PROBLEM 1.

For the buck-boost converter of Fig. 1, determine the control-to-output, \( \frac{v}{d} (s) \), transfer function, using the technique of state-space averaging. Hence determine the DC gain and the location of the poles and zeroes in terms of \( V_g, D \) and the circuit parameters, \( L, C \) and \( R \), only. (In the case of a complex pole pair, you need only determine the resonant frequency and \( Q \).)

![Fig. 1.](image)

PROBLEM 2.

The output impedance of a system may be determined by driving the output with a perturbation current source, as shown in Fig. 2, and noting the voltage variations and then forming the quotient

\[
Z_{out} = \frac{v}{i}
\]

Using this information and the small-signal mathematical model of state-space averaging, determine the output impedance, \( Z_{out} \), of the converter shown in Fig. 3.

![Fig. 2.](image)

![Fig. 3.](image)
**Small Signal State Space Averaging**

**Solution**

**Problem 1**

**During DT_3**

\[ V_j = -\frac{V}{2} \]

\[ C\frac{di}{dt} = -\frac{V}{2} \quad \Rightarrow \quad \frac{di}{dt} = -\frac{V}{2RC} \]

\[ i = \frac{V}{2} \]

\[ u + L\frac{di}{dt} = 0 \quad \Rightarrow \quad \frac{di}{dt} = -\frac{u}{L} \]

\[ i = \frac{V}{2} \quad \Rightarrow \quad u = \frac{V}{2} \]

\[ \begin{bmatrix} i \\ \frac{di}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i \\ \frac{di}{dt} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V_j \]

\[ A = DA_1 + D' A_2 = \begin{bmatrix} 0 & -\frac{1}{2C} \\ \frac{1}{2} & -\frac{1}{2C} \end{bmatrix} \]

\[ \begin{bmatrix} \frac{2}{L} \\ 0 \end{bmatrix} \]
\[
X = - A^{-1} b V_g \\
\Rightarrow X = - \begin{bmatrix}
0 & \frac{D}{L} \\
\frac{D}{L} & -\frac{1}{RC}
\end{bmatrix}^{-1} \begin{bmatrix}
\frac{D}{L} \\
0
\end{bmatrix}
V_g \\
= - \frac{LC}{D^{1/2}} \begin{bmatrix}
-\frac{1}{RC} & \frac{D}{L} \\
-\frac{D}{L} & 0
\end{bmatrix}
\begin{bmatrix}
\frac{D}{L} \\
0
\end{bmatrix}
V_g \\
= \frac{LC}{D^{1/2}} \begin{bmatrix}
-\frac{1}{RC} + \frac{D}{L} \\
-\frac{D}{L} & 0
\end{bmatrix}
V_g \\
= \frac{1}{s \left( s + \frac{1}{RC} + \frac{D}{L} \right)} \begin{bmatrix}
s + \frac{1}{RC} & -\frac{D}{L} \\
\frac{D}{L} & s
\end{bmatrix}
\begin{bmatrix}
\frac{D}{L} + 1 \\
0
\end{bmatrix}
V_g \\
= \frac{1}{s \left( s + \frac{1}{RC} + \frac{D}{L} \right)} \begin{bmatrix}
s + \frac{1}{RC} & -\frac{D}{L} \\
\frac{D}{L} & s
\end{bmatrix}
\begin{bmatrix}
\frac{D}{L} + 1 \\
-\frac{D}{RCD^{1/2}}
\end{bmatrix}
V_g \\
\frac{X}{d} = \frac{1}{s^2 + \frac{s}{RC} + \frac{D}{L}} \begin{bmatrix}
(s + \frac{1}{RC}) \frac{D}{L} + \frac{D}{RCD^{1/2}} \\
\frac{D}{L} - \frac{5D}{RCD^{1/2}}
\end{bmatrix}
V_g \\
\]
\[
\hat{y} = \hat{c} + (c_1 - c_2) X \hat{t}
\]

\[
y = v \implies c_1 = c_2 = c = \begin{bmatrix} 0 & 1 \end{bmatrix}
\]

\[
\frac{\hat{v}}{\hat{t}} = \frac{\hat{c}}{\hat{t}} = \frac{(\frac{1}{LC} - \frac{s}{RCD^{1/2}}) v_g}{s^2 + \frac{s}{RC} + \frac{1}{LC}}
\]

\[
= v_g \frac{\frac{1}{LC}}{D^{1/2}} \left( 1 - \frac{s}{RCD^{1/2}} \right)
\]

\[
\frac{\hat{v}}{\hat{t}} = \frac{v_g}{D^{1/2}} \frac{1 - s \frac{LD}{R D^{1/2}}}{\frac{LC}{D^{1/2}} + \frac{1}{D^{1/2}R} s + 1}
\]

DC gain \[ = \frac{V_g}{D^{1/2}} \]

RHP zero \[ \implies \omega_z = \frac{D^{1/2} R}{D^{1/2}} \]

Complex pole pair \[ \implies \omega_0 = \frac{D^{1/2}}{LC} \implies \omega_0 = \frac{D^{1/2}}{\sqrt{LC}} \]

\[
\frac{1}{\omega_0^2 \Omega} = \frac{L}{D^{1/2} R} \implies \Omega = \frac{D^{1/2} \sqrt{LC}}{L} = 3' \sqrt{\frac{L}{L}}
\]
**Problem 1**

\[ L \dot{x} = \begin{bmatrix} i_c \\ v_c \end{bmatrix} \quad u = \begin{bmatrix} v_g \\ i \end{bmatrix} \]

