Regular Structures
Lattice Structure for Multivalued and Binary Logic

- Realizes every binary symmetric function
- Realizes every non-symmetric function by repeating variables
- Realizes piece-wise linear multivalued functions

Patented by Pierzchala and Perkowski 1994/1999
Cell has three inputs and two outputs

Both outputs have the same function

Lattice Structure for Multivalued and Binary Logic

Multi-valued output variable

Multivalued variables
Lattice Structure for Multivalued and Binary Logic

Redness of nose in interval [3,4]

Length of beard an odd number

Redness of eyes in intervals [2,4] or [7,9]

Jeske

Al-Rabadi

Zakrevskij

Perkowski

Multi-valued output variable

Multivalued variables
Lattice Structure for Multivalued and Binary Logic

- $A > B$
- $C < D$
- $E = G$

Multi-valued output variable

- Binary

Multivalued input variables

- Jeske
- Al-Rabadi
- Zakrevskij
- Perkowski

Multivalued input variables
Lattice Structure for Multivalued and Binary Logic

Cell has 4 inputs and 2 outputs

Can we make the cell reversible?

Multivalued input variables

Multi-valued output variable

Cell has 4 inputs and 2 outputs

Can we make the cell reversible?

Multivalued input variables

Jeske

Al-Rabadi

Zakrevskij

Perkowski

Multivalued input variables
<table>
<thead>
<tr>
<th>Control</th>
<th>left</th>
<th>right</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>value</td>
<td></td>
<td>value</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>value</td>
<td>value</td>
</tr>
</tbody>
</table>

We want to make this cell reversible.

Values not separated.
Let us try to repeat control variable in output

Still not separated
Repeating variables will not help

Now it works!
This means that we added another MUX
... And we reinvented the Fredkin Gate ....!!!

• But how to use it in a Lattice?
Lattice Structure for Binary Logic

\[ F = S_{1,3} (A, B, C) \]
Reversible Lattice Structure for Binary Logic

\[ F = S^{1,3} (A,B,C) \]
Reversible Lattice Structure for Binary Logic

**Advantages**
- regular structure
- binary Fredkin Gate
- planar structure (good for Quantum Logic)
- Easy algorithmic creation
- Reasonable waste

**Disadvantages**
- Variable ordering?
- Symmetrization?
- Waste still exist

Should be patented!
Do you remember that there are other binary expansions?

**All Binary Expansions**
- Shannon - S
- Flipped Shannon - fS
- Positive Davio - pD
- Negative Davio - nD
- Flipped Positive Davio - fpD
- Flipped Negative Davio - fnD

**Ideas**
- Fredkin = <Var, S, fS>
- what about these?
  - <Var, pD, fpD>
  - <Var, nD, fnD>
  - <Var, nD, pD>
  - ... 

I checked some of them to work
Do you remember that there are other component functions of reversible gates?

- All Binary Balanced Expansions
- Linear functions - L
- Negations - N
- Majorities - M

I checked some of them to work.

Ideas
- Fredkin = <Var, S, fS>
- What about these?
  - <N, pD, fpD>
  - <Var, M, fnD>
  - <Var, nD, L>
  - ....
As you see, this opens a very broad area of research that will lead to invention of new reversible gates and regular structures that use them.

**Easy way to become a pioneer:**

- Investigate all combinations
- Use genetic programming or other search methods to build structures and map functions to them
- There is a place for many researchers
- Nobody does this research

But this was only for binary

What about multivalued, fuzzy, arithmetic or other logics?
.... And we reinvented the Fredkin Gate ....!!!

- But what about the variant with two control signals?
Multi-valued Fredkin Gate

- MVFG is described by equations:

\[
\begin{align*}
P & = A \\
Q & = B \\
R & = C \text{ if } A < B \text{ else } R = D \\
S & = D \text{ if } A < B \text{ else } S = C
\end{align*}
\]
Lattice Structure for Multivalued and Binary Logic

Cell has 4 inputs and 4 outputs

Cell is reversible!

MV and Generalized MV

Fredkin

Multi-valued output variable

waste

waste

A

B

0

1

C

D

E

G

Jeske

Al-Rabadi

Zakrevskij

Perkowski

Multivalued input variables
Multi-valued logic generates less signals

Hence it generates less waste

Of course, it generates also less power, less connections and is easier to test
The main open research problem

• The real-life functions are multi-output.
• Thus, there exists an opportunity to re-use some waste functions in other output functions.
• This is a tough problem.
• I do not know now how to solve it!

We need some group creativity
Generalized Multi-valued Fredkin Gate

Select other function of two variables

Select other pairs of VAR-type and NOT-type functions

Select other pairs of MUX-type functions
Generalized Multi-valued Fredkin Gate

• The number of these gates is astronomical

• We need both computer generation and some intelligence, simply generating them all would be a nonsense

• Very wide area of research

• It will give hints to gate designers what to look for
But this is only a beginning...
Let us go back to our fundamental invention…..

• What if we resign from oblique buses?
Buses are removed and each cell is programmed individually.....

• Some regularity is lost!