there exists a strategy profile σ^2 in which all users achieve higher or equal throughputs with respect to σ^1, and at least one user achieves a higher throughput. Hence, the value of function g in profile σ^2 would be higher than its value in profile σ^1, which is a contradiction.

Part 2. While the best Nash is always Pareto-optimal, the distance between the worst Nash and a Pareto-optimal point can be very large. We provide an example for the throughput model of Eq. (2) to prove this. Assume 2 users and 2 BSs a and b such that

$$R_{1,a} = 1, R_{1,b} = \gamma,\ R_{2,a} = \gamma, R_{2,b} = 1 \qquad (14)$$

The profile (1 in BS a, 2 in BS b) is a Nash point in which each user's throughput is equal to 1. On the other hand, the profile (1 in BS b, 2 in BS a) is a Pareto-optimal point in which each user's throughput is equal to γ ($\gamma > 1$). Thus, there exists a Pareto-optimal point in which each user increases its throughput by a factor of γ and has an average Pareto-efficiency gain of γ, increasing up to 54 in 802.11 a/g. ■

We next investigate the Pareto-efficiency of RAT selection games in class-2 throughput models. Specifically, we focus on the time-fair and proportional-fair throughput models of Eq. (4) and Eq. (5), respectively. We first prove that when each user has a similar rate across different RATs (note that different users can have different rates), all Nash points are also Pareto-optimal. Next, we provide approximations on Pareto-efficiency gains when each user has a distinct rate for each RAT.

Theorem 5. *Let G be a class-2 time-fair (or proportional-fair) RAT selection game with N users. If each user has the same rate across different RATs, then a Nash equilibrium is also Pareto-optimal.*

Proof: Assume the contrary. Let $s(i)$ denote the selected BS of user i in the Nash outcome and n_k denote the number of users on BS k in the Nash outcome. Further, let $q(i)$ denote the selected BS of user i in the Pareto outcome and p_j denote the number of users on BS j in the Pareto outcome. Assume a time-fair throughput model (similar argument holds for proportional-fair model).

From the definition of Pareto-optimality, for each user i we have that $\frac{R_i}{p_{q(i)}} \geq \frac{R_i}{n_{s(i)}}$. Therefore, at least for one user j we have $n_{s(j)} > p_{q(j)}$, and for the rest of the users (*i.e.*, $\forall$ k $\in$ **N** and $k \neq j$) we have $n_{s(k)} \geq p_{q(k)}$. These inequalities show that each BS in the Pareto-point has a smaller (or equal) number of users than in the Nash point (with at least one BS having a smaller number). However, the total number of users across all BSs is equal to N, which is a contradiction. ■

Theorem 6. *Let G be a time-fair RAT selection game with N users and M BSs. Let σ_n denote a non-Pareto-optimal Nash profile and σ_p denote a Pareto-dominant profile with respect to σ_n. Then, the average Pareto-efficiency gain of σ_p to σ_n is bounded by*

$$\begin{cases} 2 & \text{if } N \leq M \\ \frac{N+M}{N} & \text{if } N \geq M \end{cases}$$

Proof: We use the same notation as in the proof of Theorem 5. From Pareto-dominancy definition we have $R_{i,q(i)} \times f_{q(i)}(p_{q(i)}) \geq R_{i,s(i)} \times f_{s(i)}(n_{s(i)})$. From Nash equilibrium property we have $R_{i,s(i)} \times f_{s(i)}(n_{s(i)}) \geq R_{i,q(i)} \times f_{q(i)}(n_{q(i)} + 1)$. Thus, by replacing f with the corresponding value in Eq. (4), the improvement factor of user i is

$$\frac{R_{i,q(i)} \times f_{q(i)}(p_{q(i)})}{R_{i,s(i)} \times f_{s(i)}(n_{s(i)})} \leq \frac{R_{i,q(i)} \times f_{q(i)}(p_{q(i)})}{R_{i,q(i)} \times f_{q(i)}(n_{q(i)} + 1)} \qquad (15)$$

$$\leq \frac{n_{q(i)} + 1}{p_{q(i)}} \qquad (16)$$

Next, the sum of improvement factors of all users is

$$\sum_{i=1}^{N} \frac{n_{q(i)} + 1}{p_{q(i)}} \leq \sum_{p_k, p_k \neq 0} \frac{n_k + 1}{p_k} \times p_k \leq \qquad (17)$$

$$\begin{cases} 2 \times N & \text{if } N \leq M \\ (N + M) & \text{if } N \geq M \end{cases} \qquad (18)$$