**During DT_1**

\[ -V_g + L \frac{di_c}{dt} = 0 \quad \Rightarrow \quad \frac{di_c}{dt} = \frac{V_g}{L} \]

\[ i_c = \frac{c}{R} \frac{dv_c}{dt} + \frac{v_c}{R} \quad \Rightarrow \quad \frac{dv_c}{dt} = i - \frac{v_c}{C} \]

\[ V_g + L \frac{di_c}{dt} + v_c = 0 \quad \Rightarrow \quad \frac{di_c}{dt} = \frac{V_g}{L} - \frac{v_c}{L} \]

\[ i_c = \frac{c}{R} \frac{dv_c}{dt} + \frac{v_c}{R} \quad \Rightarrow \quad \frac{dv_c}{dt} = \frac{i - v_c}{C} \]

\[ \begin{bmatrix} i_C \\ v_c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_C \\ v_c \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L} \\ 0 & \frac{1}{C} \end{bmatrix} \begin{bmatrix} v_g \\ i \end{bmatrix} \]

\[ \frac{d}{dt} \begin{bmatrix} i_C \\ v_c \end{bmatrix} \begin{bmatrix} 0 & -\frac{1}{RC} \\ 0 & \frac{1}{C} \end{bmatrix} \begin{bmatrix} i_C \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i \end{bmatrix} \]

\[ A_1 \quad B_1 \quad A_2 \quad B_2 \]
\[ \hat{z} = A\hat{z} + B\hat{u} + [(A_1 - A_2)X + (B_1 - B_2)U] \hat{d} \]

\[ \hat{y} = C\hat{z} + E\hat{u} + [(C_1 - C_2)X + (E_1 - E_2)U] \hat{d} \]

where:

\[ A = DA_1 + D'A_2 \]
\[ C = DC_1 + D'C_2 \]
\[ B = DB_1 + D'B_2 \]
\[ E = DE_1 + D'E_2 \]

we neglect \( Z_{out} = \frac{\hat{y}}{\hat{c}} \)

Set \( \hat{d} = 0 \) \( \Rightarrow \)

\[ \hat{y} = \left[ C \left( S I - A \right)^{-1} B + E \right] \hat{u} \]

Now:

\[ \hat{y} = \hat{y} - \hat{y}_c = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{z} + \begin{bmatrix} 0 & 0 \end{bmatrix} \hat{u} \]

\[ C_1 = C_2 = C \]
\[ E_1 = E_2 = E \]

\[ \Rightarrow \hat{y} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{C} & S + \frac{1}{2C} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \hat{u} \]

\[ \hat{y} = \begin{bmatrix} \frac{L}{C} & 0 \\ \frac{1}{C} \end{bmatrix} \hat{u} \]

\[ \hat{u} = \begin{bmatrix} \frac{1}{L} & 0 \\ \frac{L}{C} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2L} & s + \frac{1}{2C} \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \hat{u} \]

\[ \hat{u} = \begin{bmatrix} \frac{1}{L} & 0 \\ \frac{L}{C} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2L} & s + \frac{1}{2C} \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \hat{u} \]

\[ \hat{u} = \begin{bmatrix} \frac{1}{L} & 0 \\ \frac{L}{C} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2L} & s + \frac{1}{2C} \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \hat{u} \]
\[
\mathbf{a} = \frac{1}{S^2 + S \frac{1}{RC} + \frac{1}{LC}} \left[ \begin{array}{c}
\frac{2}{L} \\
S
\end{array} \right] \left[ \begin{array}{cc}
\frac{1}{C} & 0 \\
0 & \frac{1}{C}
\end{array} \right]^{-1} \mathbf{a}
\]

\[
\mathbf{a} = \frac{1}{S^2 + S \frac{1}{RC} + \frac{1}{LC}} \left[ \begin{array}{c}
\frac{2}{L} \\
S
\end{array} \right] \mathbf{a}
\]

\[
\hat{V} = \frac{2}{LC} \hat{V} + \frac{S}{C} \hat{I} - \frac{S}{S^2 + S \frac{1}{RC} + \frac{1}{LC}} \left[ \begin{array}{c}
\frac{2}{L} \\
S
\end{array} \right] \mathbf{a}
\]

\[
\mathbf{a} \bigg|_{\hat{V}} = \frac{1}{C} \frac{S}{S^2 + S \frac{1}{RC} + \frac{1}{LC}} \left[ \frac{LC}{2 L^2} S^2 + \frac{S}{S^2 + \frac{1}{LC} \frac{1}{R} \frac{1}{C}} \right]^{-1} + 1
\]

\[
Z_{\text{out}} = \frac{1}{D^{1/2}} \frac{S L}{\frac{LC}{2 L^2} S^2 + \frac{S}{S^2 + \frac{1}{LC} \frac{1}{R} \frac{1}{C}} + 1}
\]
Review of Bode Diagrams

1. Express the gains in factored pole-zero form.

   a) \( A(s) = \)

   \[ \begin{array}{c}
   \omega_1 \\
   A_1 \\
   \omega_2 \\
   \end{array} \]

   b) \( A(s) = \)

   \[ \begin{array}{c}
   \omega_2 \\
   A_1 \\
   \omega_1 \\
   \end{array} \]

   c) \( A(s) = \)

   \[ \begin{array}{c}
   \omega_1 \\
   A_1 \\
   \omega_2 \\
   \end{array} \]

2. The accompanying graphs show experimental magnitude and phase data for a certain gain function \( A(S) \) and impedance \( Z(S) \). Draw appropriate straight-line asymptotes through the data points and hence deduce numerical values for the mid-frequency gain \( A_m \) and low-frequency impedance \( R_0 \), as well as for the poles, zeroes, and Q-factors in the corresponding analytic expressions for \( A(S) \) and \( Z(S) \).
SOLUTION

EC946/546

Power Electronics

Problem

Review of Bode Diagrams

1. Express the gains in factored pole-zero form.

\[ A(s) = \frac{A_1 \frac{\omega_1}{s}}{(1 + \frac{s}{\omega_2})(1 + \frac{s}{\omega_1})} \]

\[ A(s) = \frac{A_1 \frac{1}{s}}{(1 + \frac{s}{\omega_2})(1 + \frac{s}{\omega_1})} \]

\[ A(s) = \frac{A_1 \frac{\sqrt{\omega_2}}{s}}{(1 + \frac{s}{\omega_2})(1 + \frac{s}{\omega_1})} \]

2. The accompanying graphs show experimental magnitude and phase data for a certain gain function \( A(S) \) and impedance \( Z(S) \). Draw appropriate straight-line asymptotes through the data points and hence deduce numerical values for the mid-frequency gain \( A_m \) and low-frequency impedance \( R_0 \), as well as for the poles, zeroes, and Q-factors in the corresponding analytic expressions for \( A(S) \) and \( Z(S) \).
PROBLEM

The attached graph shows experimental data for a certain gain function $A$. Draw appropriate straight-line asymptotes through the data points and hence deduce numerical values for the low-frequency gain and for the poles and zeros in the corresponding analytic expression for $A$.

