CS 410/510spec Homework 1 – Tuesday, January 12, 2016

Please turn these in on paper in class.

Do the following Huth&Ryan exercises:

1.1.1.(b,e,i,j,k)

1.1.2.(b,f)

1.2.1.(b,g,m)

1.2.3.(l)

1.3.1.(g)

1.3.6.(b)

1.4.2.(i)

1.4.3.(c)

1.4.6.(a,d)

1.4.12.(a)

1.4.13.(c)

1.5.3.(a)

1.5.6.(b,g)

Problem A. Suppose we want to describe addition of two n-bit unsigned binary numbers $x = x_{n-1} \dots x_2 x_1 x_0$ and $y = y_{n-1} \dots y_2 y_1 y_0$, forming the result $z = z_n \dots z_2 z_1 z_0$. (Note that z potentially has an extra digit due to overflow.)

Write down a boolean formula for each z_i in terms of the x_i and y_i.

Hint: There are many possible answers, but the simplest approach, called a "ripple carry adder" is to view the operation at each bit position as a function

(z_i,c_{i+1}) = one-bit-add(x_i,y_i,c_i)

where c_i is the *carry* into position i from the addition at the previous bit $i-1$. Then your definition of z_i can mention c_i; of course, you'll also need to define c_i. To simplify notation, define $c_0 = 0$, $x_n = 0$, and $y_n = 0$.

Problem B. Some presentations of natural deduction replace the $\vee_e$ rule of H&R Figure 1 by the following rule:

$$\frac{\phi \vee \psi \quad \neg\psi}{\phi}\ \vee_{e'}$$

(a) Show that $\vee_e'$ is derivable from $\vee_e$.

(b) Show that $\vee_e$ is derivable from $\vee_e'$. Note that this is equivalent to deriving the sequent

$$\phi \vee \psi, \phi \to \chi, \psi \to \chi \vdash \chi.$$

Hint: Use two instances of $\neg_i$, one nested inside the other.