1. A two-liter cylindrical tank, 10 cm in diameter, has a piston that fits perfectly. The piston does not leak, and there is no friction between the piston and walls of the tank.

Suppose the tank is filled with water that is initially at 1 atm of pressure. How much additional weight must be placed on the piston to move the piston 1 cm?

2. Suppose the tank in the preceding exercise is filled with air that is initially at 1 atm of pressure. How much additional weight must be placed on the piston to move the piston 1 cm?

3. Calculate the speed of sound in air, and in Hydrogen at 70°F.

4. Calculate the speed of sound in air at the altitudes in the following table. Assume the specific heat ratio is $k = 1.4$.

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Temperature (°C)</th>
<th>Sound Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>−17.5</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>−49.9</td>
<td></td>
</tr>
<tr>
<td>20000</td>
<td>−56.5</td>
<td></td>
</tr>
</tbody>
</table>
5. What is the stagnation temperature and stagnation pressure on the nose of a reentry vehicle moving at a Mach number of 7 at an altitude of 200,000 ft? At 200,000 ft, the ambient temperature is 457° Rand the pressure is \(0.58 \times 10^{-2}\) inch Hg.

6. An ideal gas \((k = 1.4, R = 100\text{ ft} \cdot \text{lb}_f/\text{lb}_m/°\text{R})\) is supplied to a converging nozzle at low velocity at at 100 psia and 540°F. The nozzle discharges to atmospheric pressure, 14.7 psia. Assuming frictionless, adiabatic flow, and a mass flow rate of 2 lbm/s, calculate

a. The pressure in the exit plane in psia.

b. The velocity in the exit plane in ft/s.

c. The cross-sectional area of the exit plan in in².

7. Air flowing in a duct has a pressure of 20 psia, a Mach number of 0.6, and a flow rate of 0.5 lbm/s. The cross sectional area of the duct is \(1.5\text{ in}^2\).

a. Compute the stagnation temperature of the stream.

b. What is the maximum percent reduction in area that could be introduced without reducing the flow rate of the stream?

c. For the maximum reduction from part (b), find the velocity and pressure at the minimum area. Assume that the flow is adiabatic and friction is negligible.
Solution

1. Apply the definition of bulk modulus to finite changes in $p$ and $V$

$$E_v = -V \frac{dp}{dV} \approx V \frac{\Delta p}{\Delta V} \implies \Delta p = E_v \frac{-\Delta V}{V} \quad (1)$$

where $-\Delta V$ means that a positive pressure change occurs when $\Delta V$ is negative.

Use simple geometry of the cylinder

$$V = LA \quad \text{and} \quad \Delta V = -L \Delta x \implies \frac{-\Delta V}{V} = \frac{+\Delta x}{L} \quad (2)$$

where $A = \frac{\pi}{4} D^2$ is the area of the cylinder.

Combine Equation (1) and Equation (2)

$$\Delta p = E_v \frac{\Delta x}{L} \quad (3)$$

From the definition of pressure, $\Delta p = (W - mg)/A$, where $W$ is the added weight and $m$ is the mass of the piston. Define $W' = W - mg$ as the change in weight necessary to compress the liquid, then

$$\Delta p = \frac{W'}{A} \implies W' = A \Delta p = E_v A \frac{\Delta x}{L} \quad (4)$$

From the given geometric data

$$L = \frac{V}{A} = \frac{(2\ell) \left(1000 \text{ cm}^3\right)}{\frac{\pi}{4}(0.10 \text{ m})^2} = 0.25 \text{ m}$$

Substitute numerical values into Equation (4). From Table 1.6 in Munson, Young and Okiishi (inside book cover), $E_v = 2.15 \times 10^9 \text{ N/m}^2$

$$W' = \left(2.15 \times 10^9 \frac{\text{N}}{\text{m}^2}\right) \left(\frac{\pi}{4}(0.10 \text{ m})^2\right) \frac{0.010 \text{ m}}{0.25 \text{ m}} = 675,000 \text{ N}$$

If a small car weighs $1.5 \text{ ton} = 1.5 \times 2000 \text{ lb}_f \times (1 \text{ lb}_f/4.448 \text{ N}) = 674 \text{ N}$, then the weight of 1000 cars is required to compress the liquid by one cm!
2. Use the ideal gas law to find the relationship between changes in pressure and changes in volume.