Draw horizontal asymptotes $\circ 1$ and $\circ 2$: This fixes maximum gain $A$ at $26 \, \text{dB}$ and maximum phase shift at $\pm 92^\circ$. The true phase shift confirms the existence of a LHP zero at a low frequency. Until $\ast$, draw the magnitude characteristic with the gain and phase. We can determine the low-frequency gain ($A_0$) using $\text{from } \text{phase } + \frac{\pi}{2}$ which $k$ is the ratio of the gains. We find $A_0 \approx 12 \, \text{dB}$, this is a little high considering the data points after borrowing $\circ 3$ (at $+20 \, \text{dB/dec}$), draw in $\circ 4$ at $610 \, \text{Hz}$ and same letter. Draw in $\circ 5$ at $-20 \, \text{dB/dec}$. Draw in $\circ 6$ and $\circ 7$ conform a high-frequency pole $<7 \, \text{kHz}$, plot slope $-45 \, \text{dB/dec}$ pole. Draw in $\circ 9$.

The resulting analytic expression for $A$ is

$$A(s) = \frac{A_0 \left(1 + \frac{s}{w_0}\right)\left(1 - \frac{s}{w_2}\right)}{\left(1 + \frac{s}{w_1}\right)\left(1 + \frac{s}{w_2}\right)}$$

Where

- $A_0 = 10 \, \text{dB} \rightarrow 3.2$
- LHP zero $\omega_0 \rightarrow f_c = 17 \, \text{kHz}$
- LHP pole $\omega_1 \rightarrow f_1 = 110 \, \text{kHz}$
- LHP pole $\omega_2 \rightarrow f_2 = 6.8 \, \text{kHz}$
- RHP zero $\omega_3 \rightarrow f_3 = 100 \, \text{kHz}$
LHP Pole AT \(2\pi(200)\) \(\Rightarrow\) \(\omega_p = 2\pi(200)\)

Double LHP Zero AT \(2\pi(1000)\) \(\Rightarrow\) \(\omega_o = 2\pi(1000)\)

\(Q = 8\text{ dB} \Rightarrow 2.5 \Rightarrow Q = 2.5\)

Midband \(Z_m = 14\text{ dB} \Rightarrow 5 \Rightarrow Z_m = 5\)

\[ Z(s) = \frac{Z_m}{1 + \frac{s}{\omega_p} + \frac{s^2}{\omega_o^2}} \]
A Solved Problem

Similar to problems 8.1-8.4

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Fundamentals of Power Electronics

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**Bode Plots II**

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-20dB decade

\[ G_m \]

\[ f_1 \]

\[ f_2 \]

\[ f_3 \]

-20dB decade

**a)** Express the gain represented by the asymptotes above in factored pole-zero form. \( f_1, f_2, \) and \( f_3 \) are the corner frequencies in Hz. \( G_m \) is the magnitude of the midband asymptote, as illustrated.

**b)** Derive analytical expressions for each asymptote, using your result from part (a).

**c)** Compute the value of the asymptote at \( f = f_3 \)

**Solution to (a)**

We are given the magnitude of the midband asymptote \( G_m \).

So, reference \( G(s) \) to \( G_m \). Poles and zeros at frequencies \( f \leq f_1 \) should then be expressed in inverted form.

Poles and zeros at frequencies \( f \geq f_2 \) should be expressed in conventional non-inverted form.
So $G(s)$ contains the following terms:

$G_m$  midband gain

$\left(1 + \frac{\omega_1}{s}\right)$ inverted zero at $f = f_1$

$\omega_1 = 2\pi f_1$

$\left(1 + \frac{s}{\omega_2}\right)$ zero at $f = f_2$

$\omega_2 = 2\pi f_2$

\[\frac{1}{1 + \frac{s}{\omega_3} + \left(\frac{s}{\omega_3}\right)^2}\]

complex poles at $f = f_3$

$\omega_3 = 2\pi f_3$

with Q-factor $Q$

(Note that if the value of $Q$ is expressed in dB, then it must be converted for use in the expression for $G(s)$: $\frac{\text{dB}}{20}$)

So

$G(s) = G_m \frac{\left(1 + \frac{\omega_1}{s}\right)\left(1 + \frac{s}{\omega_2}\right)}{\left(1 + \frac{s}{\omega_3} + \left(\frac{s}{\omega_3}\right)^2\right)}$

An equivalent form that does not employ inverted zeroes:

$G(s) = G_m \frac{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}{\left(\frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_3} + \left(\frac{s}{\omega_3}\right)^2\right)}$
6) Analytical expressions for asymptotes

Given the solution to part (a)
(either version will work):

\[ G(s) = G_m \frac{(1 + \frac{\omega_1}{\omega})(1 + \frac{\omega_2}{\omega})}{(1 + \frac{s}{\omega_1}) + (\frac{s}{\omega_2})^2} \]

There are four asymptotes, one for each of the following frequency ranges:

1. \( f \leq f_1 \)
2. \( f_1 \leq f \leq f_2 \)
3. \( f_2 \leq f \leq f_3 \)
4. \( f_3 \leq f \)

There are three frequency-dependant terms:

\( (1 + \frac{\omega_1}{s}) \)
\( (1 + \frac{\omega_2}{s}) \)
\( (1 + \frac{s}{\omega_1}) + (\frac{s}{\omega_2})^2 \)

Each of these terms consists of the sum of terms.

