Modular Lazy Search for Constraint Satisfaction Problems

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Constraint Satisfaction Problems

- Ubiquitous, important, computationally hard
  - Graph coloring and matching
  - Scene labeling for vision
  - Temporal reasoning
  - Resource allocation for planning, scheduling
  - etc., etc.
- Try to simplify constraints first; then must use brute force
- Handle **binary** constraints over **finite** domains
- Assume nothing known about **structure** of constraint graph
  - n-Queens looks just like graph coloring

CSP Algorithm Zoo

MFC+Random

Backjumping

Conflict-directed Backjumping

Minimal

Forward Checking

CBJ+BM

Fail-first DVO

Backmarking

Domain-specific Heuristics

- No agreed-upon common framework.
- Many problems benefit from tailor-made **combinations** of algorithms.

“Re-use” of Imperative Code

[Kondrak94]

```
#    int FC_CBJ(z)
=    int z;
=    {
=      int h, i, j, jump,
=
=      if (z > N) {
=        solution();
=        return(N); }
#      empty(conf_set[i]);
=      for (i = 0; i < K;
=        if (domains[z][i]
=          continue;
=        v[z] = i;
=        fail = consistent
=        if (fail == 0) {
#
#          jump = FC_CBJ(z
=          if (jump != z)
=            return(jump);
=        restore(z);
=        if (fail)
=          for (j = 1; j <
=            if (checking[
#              add(j,conf_
#
#
=      for (j = 1; j < z;
=        if (checking[j][z
#          add(j,conf_set[
#      h = max(conf_set[z]
#      merge(conf_set[h],c
=      for (i = z; i >= h;
=        restore(i);
=      return(h);
```

```
il;
+)  {
);
1;
1);
;  j++)
[fail])
=
[z],j);}
+)
--)
```

Key:

= identical line

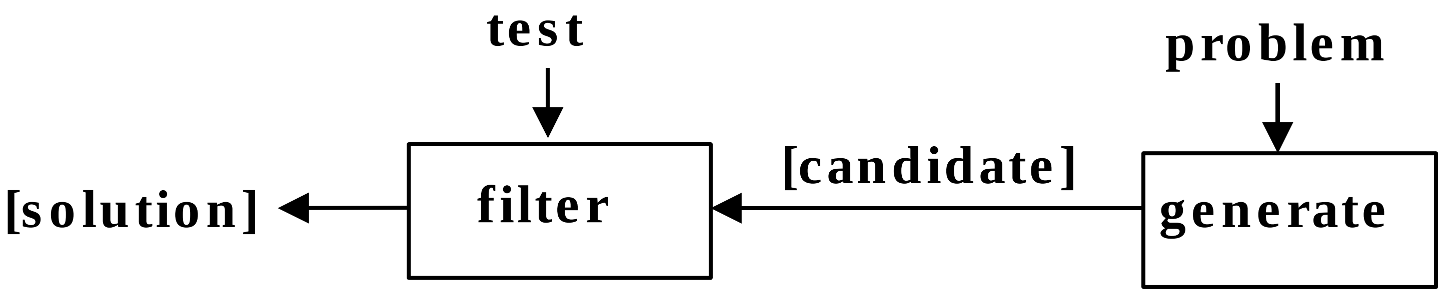
changed line

Lazy Functional Programming View

- Modularize search into separate generate & test functions…

  …communicating via explicit, but lazy, intermediate data structure.
- Simple program structure

```
generate :: problem -> [candidate]
test :: candidate -> Bool
search = (filter test) . generate
```



Binary CSPs in Haskell

- Set of variables {1,…,m}

```
type Var = Int
```

- Set of possible values {1,…,n}, same for each variable

```
type Value = Int
```

- Assignments associate variables to values

```
data Assignment = Var := Value
```

- Set of pairwise constraints on assignments
  - Defined by a symmetric **oracle** function

```
type Rel = Assign -> Assign -> Bool
```