\[ pV = mRT \implies p = \frac{mRT}{V} \]

\[ \implies \frac{p_2}{p_1} = \frac{m_2RT_2/V_2}{m_1RT_1/V_1} = \frac{T_2}{T_1} \frac{V_1}{V_2} \]

Assume that the compression process is quasistatic and thermal equilibrium with the environment is maintained. Then \( T_1 = T_2 \) and

\[ \frac{p_2}{p_1} = \frac{V_1}{V_2} \implies p_2 = p_1 \frac{V_1}{V_2} \]

From the geometry

\[ V_1 = LA \quad \text{and} \quad V_2 = (L - \Delta x)A \implies p_2 = p_1 \frac{L}{L - \Delta x} \]

From definition of pressure, \( W = p_2A \). Combining the preceding equations gives

\[ p_2 = \frac{W}{A} = p_1 \frac{L}{L - \Delta x} \implies W = p_1A \frac{L}{L - \Delta x} \]

Substitute numerical values

\[ W = \left( 101325 \text{ N/m}^2 \right) \left( \frac{\pi}{4} (0.10 \text{ m})^2 \right) \frac{0.25 \text{ m}}{0.25 \text{ m} - 0.01 \text{ m}} = 828 \text{ N} \]

828 N is the weight of

\[ m = \frac{W}{g} = \frac{828 \text{ N}}{9.8 \text{ m/s}^2} = 85 \text{ kg} \]

which is the mass of a large adult male.
3. Evaluate \( c = \sqrt{kRT} \). Use absolute temperatures and \( g_c \) to get correct units.

**Air:**

\[
R = \frac{R_u}{M} = \frac{1545 \text{ ft-lbf}}{28.97 \text{ lbm/mol}} = 53.33 \text{ ft-lbf lbm}^{-1} \text{mol}^{-1} \text{°R}
\]

\[k = 1.4\]

\[
c = \sqrt{kRT} = \sqrt{(1.4) \left(53.33 \frac{\text{ft-lbf}}{\text{lbm} \cdot \text{°R}}\right) (70 + 460 \text{°R}) \left(32.174 \frac{\text{ft-lbf}}{\text{lbm} \cdot \text{s}^2}\right)}
\]

\[
\therefore \quad c = 1130 \frac{\text{ft}}{\text{s}}
\]

**H₂:**

\[
R = \frac{R_u}{M} = \frac{1545 \text{ ft-lbf}}{2.018 \text{ lbm/mol}} = 765.6 \text{ ft-lbf lbm}^{-1} \text{mol}^{-1} \text{°R}
\]

\[k = 1.4 \quad \text{coincidentally same as air}\]

\[
c = \sqrt{kRT} = \sqrt{(1.4) \left(765.6 \frac{\text{ft-lbf}}{\text{lbm} \cdot \text{°R}}\right) (70 + 460 \text{°R}) \left(32.174 \frac{\text{ft-lbf}}{\text{lbm} \cdot \text{s}^2}\right)}
\]

\[
\therefore \quad c = 4275 \frac{\text{ft}}{\text{s}}
\]
4. Use \( c = \sqrt{kRT} \) with \( k = 1.4, \) \( R = \frac{8315 \text{ J}}{28.97 \text{ kg mol}^{-1} \text{K}} = 287 \text{ J kg}^{-1} \text{K}^{-1} \)

At 1000 m:

\[
c = \sqrt{kRT} = \sqrt{(1.4) \left( 287 \text{ J kg}^{-1} \text{K}^{-1} \right) (8.5 + 273.15 \text{ K})}
\]

\[
= 336 \sqrt{\frac{\text{J}}{\text{kg}}} = 336 \sqrt{\frac{\text{N} \cdot \text{m}}{\text{kg}}} = 336 \sqrt{\frac{\text{kg} \cdot \text{m} \cdot \text{s}^{-2}}{\text{kg}}}
\]

\[
\therefore c = 336 \text{ m s}^{-1}
\]

Repeat calculations to fill in the table.

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<td>295</td>
</tr>
</tbody>
</table>

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5. Apply formulas for stagnation pressure and temperature. Use absolute pressures and temperatures. Given pressure and temperature are ambient conditions and are the static pressure and temperature in the formulas for stagnation temperature and pressure.