Over a given frequency range, the asymptote is derived by neglecting the smallest term or terms within each sum, retaining only the term having the largest magnitude. This process leads to an exact expression for the asymptote, which may be a good approximation for the actual function \(|G(j\omega)|\).
1. For \( f \leq f_i \)

Then \( f \leq f_i < f_2 < f_3 \)
\( \omega \leq \omega_i < \omega_2 < \omega_3 \)

Consider each term:

\[
\begin{align*}
(1 + \frac{\omega}{\omega_1}) & \quad \xrightarrow{\text{justification}} \quad \|1 + j\omega\|_{s=j\omega} = \sqrt{1 + (\frac{\omega}{\omega_1})^2} \\
\text{inverted asymptote zero term for } f < f_i & \quad \xrightarrow{\text{justification}} \quad \approx \sqrt{(\frac{\omega}{\omega_1})^2} = \frac{\omega}{\omega_1} \quad \text{since for } \omega < \omega_i, \quad 1 \gg (\frac{\omega}{\omega_1})^2.
\end{align*}
\]

\[
\begin{align*}
(1 + \frac{\omega}{\omega_2}) & \quad \xrightarrow{\text{justification}} \quad \|1 + \frac{\omega}{\omega_2}\|_{s=j\omega} = \sqrt{1 + (\frac{\omega}{\omega_2})^2} \\
\text{zero term asymptote for } f < f_i & \quad \xrightarrow{\text{justification}} \quad \approx 1 \quad \text{since for } \omega < \omega_i, \quad 1 \gg (\frac{\omega}{\omega_2})^2 \quad \text{so} \quad (1 + \frac{\omega}{\omega_2}) \approx 1 \quad \text{for } f < f_i
\end{align*}
\]

\[
\begin{align*}
(1 + \frac{\omega}{\omega_3} + (\frac{\omega}{\omega_3})^2) & \quad \xrightarrow{\text{justification}} \quad \|1 + \frac{\omega}{\omega_3} + (\frac{\omega}{\omega_3})^2\| = \sqrt{(1 - (\frac{\omega}{\omega_3})^2 + (\frac{\omega}{\omega_3})^4} \\
\text{pole term asymptote for } f < f_i & \quad \xrightarrow{\text{justification}} \quad \approx \sqrt{1} = 1 \quad \text{since for } \omega < \omega_i, \quad 1 \gg (\frac{\omega}{\omega_3})^2 \quad \text{and} \quad 1 \gg \frac{\omega}{\omega_3}. \\
\quad \text{so} \quad (1 + \frac{\omega}{\omega_3} + (\frac{\omega}{\omega_3})^2) \approx 1 \quad \text{for } f < f_i
\end{align*}
\]

Composite: \( G(s) = G_m \left( \frac{\frac{\omega_1}{\omega}}{(1 + \frac{\omega}{\omega_2} + (\frac{\omega}{\omega_3})^2)} \right) \rightarrow G_m \left( \frac{\omega_1}{\omega} / (\frac{\omega}{\omega_3}) \right) = G_m \frac{\omega_1}{\omega}
\]

let \( s = j\omega \) and take magnitude: \( \left( \frac{G_m \omega_1}{\omega} \right) = \left( \frac{G_m f_i}{f} \right) \) is the expression for the magnitude asymptote for \( f < f_i \).
2. for \( f_1 \leq f \leq f_2 \)

Then
\[
G(s) = G_m \frac{(1 + \frac{\omega_1^2}{s})(1 + \frac{\omega_2^2}{s})}{(1 + \frac{\omega_1^2}{s \omega_3^2} + (\frac{\omega_2^2}{s \omega_3^2})^2)} \rightarrow G_m \frac{(1)(1)}{(1)} = G_m
\]

3. for \( f_2 \leq f \leq f_3 \)

Then
\[
G(s) = G_m \frac{(1 + \frac{\omega_1^2}{s})(1 + \frac{\omega_2^2}{s})}{(1 + \frac{\omega_1^2}{s \omega_3^2} + (\frac{\omega_2^2}{s \omega_3^2})^2)} \rightarrow G_m \frac{(1)(\frac{\omega_2}{\omega_3})}{(1)} = G_m \frac{\omega_2}{\omega_3}
\]

which has magnitude \( G_m \frac{\omega_2}{\omega_3} = G_m \frac{f_2}{f_3} \)

4. for \( f_3 \leq f \)

Then
\[
G(s) = G_m \frac{(1 + \frac{\omega_1^2}{s})(1 + \frac{\omega_2^2}{s})}{(1 + \frac{\omega_1^2}{s \omega_3^2} + (\frac{\omega_2^2}{s \omega_3^2})^2)} \rightarrow G_m \frac{(1)(\frac{\omega_2}{\omega_3})}{(\frac{\omega_2}{\omega_3})^2} = G_m \frac{\omega_3^2}{s \omega_2}
\]

which has magnitude \( G_m \frac{\omega_3^2}{s \omega_2} = G_m \frac{f_3^2}{f_3 f_2} \)
Summary of analytical expressions for asymptotes:

\[ \frac{G_m f_1}{f} \quad \frac{G_m f}{f_2} \quad \frac{G_m f_3^2}{f f_2} \]

1. Compute the value of the asymptote at \( f = f_3 \), and estimate the value of the actual magnitude.
2. Plug into expression for either adjacent asymptote (either the \( f_2 \leq f \leq f_3 \) or the \( f_3 \leq f \) asymptote).
3. Result is
   \[ \frac{G_m f_3}{f_2} \]

At \( f = f_3 \), the magnitude of the actual curve is given approximately by
\[ \frac{Q G_m f_3}{f_2} \]

This neglects the (very small) deviation of the actual curve from the asymptotes caused by the zeroes at \( f_1 \) and \( f_2 \), but it includes the (significant) deviation caused by the Q-factor of the complex poles.
Design of a buck regulator with specified closed-loop output impedance:

\[ V_s = 100\text{V} \]

\[ I_s = 50\text{kHz} \]

\[ R_L \]

\[ C = 200\mu\text{F} \]

\[ I_{\text{LOAD}} \]

\[ R_{\text{LOAD}} \]

\[ Z_{\text{out}} \]

\[ \text{PWM} \]

\[ D = \frac{v_c}{V_m} \]

\[ V_m = 4\text{Vrms} \]

\[ G_c(s) \]

\[ k = 0.1 \]

\[ V_{\text{ref}} = 5\text{V} \]

It is desired to design a compensation network \( G_c(s) \) such that the closed-loop output impedance of the above regulator system is less than 0.2 Ohms over the entire frequency range 0-20kHz. Also, to ensure that the transient response is well-behaved, the \( Q \) of the closed-loop system must be less than 1 -> 0dB. The load current \( I_{\text{LOAD}} \) can vary from 5 Amperes to 50 Amperes, and the above specifications must be met for every value of \( I_{\text{LOAD}} \) in this allowed range. For simplicity, you may assume that \( V_g \) does not vary.