  - If oracle returns true, assignments are **consistent**
  - Each call on this function is a **constraint check**
- Problem: `type CSP = CSP{vars::Int,vals::Int,rel::Rel}`

States and Solutions

- A **state** is a set of assignments

```
type State = [Assignment]
```

- A state that assigns all variables is **complete**.

```
complete :: CSP -> State -> Bool
complete CSP{vars} as = (length as == vars)
```

- A state is **consistent** if every pair of assignments is.

```
consistent :: CSP -> State -> Bool
consistent CSP{rel} [] = True
consistent CSP{rel} (a:as) =
         (all (rel a) as) && (consistent as)
```

- A **solution** is a complete, consistent state.

```
solution :: CSP -> State -> Bool
solution csp as = (complete csp as)
                      && (consistent csp as)
```

n-Queens Problem

- Assume one queen per column.
- Variables model rows; values model columns.

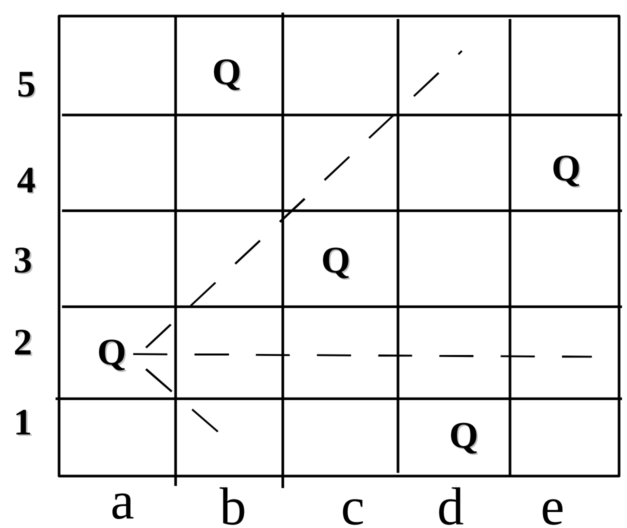
```
queens :: Int -> CSP
queens n = CSP{vars = n, vals = n, rel = safe}
   where safe (c1 := r1) (c2 := r2) =
         (r1 /= r2) && abs (c1-c2) /= abs(r1-r2)
```

- Obtaining **all** solutions

```
solver :: CSP -> [State]
solver (queens 5)) ->
  [[e:=4,d:=1,c:=3,b:=5,a:=2],
    …]
```

