1. Write a MATLAB program to find the value of $x$ that satisfies $\cos(x) = x$ using fixed point iteration. Use the notes from lecture 1 as a guide. The iterative formula is $x_{k+1} = \cos(x_k)$.

Your solution should include the following components.

- Listing of your MATLAB program,
- Brief discussion of your choice of convergence tolerance (numerical value) and convergence criterion (i.e., absolute or relative),
- A print-out of the values of $x_k$ during the iterations,
- Values of $E_{\text{abs}} = \cos(\tilde{x}) - \tilde{x}$ and $E_{\text{rel}} = (\cos(\tilde{x}) - \tilde{x})/\tilde{x}$ at convergence.

2. Transcribe the MATLAB program from the class notes for evaluating the series approximation to $\sin(x)$. Plot the number of terms required for convergence over the range $0 \leq x \leq 2\pi$. Your solution should include the program listing, the plot, and a brief (2-3 sentences) discussion of the trend in the data.

3. (5 points) Estimate the diffusion time scale for thermal energy transport through an oak table 1.5 cm thick if the thermal diffusion coefficient is $1.8 \times 10^{-7}$ m$^2$/s. Estimate the transit time for a sound pulse through a 1.5 cm layer of oak if the sound speed in oak is 3800 m/s. How does time for transmission of a sound wave across a layer of oak compare to the time it takes that same layer to reach steady state after a step change in thermal boundary condition?

ME 548 Students also do:

4. (20 points) The analytical solution to

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq x \leq L, \quad t > 0 \quad (1)$$

$$u(x, 0) = \begin{cases} U_0 & 0 \leq x \leq x_c \\ 0 & x_c < x \leq L \end{cases} \quad (2)$$

$$u(0, t) = U_0, \quad u(L, t) = 0 \quad (3)$$

where $\alpha$ and $0 < x_c/L < 1$ are constants is a variation on the exact solution given in Chapter 14 of the notes. The real work is in finding the coefficients $A_n$ in the Fourier expansion for the initial condition.

Hint: The inhomogeneous boundary condition at $x = 0$ can be removed by finding the solution to a related problem. Define

$$u(x, t) = u_{\text{ss}} - v(x, t) \quad (4)$$

where $u_{\text{ss}}$ is the solution to the steady state problem

$$u_{\text{ss}} = U_0 \left(1 - \frac{x}{L}\right) \quad (5)$$
a. Substitute Equation (4) into Equations (1) through (3) to show that 
$v(x,t)$ is a solution to a homogeneous problem with a new initial 
condition.

b. What is the formula for the constants, $A_n$, in the analytical solution?

c. What are the first five terms, $A_n$, $n = 1, 2, \ldots, 5$ when $\alpha = 0.1$, 
$x_c = 2$, and $L = 5$.

d. Modify one of the MATLAB codes given with the lecture notes so that 
the exact solution is evaluated and plotted for $t = \ldots$.

Start with the equations given in Chapter 14. Do not derive the separate 
of variables solution from the beginning.