Since the state-space averaging method is valid only for frequencies sufficiently less than the switching frequency, you should choose the loop gain crossover frequency \( f_c \) to be no greater than \( f_s/4 = 12.5\text{kHz} \), to be sure that your model is valid.

1. Draw the open-loop output impedance \( Z_{\text{out}}(s) \). Over what range of frequencies is the output impedance specification not met? Hence, deduce how large the minimum loop gain \( T(s) \) must be such that the closed-loop output impedance meets the specifications, and choose a suitable crossover frequency.
2. Design a compensation network $G_c(s)$ such that all specifications are met. You must give:

1) Your choice for the transfer function $G_c(s)$
2) The worst-case closed-loop $Q$.
3) Bode plots of the loop gain $T(s)$ and closed-loop $Z_{out}(s)$ for load currents of 5A and 50A. What effect does variation of $R_{LOAD}$ have on the closed-loop behavior?

**Hint:** For this question, from the small-signal state space, average mathematical model draw an equivalent circuit model. Alternatively, use as your circuit model the canonical model.
Replace open loop converter with its canonical model

\[ e(s) = \frac{v_o}{s} ; \quad m = D \]

**Fig. 1**

1) Find \( Z_0 \) (open loop)

Null all independent sources

\[
Z_0 = \frac{R}{1 + \frac{1}{sC}} \parallel \frac{1}{s} \parallel \frac{1}{s} \parallel R
\]

\[
= \frac{(R + sL) R}{1 + sRC}
\]

\[
= \frac{(R + sL) R}{1 + sRC}
\]

\[
= \frac{(R + sL)}{(R + sL) + \frac{R}{1 + sRC}}
\]
\[
Z_0 = \frac{(R_L + sL)}{R_L + sRCR_L + s^2RCL + sL + R}
\]

\[
= \frac{R_L}{R + R_L} \left( 1 + \frac{sL}{R_L} \right)
\]

\[
L = 1.0 \text{ mH} ; \quad C = 200 \mu\text{F} ; \quad R_L = 4 \text{ m}\Omega
\]

\[
V_0 = 50 \text{ V} \quad E_{load} = 5 \text{ V to } 50 \quad \Rightarrow \quad R = 1 \text{ to } 10 \Omega
\]

\[
Z_0 = 5 \times 10^{-3} \left( 1 + \frac{sL}{R_L} \right)
\]

\[
l = \frac{10^{-3}}{R}
\]

dc impedance = \( 4 \times 10^{-3} \Omega \)

\[
\omega_0 = \sqrt{\frac{1}{0.2 \times 10^{-6}}} = 2236 \quad \Rightarrow \quad f_0 = 3.56 \text{ Hz}
\]

\[
\frac{1}{\omega_0 R} = 10^{-3} \Rightarrow Q = \frac{R}{\omega_0 10^{-3}} = \frac{R}{2.236} \quad \begin{cases} R = 1 \Rightarrow Q = 0.45 \\ R = 10 \Rightarrow Q = 4.5 \end{cases}
\]
We require \(|Z_0| < 0.2\) for \(0 < f < 20 \times 10^3\).

\[
\Rightarrow \frac{1}{\sqrt{L}} < 0.2
\]
\[
\Rightarrow \frac{1}{2\pi \times 10^{-3}} < 0.2
\]
\[
\Rightarrow f < \frac{0.2}{2\pi \times 10^{-3}} = 32 \text{ Hz}
\]

Also,

\[
\frac{1}{\sqrt{C}} < 0.2
\]
\[
\Rightarrow f > \frac{1}{2\pi \times 0.2 \times 200 \times 10^{-6}} = 3979 \text{ Hz}
\]

Thus, the output impedance spec is met not for the frequency range

\[32 < f < 3979 \text{ Hz}\]
Magnitude of $Z_0$ at intersection of asymptotes:

at $\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \sqrt{\frac{1}{LC}}$

$\Rightarrow f_0 = 356 \, \text{Hz}$

$2\pi f_0 L = 2\pi (356) \times 10^{-3} = 2.24 \, \text{m}$

The $Q$ can be as high as 4.5

$\Rightarrow |Z_0|_{\text{max}} = 4.5 \times 2.24 = 10.2$

we need to reduce $|Z_0|_{\text{max}} < 0.2$.

Now in the presence of feedback (with loop gain $T(s)$) the output impedance $Z_{of}$ becomes

$$Z_{of} = \frac{Z_0}{1 + T(s)}$$

For $T$ large we require

$$T = \frac{Z_0}{Z_{of}} = \frac{10}{0.2} = 50$$
From Fig. 4:

\[ T(\xi) = \frac{\xi}{\xi_0} \frac{\xi}{\partial \xi} G_c(\xi) \]  

Now \( \frac{\xi}{\xi_0} = \frac{1}{4} \) \( k = 0.1 \)

\[
\frac{\xi}{\partial \xi} = V_g \frac{R}{R_\kappa + R} \frac{1}{s^2 C_\kappa \left( \frac{R}{R_\kappa + R} \right) + s \left( \frac{R_\kappa C + L}{R} \right) \left( \frac{R}{R_\kappa + R} \right) + 1} \]

\[
\Rightarrow T(\xi) = G_c(\xi) \frac{2.5}{s^2 (0.2 \times 10^{-6}) + s \frac{0.2}{\kappa} + 1} \]

\[
\left| \frac{T(\xi)}{G_c(\xi)} \right| = 2.5 \quad f_p = 356 \text{ Hz} \]
Let us choose $G_c(s)$ to be of the form

$$G_c(s) = \frac{G_m}{1 + \frac{\frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}}$$

this is lead-lag compensation.

\[T(s)\] becomes

$$T(s) = \frac{T_m}{1 + \frac{s}{\omega_z} + \frac{s^2}{\omega_c^2}}$$

where $\omega_c < \omega_2 < \omega_p$ and $T_m = k M_d G_m V_y > 50$

We can sketch $|T(s)|$, $|1 + T(s)|$, $|Z_o|$ and $|Z_{of}|$ for the two cases $\beta_{eq} = 1$ and $10 \beta$. 
Chose of compensation parameter: \( G_c(s) = G_m \frac{1 + \frac{s}{\omega_n}}{1 + \frac{s}{\omega_p}} \)

we require \( T_m = T_M \quad \Rightarrow \quad G_m \gg 50 \)