The average gain is derived by dividing the above by N. ■

Theorem 6 provides a tight bound on the average Pareto-efficiency gain of time-fair RAT selection games. In order to observe this, consider a 2 player example with 2 BSs a and b with the following rates

$$R_{1,a} = 1, R_{1,b} = 2 - \epsilon,\ R_{2,a} = 2 - \epsilon, R_{2,b} = 1 \qquad (19)$$

The profile (1 in BS a, 2 in BS b) is a Nash profile, in which each user's throughput is 1. The profile (1 in BS b, 2 in BS a) is a Pareto-dominant profile, in which each user's throughput is $2 - \epsilon$. The average Pareto-efficiency gain is equal to $2 - \epsilon$, which can become arbitrarily close to 2 as $\epsilon \to 0$.

Theorem 7. *Let G be a proportional-fair RAT selection game with N users and M BSs. Let σ_n denote a non-Pareto-optimal Nash profile and σ_p denote a Pareto-dominant profile. Then, the average Pareto-efficiency gain of σ_p to σ_n is bounded by*

$$\begin{cases} 2 \times (1 + \mathit{ln(N)}) & \text{if } N \leq M \\ \frac{N+M}{N} \times (1 + \mathit{ln(N)}) & \text{if } N \geq M \end{cases}$$

Proof: We use the same steps as in the proof of Theorem 6. By placing the proportional-fair throughput model of Eq. (5) in Eq. (15), the improvement factor of user i is

$$\leq \frac{n_{q(i)} + 1}{p_{q(i)}} \times \frac{\sum_{k=1}^{p_{q(i)}} \frac{1}{k}}{\sum_{k=1}^{n_{q(i)}+1} \frac{1}{k}} \qquad (20)$$

The bound is next achieved due to the following inequality

$$\frac{\sum_{k=1}^{p_{q(i)}} 1/k}{\sum_{k=1}^{n_{q(i)}+1} 1/k} \leq \sum_{k=1}^{N} 1/k \leq (1 + \ln(\mathrm{N})) \qquad (21)$$

■

VI. PERFORMANCE EVALUATION

In this section, we study the performance of RAT selection games through measurement-driven simulations. We first perform hundreds of measurements to obtain SNR values of multiple wireless access technologies in an indoor building. We next analyze the performance of these games in realistic environments.

Measurement Driven Simulations. We use the *field test* application in the iPhone to obtain information on the number of wireless towers, their frequency of operation and technology, and the received SNR at the receiver. Our measurements were conducted over AT&T's cellular network. We randomly select 100 locations spread across three floors of a large university building. The measured SNR value across all locations is between -68 dBm and -104 dBm. Each user in these locations has access to UMTS/HSPA, while many locations also have access to HSPA+. The average number of towers observed across the users (locations) is 4.

In addition to cellular statistics, we also measure the received SNR of the Wi-Fi BSs, their frequency of operation and technology (802.11 a/b/g). The average number of Wi-Fi BSs observed across all the users is 5. These SNR values are then converted to a data-rate based on the SNR-Rate table of the corresponding technology, and are fed to our simulation.

Equilibrium Analysis. Figs. 5(a) and 5(b) correspond to the number of equilibria and their Pareto-optimality, respectively. With M BSs and N users, there exists M^N system states, defined as the set of BSs and the users connected to them. We consider 9 users each with 3 RATs: 2 Wi-Fi RATs and a 3G RAT. Thus, the total number of system states is $3^9 = 19683$. We randomly select these 9 users from our database of 100 users (locations), and repeat this selection for 20 times. For each realization, we consider all system states and count the number of Nash equilibria, and their Pareto-optimality. Note that while the number of Nash equilibria is dependent on η in our RAT selection algorithm, the number of Pareto-optimal points is not, and averages to 6033 across all realizations.