**Given:**

\[
\begin{align*}
Ma &= 7, \quad T = 457^\circ R, \\
p &= (0.58 \times 10^{-2} \text{ inch Hg}) \left( \frac{14.7 \text{ psia}}{29.92 \text{ inch Hg}} \right) = 0.00285 \text{ psia}
\end{align*}
\]

Compute stagnation temperature. \( k = 1.4 \) for air.

\[
T_o = T \left( 1 + \frac{k - 1}{2} Ma^2 \right) = 457^\circ R \left( 1 + \frac{0.4}{2} 7^2 \right) = 457^\circ R (10.8)
\]

\[\therefore \quad T_o = 4940^\circ R \quad \text{Hot!}\]

Compute stagnation pressure.

\[
p_o = p \left( \frac{T_o}{T} \right)^{k/(k-1)} = (0.00285 \text{ psia}) (10.8)^{1.4/0.4}
\]

\[\therefore \quad p_o = 11.83 \text{ psia}\]

**Note:** The given pressure could not be a gage pressure because that would imply that \( p \approx 14.7 \text{ psia} \) at 200,000 ft.
6. Assume pressure at the exit plane is equal to the pressure of the surroundings. Use subscript “e” to designate conditions at the exit plane.

\[ p_e = 14.7 \text{ psia} \]
\[ \frac{p_e}{p_0} = \frac{14.7 \text{ psia}}{100 \text{ psia}} = 0.147 \]
\[ \dot{m} = 2 \text{ lb}_m/\text{s} \]

Since \( \frac{p_e}{p_0} < 0.528 \) the flow is choked. Therefore, \( \text{Ma} = 1 \) at the smallest area along the flow path, which is the exit plane.

a. \( \text{Ma} = 1 \implies p = p^* \) at the exit.

\[ p_e = p^* = 0.528p_0 \quad \text{for} \quad k = 1.4 \]
\[ = 0.528(100 \text{ psia}) \]
\[ \therefore \quad p_e = 52.8 \text{ psia} \]

b. \( \text{Ma} = 1 \) at the exit means \( V_e = c = \sqrt{kRT} \) and \( T = T^* \).

Compute \( T^* \) and then \( V_e \).

\[ \frac{T^*}{T_0} = 0.833 \implies T^* = (0.833)(540 + 460 ^\circ \text{R}) = 833 ^\circ \text{R} \]

Recall that \( R = 100 \text{ ft} \cdot \text{lb}_f/\text{lb}_m/^{\circ} \text{R} \) for the gas (not air)

\[ V_e = \sqrt{(1.4) \left(100 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m^{\circ} \text{R}} \right) (833 \circ \text{R}) \left( \frac{32.174 \text{ ft} \cdot \text{lb}_f}{\text{lb}_m^{s^2}} \right)} \quad \therefore \quad V_e = 1937 \text{ ft/s} \]

c. \( \dot{m} = \rho_e V_e A_e = 2 \text{ lb}_m/\text{s} \). Use ideal gas law to compute \( \rho_e \) and then solve for \( A_e \).

\[ A_e = \frac{\dot{m}}{\rho_e A_e}, \quad \rho_e = \frac{p_e}{RT_e} \implies A_e = \frac{\dot{m}RT_e}{p_e V_e} \]

\( p_e \) is known from the solution to part (a). \( T_e \) and \( V_e \) values are
known from solution to part (b).

\[ A_e = \frac{(2 \text{ lb}/s)}{\left(52.8 \frac{\text{lb}}{\text{in}^2} \frac{144 \text{in}^2}{\text{ft}^2} \right)} \left( 100 \frac{\text{ft} \cdot \text{lb}}{\text{lb} \cdot \text{R}} \right) \left( 833 \degree \text{R} \right) = \left( 0.01131 \text{ ft}^2 \right) \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) \]

\[ \therefore A_e = 1.63 \text{ in}^2 \]

d. \( \dot{m} = \rho_e V_e A_e = 2 \text{ lb}_m/s. \) Use ideal gas law to compute \( \rho_e \) and then solve for \( A_e \).

\[ A_e = \frac{\dot{m}}{\rho_e A_e}, \quad \rho_e = \frac{p_e}{RT_e} \quad \Rightarrow \quad A_e = \frac{\dot{m}RT_e}{p_e V_e} \]