\( h = 0.1 \ ; \ \alpha_d = \frac{1}{5} \ ; \ \nu_g = 100 \)

\( \Rightarrow \quad G_m > \frac{50 \times 4 \times 10}{100} = 20 \quad \Rightarrow \quad 26 \text{ dB} \)

Let us choose a unity gain crossover frequency of \( f_c = 10 \text{ kHz} \). Let us place \( f_z = 800 < \frac{f_c}{10} \)

The phase margin can be controlled by placing \( f_p \) appropriately:

\( \phi_p = \text{phase margin} = 180^\circ - \left[ -\tan^{-1} \left( \frac{\nu_g}{f_z} \right) + \tan^{-1} \left( \frac{10^3}{f_p} \right) + 180^\circ \right] \)

\( \angle \phi_p \) phase contribution due to double pole at \( w_0 \)

\[ \phi_m = 85 - \tan^{-1} \left( \frac{10^3}{f_p} \right) \]

\( \Rightarrow \quad \tan^{-1} \left( \frac{10^3}{f_p} \right) = -\phi_m + 85 \quad \Rightarrow \quad f_p = \frac{10^3}{\tan \left[ 85 - \phi_m \right]} \)

for a phase margin of \( \phi_m = 60^\circ \)

\( \Rightarrow \quad f_p = 21,445 \)

For \( \phi_p = 60^\circ \Rightarrow \) closed loop \( Q = -1.75 \text{ dB} \)

(this figure is independent of the load)
\( T_2 = \text{Gain} \text{ at } \omega_2 \quad \Rightarrow \quad \frac{\omega_c}{\omega_2} = \frac{10,000}{500} = 20 \)

\( T_m = \text{Gain} \text{ at } \omega_p \quad \Rightarrow \quad T_2 \left( \frac{\omega_p}{\omega_2} \right)^2 = T_m \left( \frac{\omega_p}{\omega_0} \right)^2 = 63 \left( \frac{356}{500} \right)^2 = 12.5 \left( \frac{356}{500} \right)^2 = 63 \left( \frac{356}{500} \right)^2 = 12.5 \)

\( \left( \text{Check: } T_m \left( \frac{\omega_p}{\omega_2} \right)^2 = 63 \left( \frac{356}{500} \right)^2 = 12.5 \right) \)

\( T_m = \frac{1}{M} \text{Mr} \text{Vg} G_m \)

\( \Rightarrow \quad G_m = \frac{T_m}{\frac{1}{M} \text{Mr} \text{Vg}} = \frac{63}{0.1 \times \frac{1}{4} \times 100} = 25.2 \rightarrow 28 \text{ dB} \)

\( G_m = 25 \)

\( \omega_2 = 2\pi (800) \)

\( \omega_p = 2\pi (21,945) \)

\( \text{Check: } \eta \quad \Rightarrow \quad \frac{2.24}{1 + T_m} \frac{\omega_p}{\omega_0} = \frac{2.24}{64} \frac{2\pi (356)}{2\pi (500)} = 0.079 \Delta < 0.2 \Delta \)
Appendix:

**Semi-Graphical Method for Constructing Bode Plot of \( Z_0 \)**

\[
Z_0 = \frac{(R_L + sL)}{sL / R}
\]

**RL+SL:**

\[
\begin{align*}
\text{Take the bigger of the two at crossover } R_L &= sL \\
\Rightarrow \frac{\omega_z}{2\pi} &= \frac{R_L}{L} = \frac{4 \times 10^{-3}}{1 \times 10^{-3}} \\
\Rightarrow \omega_z &= \frac{4}{2\pi} = 0.64 \text{ Hz}
\end{align*}
\]

\[
\begin{align*}
|RL+sL| &\Rightarrow
\end{align*}
\]
\[(L_a + L_b) \parallel \frac{1}{L_c} \parallel R\] parallel combination, take the smallest quantity.

![Diagram with labels and figure numbers]

at crossover #1: \[SL = R\]
\[\Rightarrow \quad \omega_1 = \frac{R}{L} = \begin{cases} 10^3, & R = 1 \\ 10^4, & R = 10 \end{cases}\]

at crossover #2: \[\frac{1}{L_c} = R\]
\[\omega_2 = \frac{1}{RC} = \begin{cases} 5 \times 10^3, & R = 1 \\ 5 \times 10^2, & R = 10 \end{cases}\]

From the values for \(\omega_1\) and \(\omega_2\), we see that the poles will change as \(R\) changes. Clearly, for \(R = 1\), Fig A applies as \(\omega_1 < \omega_2\). However, for \(R = 10\), Fig B applies as \(\omega_2 < \omega_1\).
At the crossing point of the asymptotes

\[ sh = \frac{1}{sc} \quad \Rightarrow \quad w_0 = \frac{1}{\sqrt{sc}} \]

the impedance is \( w_0 R \) \( \Rightarrow \) \( \frac{R}{\sqrt{sc}} = \sqrt{\frac{R}{c}} = \text{characteristic impedance} \)

\[ Q = \frac{R}{\sqrt{sc}} \quad \Rightarrow \quad \begin{cases} \frac{1.25}{2.25} = 0.55 & \text{for } R = 1 \\ \frac{10}{2.25} = 4.5 & \text{for } R = 10 \end{cases} \]

From the Bode plot of Fig. 3 we can write the factored pole-zero expression for \( Z_0 \) as

\[ Z_0(s) = \frac{R\pi}{1 + \frac{s}{\omega_2}} \quad \frac{1 + \frac{s^2}{\omega_0^2}}{1 + \frac{s}{\omega_2} + \frac{s^2}{\omega_0^2}} \]

\[ \omega_2 = \frac{R\pi}{2} = 2\pi (0.64) \]

\[ \omega_0 = \sqrt{\frac{R}{c}} = 2\pi (3.56) \]

\[ Q = \frac{R}{\sqrt{sc}} \quad \begin{cases} 0.55 & \text{for } R = 1 \\ 4.5 & \text{for } R = 10 \end{cases} \]

\[ R\pi = 0.0067 \]
PROBLEM 1.
For the converter of Fig. 1, (assuming continuous conduction operation),

(a) Determine an expression for \( Z_{in} \), the linearized, average impedance seen by the source \( V_g \).