Fig. 5(a) depicts the total number of equilibria as a function of η for 3 different throughput models. With throughput-fair, the throughput model of *all technologies* is according to the relationship in Eq. (2), while in time-fair, the throughput model of *all technologies* is according to the relationship in Eq. (4). In the mixture mode, all Wi-Fi RATs are throughput-fair, while the 3G RATs are time-fair. With $\eta = 1$, there is an average of 200, 4 and 8 Nash equilibria in the throughput-fair, time-fair, and mixture models, respectively. Thus, only a very small number of states form the equilibria in these games. As η increases, the number of equilibria increases rapidly, and the gap between time-fair and throughput-fair models decreases.

Fig. 5(b) depicts the number of Pareto-optimal and non-Pareto-optimal equilibria as a function of η in the mixture model. We observe that by varying η, the ratio between Pareto and non-Pareto equilibria remains similar, while the individual values increase. Since increasing η can significantly increase the number of equilibria, it has the potential to reduce convergence times without compromising Pareto-efficiency gains, shown later.

Average Pareto-Efficiency Gain. We next evaluate the Pareto-efficiency gains of Pareto-dominant points with respect to Nash equilibria. We consider prior configuration setup with 9 users and 3 RATs. For each Nash equilibrium, we consider the set of Pareto-dominant points and measure the average Pareto-efficiency gain for each Pareto-dominant point, as well as the cardinality of the Pareto-dominant set. Figs. 5(c) and 5(d) depict the corresponding CDF plots across all Nash points. We observe that the average Pareto-efficiency gain in the time-fair model is close to 1, suggesting that in time-fair models Nash points are mostly close to the Pareto-dominant points. The situation in the throughput-fair model is quite the contrary, in which for a small number of Nash points (less than 1% in Fig. 5(c)) the average Pareto efficiency gain can be as high as 10 with a large number of Pareto-dominant points.

Fig. 5(e) depicts the impact of increasing η on the average Pareto-efficiency gains of the mixture model. As η increases, the number of equilibria increases rapidly. However, Fig. 5(e) shows that limiting η to less than 2 only slightly increases the average Pareto-efficiency gains.

Convergence Time. Figs. 5(f) and 5(g) depict the impact of system parameters (number of users/RATs/η) on the average number of per-user RAT switchings and the maximum convergence time in the mixture model. Here we randomly select a given number of users from our user database and execute our RAT selection algorithm. The randomization parameter (p) and the frequency of measurement prior to switching (T) are set to $\frac{1}{2}$ and 4, respectively. The simulation is repeated for 300 initialization points.

Fig. 5(f) shows that increasing the number of RATs from 2 to 5, slightly increases the average number of per-user switchings. Similarly, the number of users has a small impact on the number of per-user switchings. Fig. 5(g) shows a similar trend on the maximum convergence time. Figs. 5(f) and 5(g) also show that by increasing η to 2, the average number of per-user switchings decreases by 1. Thus, the average number of per-user time-slots to reach convergence decreases significantly. Note that a small increase in η does not cause serious degradation in average Pareto-efficiency gains (as observed in Fig. 5(e)), and therefore one can select an appropriate η value for a given network to balance between *convergence time* and *the desirability of the equilibria*.

Further, note that the total number of states (M^N) provides an upper bound on the maximum convergence time. However, our results in Figs. 5(f) and 5(g) show that with appropriate parameter selection (η, p, T), the number of concurrent switchings and oscillations would be negligible, and the system will converge to an equilibrium in a very small number of steps.

Impact of Noisy Measurements. Since the RAT selection algorithm relies on correct throughput prediction on RATs, sensitivity to noisy estimates can become a bottleneck. In Fig. 5(h) we plot the impact of such noise on the average number of switchings for the mixture model. We model the noise by assuming that the predicted throughput is according to a Gaussian distribution in which the mean is equal to the actual throughput and the standard deviation is equal to the product of the noise value and the actual throughput.

Fig. 5(h) shows that increasing the noise power increases the average number of switchings. Further, it is possible for some of the users to keep on switching without reaching convergence. This problem can be addressed by adapting the η value according to the noise power. By increasing the η value, a user requires higher throughput values to make a change, compensating for noisy throughput estimates.