- Obtaining **one** solution

```
head (solver (queens 5))
```



Tree Search

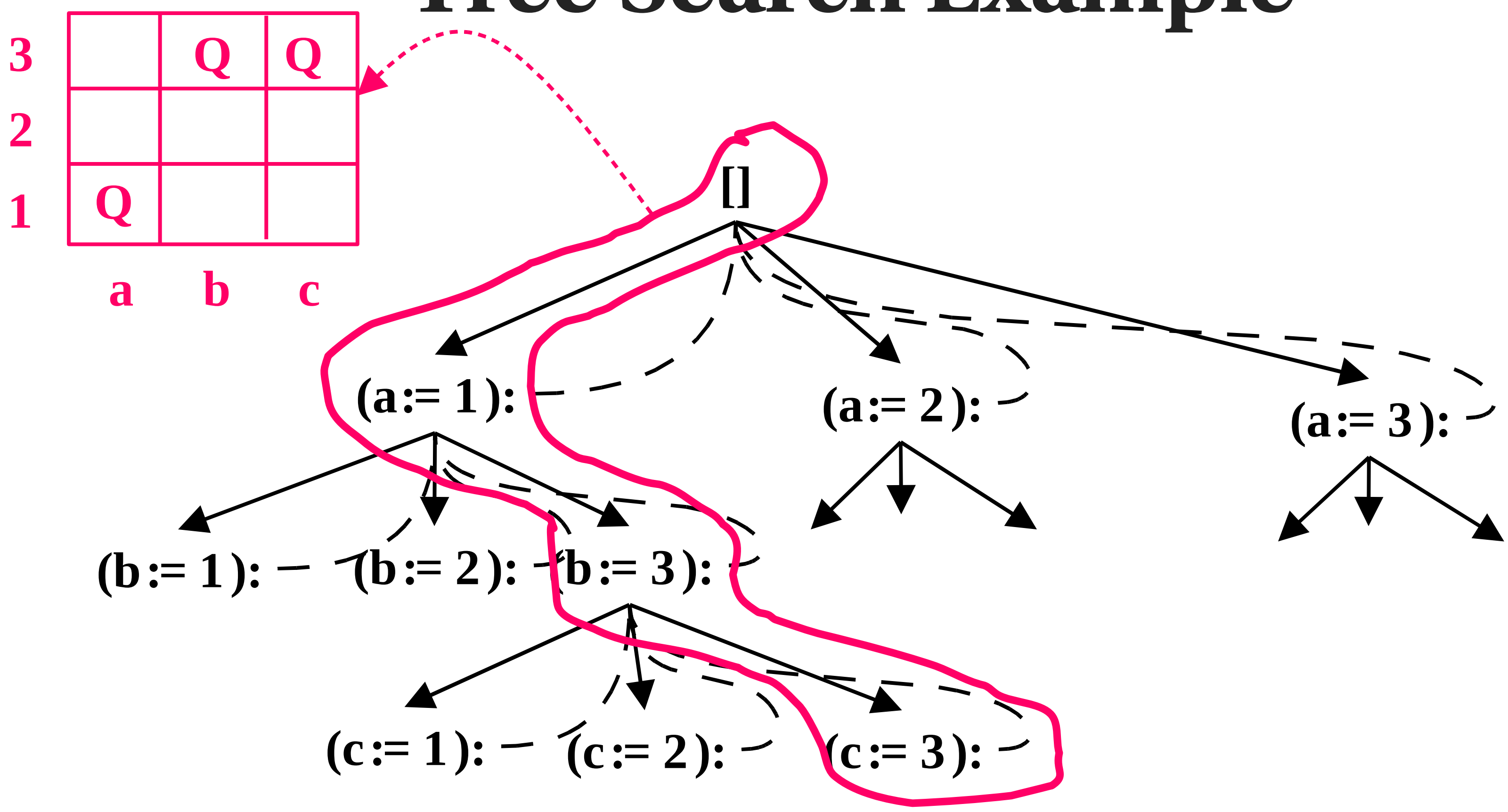
```
data Tree a = T a [Tree a]

mkTree :: CSP -> Tree State
pruneTree ::(State -> Bool) -> Tree State -> Tree State
leaves :: Tree State -> [State]

solver :: CSP -> [State]
solver csp = (filter (complete csp) .
              leaves .
              pruneTree (not . (consistent csp)) .
              mkTree) csp
```