(b) Sketch the magnitude straight-line asymptote of \( Z_{in} \), noting the salient features such as the values of the break frequencies, level of constant impedance values and peaking levels.

![Fig. 1](image-url)
(10 marks) PROBLEM 3.

(5) a) The transfer function, \( A(s) \), of a converter was determined to be

\[
A(s) = A_m \frac{1 - s/\omega_z}{(1 + s/\omega_p)(1 + s/\omega_c)}
\]

\( A_m = 1.53 \)
\( \omega_z = 2\pi(2.29)kHz \)
\( \omega_p = 2\pi(16.4)Hz \)
\( \omega_c = 2\pi(7.69)kHz \)

A compensation network with transfer function, \( G_c(s) \), where

\[
G_c(s) = G_m\left(1 + \frac{\omega_1}{s}\right)
\]

\( G_m = 30 \)
\( \omega_1 = 2\pi(9.5)Hz \)

completes a feedback loop as shown in Fig. 2. Using a graphical technique or otherwise, determine the factored pole-zero expression for the feedback factor, \( 1 + T \), where \( T \) is the loop gain. On a sketch of \( |1+T| \) label the numerical values of any poles and zeroes and constant gain regions.

(5) b) The open loop output impedance, \( Z_o \), of the converter is given as

\[
Z_o = R_m \frac{1}{1 + s/\omega_p}
\]

\( R_m = 4.88\Omega \)
\( \omega_p = 2\pi(16.4)Hz \)

Using a graphical technique or otherwise, determine the factored pole-zero expression for the closed loop output impedance, \( Z_{of} \), for the loop gain given in part (a). On a sketch of \( |Z_{of}| \) label the numerical values of any poles and zeroes and constant gain regions.
PROBLEM 4.

The control-to-output transfer function of a certain converter is given by the following

\[
\frac{v}{d} = \frac{V_g}{D^{2R}} \frac{1 - s \frac{L}{D^{2R}}}{1 + s \frac{L}{D^{2R}} + s^2 \frac{LC}{D^{2R}}}
\]

For the values, \(D = 0.5, R = 10, V_g = 30, L = 160\mu H\) and \(C = 160\mu F\).

a) Draw a Bode plot (magnitude and phase) using asymptotic straight line segment approximations. Label all salient features, such as, frequency breakpoints, constant gain/phase regions, etc.

b) Using your Bode plot, or otherwise, determine the unity gain frequency and the phase shift at this frequency.
Problem 1:

\[ Z_{\text{in}} = \frac{V_g}{I_g} \]

Now \[ I = A \hat{i}^2 + B \hat{j}^2 + B \hat{k} \]

\[ \hat{j} = C \hat{i} + E \hat{j} + E \hat{k} \]

Now \[ \hat{j} = 0 \] and let \[ \hat{j}^* = \hat{j} \]

\[ \frac{\hat{j}^*}{\hat{j}} = \frac{1}{Z_{\text{in}}} = \frac{C \hat{i} + E}{E} \]

where \[ \frac{1}{\hat{j}} = (E^T - A)^{-1} B \]

Now let's find the appropriate \( A, B, C + E \)

\[ A = DA + D' A_2 \]
\[ B = DB + D' B_2 \]
\[ C = D C_1 + D' C_2 \]
\[ E = DE_1 + D' E_2 \]
During $\Delta T_3$

\[ y - \frac{L}{C} \frac{di}{dt} + v = 0 \]

\[ \theta \frac{di}{dt} = -\frac{v}{L} + \frac{v_y}{L} \]

\[ \frac{di}{dt} = -\frac{v}{L} \]

\[ i = \frac{c}{L} \frac{dv}{dt} + \frac{v}{R} \]

\[ \theta \frac{dv}{dt} = \frac{v}{L} - \frac{v}{R_C} \]

\[ \frac{dv}{dt} = \frac{i}{c} - \frac{v}{R_C} \]

Let $x = [i \ v]^T$

\[ A_1 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_C} \end{bmatrix} \]

\[ B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \]

\[ A_2 = \begin{bmatrix} 0 & \frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_C} \end{bmatrix} \]

\[ B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} v_y \]

\[ \dot{y} = \begin{bmatrix} 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} v_y \]
A = A_1 + A_2

B = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}

C = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}

E = E_1 + E_L = 0

Now \frac{\partial^2 y}{\partial x^2} = \frac{1}{Z_{in}} = \frac{C}{c^2} \left( sL + A \right)^T B \cdot y + E

\frac{D}{Z_{in}} = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s + \frac{1}{\frac{1}{L}c} & -\frac{D}{L} \\ -\frac{D}{L} & s + \frac{1}{\frac{1}{L}c} \end{bmatrix} \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}

\frac{D}{\frac{s(s + \frac{1}{L}c)}{\frac{s}{LC} + \frac{1}{LC}}}

= \begin{bmatrix} D (s + \frac{1}{\frac{1}{L}c}) & \frac{-D}{L} \\ -\frac{D}{L} & s + \frac{1}{\frac{1}{L}c} \end{bmatrix} \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}

\frac{\frac{D^2}{L^2} (s + \frac{1}{\frac{1}{L}c})}{\frac{s^2}{LC} + \frac{1}{LC}}
\[
\frac{1}{2i \omega_c} = \frac{D^2}{R} \left( \frac{1 + RCS}{LCs^2 + \frac{s^2}{R} + 1} \right)
\]

\[
\Rightarrow Z_{in} = \frac{R}{D^2} \left( \frac{LCs^2 + \frac{s^2}{R} + 1}{1 + RCS} \right)
\]

\[
\Rightarrow \beta \omega_c + \omega_p = \frac{1}{2c}
\]

\[
\text{zeros} \quad \omega_z = \frac{1}{\sqrt{LC}} \quad Q = \frac{\frac{s}{c}}{\sqrt{\epsilon}}
\]
b) Two cases

