Sample solution to Homework 5 Exercise 4

Mystery Proof.

Answer: The problem is in the T-App case, in the attempted use of induction on the result of the substitution.

"By the Substitution Lemma, $\vdash [\mathtt{x} \mapsto \mathtt{v}_2]\mathtt{t}_{11} : \mathtt{T}_2$. Finally, by induction on this derivation, there exists a $\mathtt{v}$ such that $[\mathtt{x} \mapsto \mathtt{v}_2]\mathtt{t}_{11} \Downarrow \mathtt{v}$ and $\vdash \mathtt{v} : \mathtt{T}_2$."

The proof is structured as a structural induction on the typing deriviation $\vdash \mathtt{t} : \mathtt{T}$. This means we're allowed to use the induction hypothesis on *sub-derivations* of this derivation. But $\vdash [\mathtt{x} \mapsto \mathtt{v}_2]\mathtt{t}_{11} : \mathtt{T}_2$ is *not* such a sub-derivation, since $[\mathtt{x} \mapsto \mathtt{v}_2]\mathtt{t}_{11}$ is not (in general) a subterm of $\mathtt{t}$.

Exactly the same problem occurs with proving normalization using a small-step semantics. This is the whole reason for introducing the more complicated logical relations proof method used in Chapter 12.