collect → `leaves .`

prune → `pruneTree (not . (consistent csp)) .`

generate → `mkTree) csp`

Tree Search Example



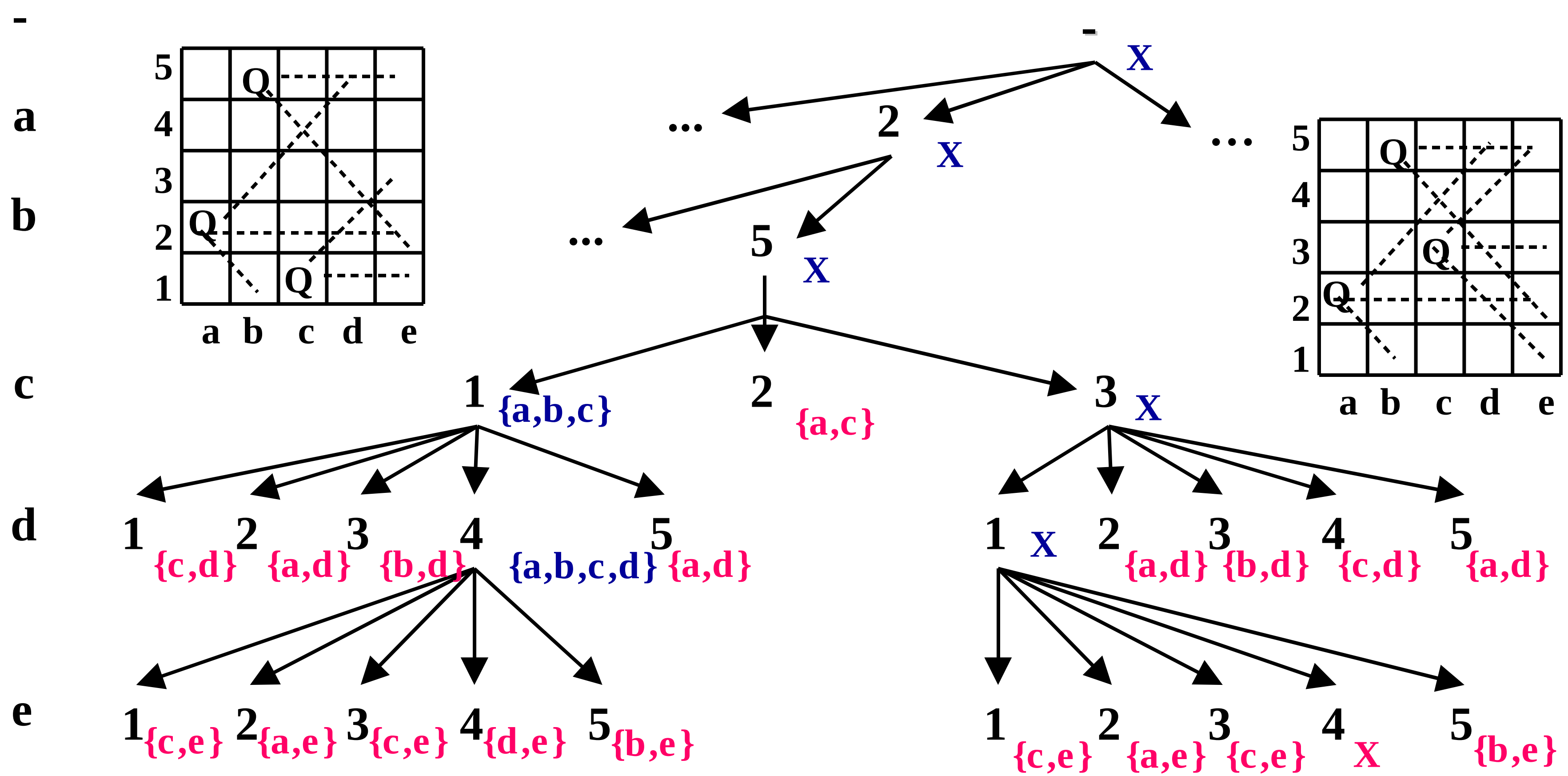
- Equivalent to ordinary imperative **backtracking** algorithm.
- Tree is isomorphic to **activation history** tree for recursive implementation.

Organizing the Zoo with Conflict Sets

- A **conflict set (CS)** for a state S is:
  - a non-empty subset of the variables in S, such that
  - if S' is any **solution** state, then there is at least one variable x in CS such that $S(x) \neq S'(x)$.

  I.e., at least one of the variables in CS "must change its value" to reach a solution.
- A state can be extended to a solution iff it has no CS.
- If we know a CS for a state, we can safely prune its sub-tree.
- Many interesting algorithms can be phrased as conflict-set computations, allowing them to be classified and combined.