1) \( \omega_p < \omega_z \)

2) \( \omega_p > \omega_z \)
Problem 3

\[
T(s) = A(s) G_c(s) = A_m G_m \frac{(1 + \omega_d / \omega_1)}{(1 + \omega_d / \omega_p)} \frac{(1 - \omega_d / \omega_2)}{(1 + \omega_d / \omega_c)}
\]

\[
T_m = A_m G_m = 30 \times 1.53 = 45.9 \Rightarrow 33 \text{ dB}
\]
\( \omega_0 \) is freq at loop gain of 1 (0 dB)

\[
1 = \frac{T_m \omega_p}{\omega_0}
\]

\( \Rightarrow \omega_0' = T_m \omega_p = 45.9 \times 2\pi (16.4) = 2\pi (753) \text{ Hz} \)

Gain at freq \( \omega_z \):

\[
T_2 = \frac{T_m \omega_p}{\omega_z} = \frac{45.9 \times 16.4}{2290} = 0.33 \rightarrow -9.7 \text{ dB}
\]

\[|1+T_m| = 46.9 \rightarrow 33.4 \text{ dB} \]
From the plot of $(1+T)$ we can write

\[ 1 + T(s) = \frac{(1 + T_m)(1 + \frac{\omega_r}{\omega_p})}{(1 + \frac{s}{\omega_p})} \]

\[ 1 + T_m = 46.9 \quad \rightarrow \quad 33.4 \]

\[ \omega_r = 2\pi (9.5) \text{ Hz} \]

\[ \omega_p = 2\pi (16.4) \text{ Hz} \]

\[ \omega_o = (1 + T_m) \omega_p = 2\pi (769) \text{ Hz} \]

\[ Z_0 = \frac{\beta_m}{1 + \frac{s}{\omega_p}} \]

\[ \beta_m = 4.88 \Omega \]

\[ \omega_p = 2\pi (16.4) \text{ Hz} \]

See plot, over the page.

\[ Z_{ef} = \frac{Z_0}{1+T} \quad \rightarrow \text{ from feedback theory} \]
From the plot of $|Z_f|$ we can write

$$Z_{of} = \frac{R_{om}}{1 + T_m} \frac{1}{(1 + \frac{\omega_f}{\frac{4.88}{46.9}})(1 + \frac{\omega_o}{\omega_p})}$$

with

$$\omega_f = 2\pi (4.5) \text{ Hz}$$

$$\omega_o = (1 + T_m) \omega_p = 2\pi (769) \text{ Hz}$$
Problem 4

Solution

1) This problem is taken from the lecture notes.

\[ \frac{\hat{C}}{\hat{D}} = \frac{V_0}{\hat{D}^2} \left[ 1 - \frac{s \frac{L}{D^2 R}}{1 + s \frac{\frac{L}{D^2 R} + s^2 \frac{C}{D^2}}{1 + \frac{s}{2\omega_0} + \left( \frac{s}{\omega_0} \right)^2} } \right] = \frac{C_0}{\omega_2} \]

where \( C_0 = 120 \Rightarrow 41.6 \text{ dB} \)

\[ f_2 = 2.5 \text{ kHz} \quad (\text{RHP}) \quad (15,708 \ \text{rad/s}) \]

\[ f_0 = 500 \text{ Hz} \quad (3,172 \ \text{rad/s}) \]

\[ Q = 5 \Rightarrow 14 \text{ dB} \]
1) The magnitude along the -20dB slope away from the resonant frequency is given by
\[ G_{dB} \left( \frac{\omega_0}{\omega} \right)^2 \]

\[ \text{at } f_2, \quad \left| \frac{\omega}{\omega_0} \right| f_2 = 2.5 \text{kHz} \]

\[ \therefore 120 \left( \frac{500}{2500} \right)^2 = 4.8 \]

The magnitude along the -20dB slope is given by
\[ G_{f_2} \frac{\omega_0}{\omega} \]

\[ \text{the unity gain frequency } = f_u = G_{f_2} f_2 \]

\[ = 4.8 \times 2500 \]

\[ = 12 \text{ kHz} \quad (75900 \text{ rad/s}) \]

Using MATLAB (see program (c) for) we find \( f_u = 12.202 \text{ kHz} \)

To determine the phase we realize that since 12kHz is very much higher than the double pole the phase contribution is very close to -180 degrees =)

\[ \text{phase at } 12 \text{kHz} = -180 - \tan^{-1} \left( \frac{12000}{2500} \right) \quad \text{the contribution from the RHP zero} \]

\[ = -180 - 78.23 \]

\[ = -258.23 \quad \text{MATLAB gives } -258.01 \]
Alternatively using the straight line asymptote approach for the phase we realize the phase at \( f_2 = \frac{-180}{-45} = -225 \) deg.

Phase due to phase due to

\[ + \text{ double pole} \quad \text{zero} \]

Now at \(-45^\circ/\text{dec} \) slope the phase change from \( f_2 = 2500 \text{ Hz} \) to \( f_u = 12000 \text{ Hz} \) is given by

\[
-45^\circ \times \log_{10} \frac{12000}{2500} = -30.66^\circ
\]

:. Total phase = \(-225 - 30.66\)

\[ = -255.66^\circ \quad \text{thus a slight error from } -258^\circ \quad \text{is introduced using the straight line approximations} \]
% This m-file plots the magnitude and phase response of the transfer
% function given in Question 4.
% The unity gain frequency and the phase at this frequency are also
% determined.

vg=30; r=10; d=0.5; l=160e-6; c=160e-6;
dp=1-d;

m=vg/(dp*dp);
a0=1; a1=-1/(dp*dp*r); a2=0;
b0=1; b1=1/(dp*dp*r); b2=(l*c)/(dp*dp);

num=m*[a2 a1 a0];
den=[b2 b1 b0];

w=logspace(2, 6, 1000);
[mag, phase]=bode(num, den, w);

[Gm, Pm, Wcg, Wcp]=margin(mag, phase, w)
magdb=20*log10(mag);
semilogx(w, magdb)
pause
semilogx(w, phase)
pause
f Unity=Wcp/(2*pi) % the unity gain frequency
phase u=-180+Pm % the phase at the unity gain frequency