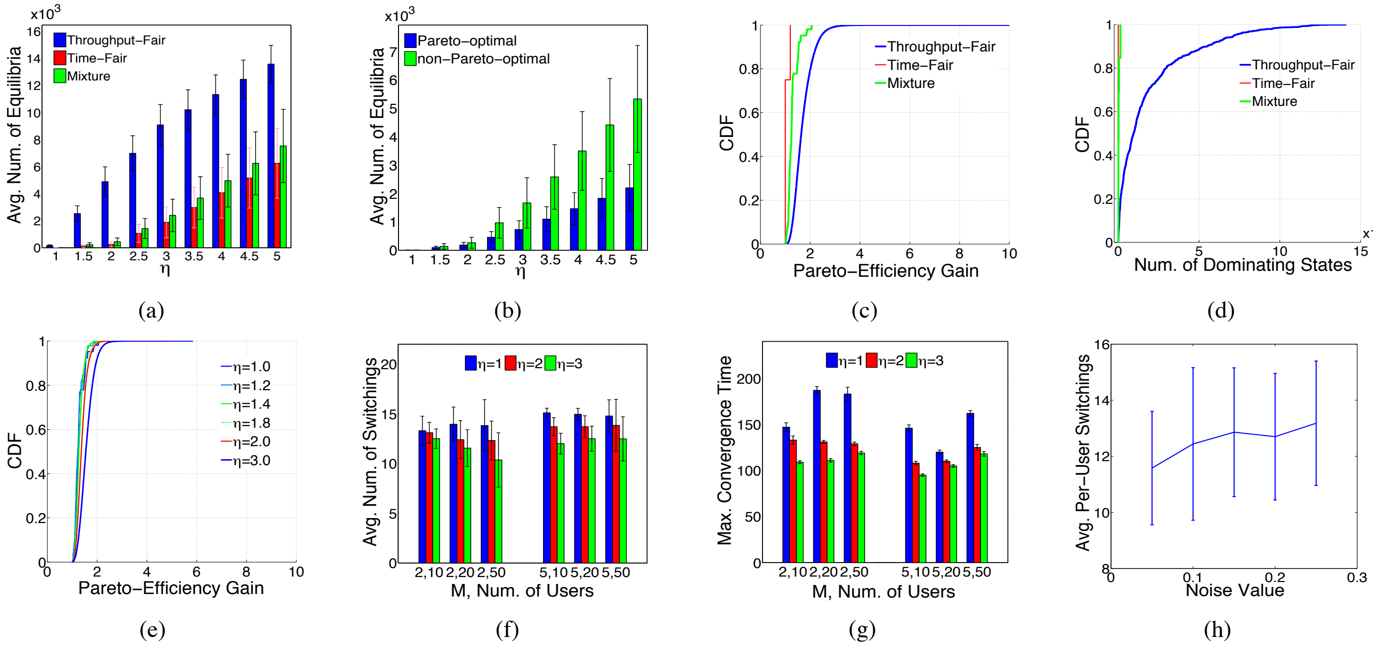

Fig. 5. (a) Average number of equilibria with 9 users and 3 RATs, (b) Pareto-optimal/non-Pareto-optimal equilibria, (c) CDF of Pareto-efficiency gain; (d) CDF of the cardinality of Pareto-dominant sets; (e) Impact of η on Pareto-efficiency gain; (f) Average number of per-user switchings with varying users/RATs/η; (g) Maximum convergence time with varying users/RATs/η; (h) Impact of noisy throughput estimations on the average number of per-user switchings.

VII. Conclusions

We studied the dynamics of RAT selection games in heterogeneous wireless networks. We investigated the convergence properties of these games and introduced hysteresis as a system parameter that can guarantee convergence. We also provided tight bounds on the average Pareto-efficiency gains of RAT selection games. Finally, through measurement-driven simulations we showed that RAT selection games converge to Nash equilibria within a small number of switchings.

VIII. Acknowledgements

This research was in part supported by a grant from Intel. The authors would like to thank Nageen Himayat and Rath Vannithamby at Intel Labs for the useful discussions.

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