1. As depicted by the sketch below, flow in an infinite plane is driven by a large sheet of material a distance $h$ above a solid wall. The sheet moves with velocity, $U$, and the velocity field is $\mathbf{u} = \hat{e}_x U y / h$. Find the equation of the streamlines in this flow. Locate the streamline that divides the total flow rate into two equal parts.

2. The velocity components for a two-dimensional (plane) incompressible flow are

$$v_r = \frac{a}{r} + \frac{b}{r^2} \cos \theta \quad v_\theta = \frac{b}{r^2} \sin \theta$$

where $a$ and $b$ are constants. Determine the corresponding stream function, $\psi = \psi(r, \theta)$. *Hint:* Set $\psi = \psi_0 =$ constant at any convenient location.

3. Incompressible, inviscid flow around a circular cylinder of radius $a$ has the stream function

$$\psi = \left( \frac{a^2}{r} - r \right) U \sin \theta$$

where $U$ is the free stream velocity.

   a. Obtain expressions for the $v_r$ and $v_\theta$ velocity components.
   
   b. Prove that $v_r = 0$ at $r = a$. Why is this condition important?
   
   c. What is the magnitude of the velocity at $r = a$? If the flow were viscous, what boundary condition would be necessary at $r = a$.

4. Given the velocity field from the previous problem, use Bernoulli’s equation to determine the pressure field, $p(r, \theta)$ in the flow around the cylinder. As necessary for the application of Bernoulli’s equation, neglect viscous affects. Also assume that the flow is steady and incompressible, and that body forces are negligible. Let the pressure far from the cylinder be $p_0$. What is the pressure field on the surface of the cylinder $p(a, \theta)$