Quiz 1
October 19, 2005

ECE 311: Feedback and Control

- Do not begin to look at the following exam problems until instructed to do so.
- Write your name and student number on this exam.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- You will have 100 minutes to complete the exam.
- Read the questions carefully, and clearly show your approach in answering each question.
- If you have extra time, double check your answers.
- You are not permitted to use a calculator during the exam.
- Neither books nor notes, save for one single-sided page of notes are permitted to be used during the exam.

Student name: 

Student number: 

Problem 1: __________ / 10
Problem 2: __________ / 10
Problem 3: __________ / 10
Problem 4: __________ / 10

Total: __________ / 40
Problem 1:
Find the transfer function of the following system.

\[ \Delta = 1 - (b_i + d_j + f_k + b_i d_j + b_i f_k + d_j f_k) + (b_i d_j + b_i f_k + d_j f_k) - b_i d_j f_k \]

\[ P = a b c d e f g h \]

\[ b_1 = 1 \]

Maclaurin Rule:

\[ \frac{Y}{R} = \frac{a b c d e f g h}{1 - (b_i + d_j + f_k + b_i d_j + b_i f_k + d_j f_k) + (b_i d_j + b_i f_k + d_j f_k) - b_i d_j f_k} \]
Problem 2:
Determine the time-domain response of a system described by:

\[ T(s) = \frac{10}{s + 2} \]

Also, \( \frac{dy}{dt} = -28(1) \)

\[
\begin{align*}
\Rightarrow \quad y(0) &= \frac{10}{s + 2} \cdot \left( \frac{5s + 10}{5s + 10} \right) + \frac{5}{5} \\
\Rightarrow \quad b_0 &= 2 \quad b_1 = 0 \\
\Rightarrow \quad a_0 &= 2 \\
\Rightarrow \quad r(t) &= 1 - 2a(1) \\
\Rightarrow \quad y(t) &= 10e^{-2t} + 5u(t) \\
\Rightarrow \quad y(0) &= 5
\end{align*}
\]
**Problem 3:**
Consider the polynomial:

\[ p(s) = s^5 + s^4 + 2s^3 + 2s^2 + s + 1 \]

Using the Routh-Hurwitz test:

a) determine the root types (LHP, RHP, IA),
b) determine the actual root locations.

\[
\begin{array}{c|ccc}
\text{row} & s^5 & s^4 & s^3 \\
\hline
0 & 1 & 2 & 1 \\
1 & s^4 & 1 & 2 & 1 \\
2 & s^3 & 0 & 0 & 0 \\
3 & s^2 & 0 & 0 & 0 \\
4 & s^1 & & & \\
5 & s^0 & & & \\
\end{array}
\]

\[ P_{\text{div}}(s) = s^4 + 2s^2 + 1 \]

\[ \begin{array}{c|cccc}
\text{row} & s^5 & s^4 & s^3 & s^2 + 1 \\
\hline
0 & 1 & 2 & 1 & 1 \\
1 & s^4 & 1 & 2 & 1 \\
2 & s^3 & 4 & 0 & 0 \\
3 & s^2 & 1 & 1 & 0 \\
4 & s^1 & 2 & 0 & \text{from } \frac{d}{ds} \frac{dP_{\text{div}}(s)}{ds} \\
5 & s^0 & 1 & \text{no sign changes } \Rightarrow \text{no RHP roots} \\
\end{array}
\]

\[ \Rightarrow 4 \text{ IA, 1 LHP (0 RHP)} \]

b) \[ P_{\text{div}}(s) = s^4 + 2s^3 + 1 = (s^2 + 1)^2 \]

\[ \Rightarrow s^2 = -1 \Rightarrow s = \pm j \text{ twice} \]

\[ P_{\text{div}}(s): \]

\[ \begin{array}{c|cc}
\text{row} & s^2 & 1 \\
\hline
0 & & \\
1 & s^2 & 1 \\
2 & & \text{last factor was } s + 1 \\
\end{array} \]

\[ \Rightarrow s^2 = -1 \Rightarrow s = \pm j \]

Roots are: \( \pm j, \pm j, -1 \)
Problem 4:
Find the nature (dampedness) of the second-order system response for the system shown below for different $k$ as given below in (a), (b) and (c). For underdamped response, determine the exponential constant, oscillation frequency, damping ratio and undamped frequency.

\[
\begin{align*}
R(s) & \rightarrow \frac{k}{s^2 + 4s + k} \rightarrow Y(s) \\
\text{roots} & = -2 \pm \frac{\sqrt{16-4k}}{2} = -2 \pm \sqrt{(4-k)}
\end{align*}
\]

a) $k=2$
\[
\begin{align*}
\text{roots} & = -2 \pm \sqrt{4-2} = -2 \pm \sqrt{2} \Rightarrow \text{real and distinct} \\
\Rightarrow & \ \text{overdamped}
\end{align*}
\]

b) $k=4$
\[
\begin{align*}
\text{roots} & = -2 \pm \sqrt{(4-4)} = -2, -2 \Rightarrow \text{repeated} \\
\Rightarrow & \ \text{critically damped}
\end{align*}
\]

c) $k=8$
\[
\begin{align*}
\text{roots} & = -2 \pm \sqrt{4-8} = -2 \pm j\sqrt{4} = -2 \pm 2j \\
\Rightarrow & \ \text{complex roots}
\end{align*}
\]
\[
\begin{align*}
\text{s}^2 + 4s + 8 & = s^2 + 2j\omega_n s + \omega_n^2 \\
\Rightarrow & \ \omega_n = \sqrt{8} = 2\sqrt{2}, \\
2 \omega_n & = 4 \\
\Rightarrow & \ \gamma = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}. \\
\omega & = \omega_n \sqrt{1-\gamma^2} \\
& = 2\sqrt{2} \sqrt{1-\frac{1}{2}} \\
& = \frac{2\sqrt{2}}{\sqrt{2}} = 2 \text{ rads/s} \\
\gamma & = 2
\end{align*}
\]