This is a closed-notes, closed-book exam.

1. PDA construction

   (a) Construct a PDA accepting the language
   
   \[ A = \{ w \mid w \text{ has an equal number of } a\text{'s and } b\text{'s} \} \]

   (b) Justify your construction
   (c) Illustrate a computation of your machine on the string \( aabbbba \).

2. Not Regular

   Consider the language
   
   \[ A = \{ w \mid w \text{ has an equal number of } a\text{'s and } b\text{'s} \} \]

   Use this language to demonstrate three techniques for showing that \( A \) is not regular.

   (a) Show \( A \) is not regular using the pumping lemma.
   (b) Show \( A \) is of infinite index.
   (c) Show \( A \) is not regular by using closure properties and the fact that
   \( \{ a^i b^i \mid i \geq 0 \} \) is not regular.

3. Shuffle Let \( A, B \subseteq \Sigma^* \) be languages. Define the shuffle of \( A \) and \( B \), \( A \odot B \) as follows:

   \[ A \odot B = \{ x_1 y_1 \cdots x_k y_k \mid x_1 \cdots x_k \in A \text{ and } y_1 \cdots y_k \in B, x_i, y_i \in \Sigma^* \} \]

   For example, \( \{000\} \odot \{111\} \) includes the strings \( 000111, 111000, 101010, \)
   \( 010101, 011100, \ldots \).

   Define the shuffle closure of \( A \), \( A^{\odot} \), as follows:

   \[ A^{\odot_0} = \{ \epsilon \} \]
   \[ A^{\odot_{n+1}} = A^{\odot_n} \odot A \]
   \[ A^{\odot} = \bigcup_{i \geq 0} A^{\odot_i} \]

   (a) Show the regular sets are closed under shuffle (\( \odot \)).
   (b) Show the regular sets are not closed under shuffle closure (\( \odot \)).