Conflict Set Labeling Example



- Earliest Conflict
- Union Rule

Generic Solver in Haskell

- Parameterized by conflict set labeling mechanism

```
type ConflictSet = [Var]
type Labeler = CSP -> Tree State ->
                      Tree (State,ConflictSet)
```

- Labeling just adds extra stage to solver's "lazy pipeline"

```
search :: Labeler -> CSP -> [State]
search labeler csp =
  (filter complete . map fst . leaves .
     prune (not.null.snd) . labeler csp . mkTree) csp
```

- Example: simple backtracking uses a trivial labeler

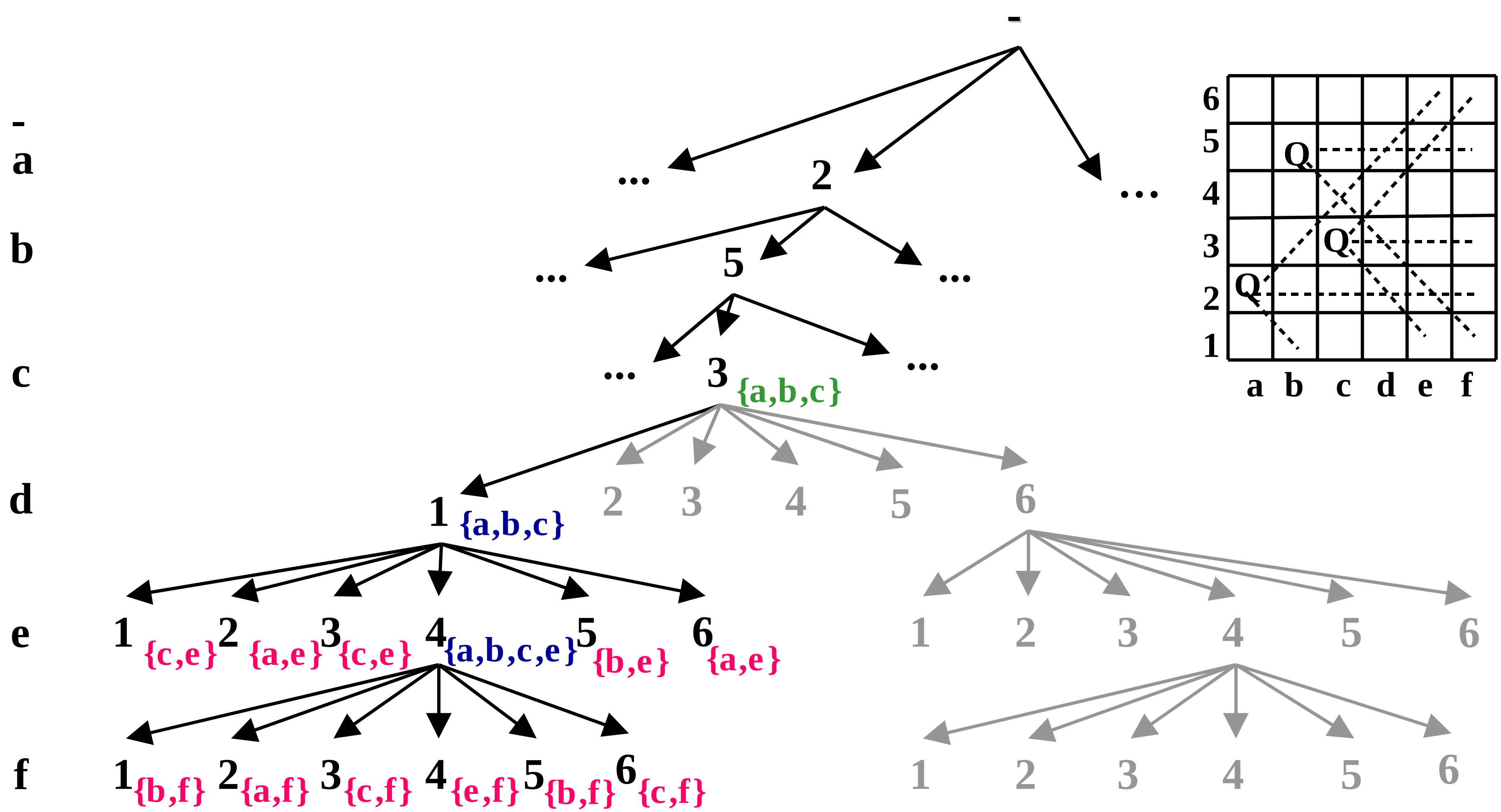
```
bt :: Labeler
bt csp = mapTree f
           where f s = (s,inconsistency csp s)
btsolver = search bt
```

