1. If at first you don’t succeed...

   (a) [10 points] Define a PDA that recognizes palindromes (strings that read the same written forward and backward, such as “Wasilla’s all I saw” or “a man a plan a canal panama”) by final state and empty stack.

   (b) [15 points] Show that your machine accepts the following strings.

      i. $\epsilon$
      ii. 010
      iii. 0110

2. [25 points] Prove that the set of palindromes is not regular.

3. [25 points] From first principles prove that $A_{TM}$ is undecidable.

4. [25 points] In the proof of the incompleteness theorem we needed the notion of representability to show that there must be a formula in the theory of arithmetic that corresponds to the Kleene $T$ predicate (or equivalently Sipser’s $\phi_{M,w}$).

   (a) [5 points] Sketch the definition of representability.

   (b) [10 points] Discuss how representability is used in the incompleteness argument presented in class. Address the questions:

      i. Is it needed to argue that truth is undecidable?
      ii. Is it needed to formulate an unprovable sentence?

   (c) [10 points] In class we sketched an induction proof that all the partial recursive functions were representable. Show that if $g$ is a representable $k$-ary function and $h_1, \ldots, h_k$ are representable $l$-ary functions that $f = go[h_1, \ldots, h_k]$ is representable.