Conflict-directed Backjumping

- Complicated algorithm, usually phrased as "jumping back" to a state further up the recursion stack; hard to show correct.
- We can give a purely **local**, **declarative** description.
- Use union rule plus one other fact:
  - If a node A has a known conflict set CS that does not contain the variable assigned at A, then CS is also a conflict set for A's parent.
- View CBJ as way to **improve** an existing CS labeling

```
cbj :: CSP -> Tree (State,ConflictSet) ->
              Tree (State,ConflictSet)
cbjsolver = search cbjbt
               where cbjbt = cbj csp . bt csp
```

Backjumping Example



Some Other Algorithms

- **Forward checking, backmarking** and related algorithms compute CSs for all **future** assignments at each node.

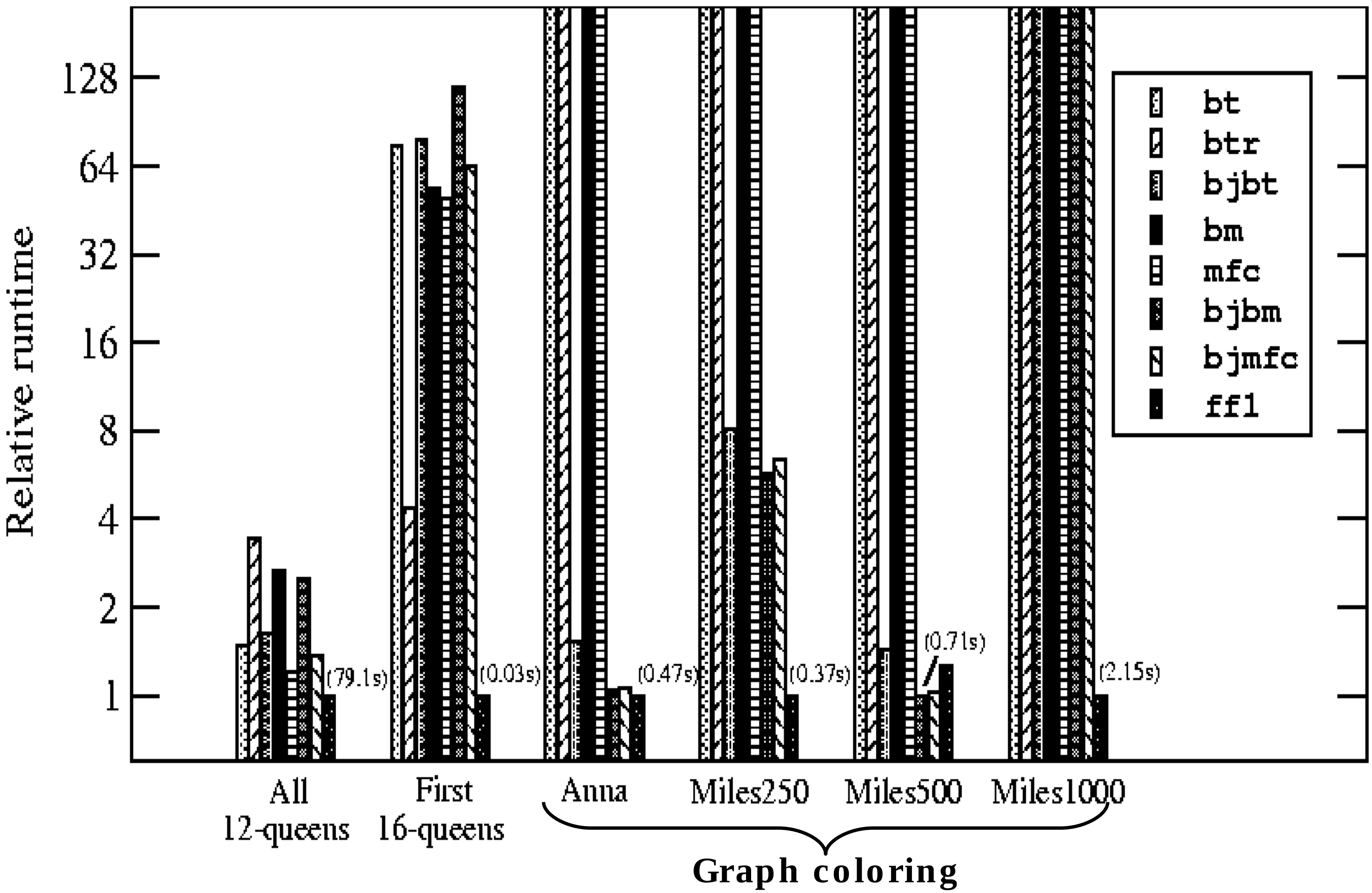
```
storeConflicts:: CSP -> Tree State ->
                   Tree (State, Cache ConflictSet)
bm csp = extractConflicts . storeConflicts csp
```

- **Value-ordering heuristics** change the order of branches to put more promising branches on the left.

```
hrandom :: Seed -> Tree a -> Tree a
btr :: Seed -> Labeler
btr seed csp = bt csp . hrandom seed
```

- **Fail first dynamic variable ordering** requires just slightly richer framework.
- Trivial to **mix and match** by composing labelers**.**

Runtime Comparison



Performance of Modular Lazy CSP

- Compared to imperative algorithms:
  - Same number of consistency checks
  - Roughly **same space** (polynomial in problem size) after plugging “space leaks”
  - Roughly **30X slower** than optimized C (on kernel)
- Compared to manually fused Haskell code
  - Roughly **4X slower** (on kernel)
- But **fast enough** to allow experimentation with different combinations of algorithms and heuristics.
  - Can then recode in imperative style if desired
  - Constant factors don’t matter much anyhow.

Fusion by Rewrite Rules

- Search pipeline generates **lots** and **lots** of tree nodes.

```
search ≈ leaves . prune . label . mkTree
```

- Can reimplement Tree ADT in terms of highly regular **producer** and **consumer** functions:

```
data Tree a = T a [Tree a]
foldTree :: (a -> [b] -> b) -> Tree a -> b
buildTree :: (∀b.(a->[b]->b)->b) -> Tree a
buildTree g = g T
```

- Simple rewrite **rule** describes fusion

```
∀ k,g. foldTree k (buildTree g) = g k
```

  to avoid building intermediate nodes
- Glasgow Haskell Compiler (GHC) has prototype mechanism to specify and apply rules.
- Improves speed of kernel by >3X, almost to hand-fused Haskell, **without** changing search application code at all.

Space Leaks

- Space behavior of lazy programs is not compositional.
- Tiny changes in the way a tree producer is **used** can easily change program's space from linear to exponential.
- Our (ignorant) development cycle:
  - Code (hoping for the best)
  - Profile (awkward in practice, but tools can be improved)
  - Ponder for awhile (or ask a guru – not too useful)
  - Fiddle with the code and try again
- Improving this story is a major research challenge.
  - More important than shaving constant factors with better optimizing compilers.

Conclusions & Future Work

- Using modular lazy framework can **clarify** algorithms and their key invariants.
- New **combinations** of algorithms for particular problems can be easily expressed -- often with just one line of code.
- Useful **experiments** can be conducted, despite the overheads due to laziness.
- Future work:
  - More sophisticated algorithms
  - Tools/ideas for **space behavior** and **selective laziness**
  - Selling to constraints community (without functional programming?)