\( p_e \) is known from the solution to part (a). \( T_e \) and \( V_e \) values are known from solution to part (b).

\[ A_e = \frac{(2 \text{ lb}/s)}{\left(52.8 \frac{\text{lb}}{\text{in}^2} \frac{144 \text{in}^2}{\text{ft}^2} \right)} \left( 100 \frac{\text{ft} \cdot \text{lb}}{\text{lb} \cdot \text{R}} \right) \left( 833 \degree \text{R} \right) = \left( 0.01131 \text{ ft}^2 \right) \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) \]

\[ \therefore A_e = 1.63 \text{ in}^2 \]
7. **Known:** $k = 1.4$, $Ma = 0.6$, $\dot{m} = 0.5 \text{ lb}_m/\text{s}$, $A = 1.5 \text{ in}^2$, $p = 20 \text{ psia}$.

a. We need to find $T$ before we can compute $T_0$. Start by calculating $p_0$.

$$\frac{p}{p_0} = \left[ \frac{1}{1 + \frac{k-1}{2Ma^2}} \right]^{k/(k-1)} = \left[ \frac{1}{1 + \frac{0.4}{2}(0.6)^2} \right]^{1.4/0.4} = 0.784$$

$$\Rightarrow p_0 = \frac{p}{0.784} = \frac{20}{0.784} \Rightarrow p_0 = 25.51 \text{ psia}$$

Since $\dot{m}$ is given the formula for the *maximum* flow rate is useful

$$\dot{m}_{\text{max,air}} = \frac{0.6847p_0 A^*}{\sqrt{RT_0}} \quad (\star)$$

Both $\dot{m}_{\text{max,air}}$ and $A^*$ are unknown. However, since $A$ and $Ma$ are known we can compute $A^*$ from

$$\frac{A}{A^*} = \frac{1}{Ma} \left[ \frac{2}{k+1} \left( 1 + \frac{k-1}{2Ma^2} \right)^{(k+1)/(2(k-1))} \right] = \frac{1}{0.6} \left[ \frac{2}{2.4} \left( 1 + \frac{0.4}{2}(0.6)^2 \right) \right]^{2.4/0.8}$$

$$\therefore \frac{A}{A^*} = 1.1882 \quad \text{and} \quad A^* = \frac{1.5 \text{ in}^2}{1.1882} = 1.262 \text{ in}^2$$

But since $\dot{m}$ does not vary along the duct

$$\dot{m}_{\text{max,air}} = \dot{m} = 0.5 \frac{\text{lb}_m}{\text{s}}$$

Solve Equation $(\star)$ for $T_0$

$$\sqrt{T_0} = \frac{0.6847p_0 A^*}{\dot{m}_{\text{max,air}} \sqrt{R}} \Rightarrow T_0 = \left( \frac{0.6847p_0 A^*}{\dot{m}_{\text{max,air}}} \right)^2 \frac{1}{R}$$

Everything on the right hand side is known. A factor of $g_c$ is needed for the units to work out

$$T_0 = \left[ \frac{0.6847 \left( 25.51 \frac{\text{lb}_m}{\text{in}^2} \right) (1.262 \text{ in}^2)}{0.5 \frac{\text{lb}_m}{\text{s}}} \right]^2 \frac{32.174 \frac{\text{lb}_m \text{ft}}{\text{lb}_m \text{s}^2}}{53.33 \frac{\text{lb}_m \text{ft}}{\text{lb}_m \text{s}^2 \text{R}}} \therefore T_0 = 521 \text{ R}$$

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b. $A^*$ is the minimum possible area if the mass flow rate is to remain unchanged. The actual area is $A$. Therefore the largest percent reduction in area is

$$100 \times \frac{A - A^*}{A} = 100 \times \frac{1.5 - 1.262}{1.5} = 16 \text{ percent}$$

c. Find $V$ and $p$ at $A = A^*$

$$V^* = \sqrt{kRT^*} = \sqrt{kRT_0 \frac{T^*}{T_0}}$$

$$= \sqrt{(1.4) \left( \frac{53.33 \text{ ft} \cdot \text{lb}}{\text{lb}_m \circ R} \right) (521 \circ R) (0.8333) \left( \frac{32.174 \text{ ft} \cdot \text{lb}}{\text{lb}_m \text{s}^2} \right)}$$

$$\therefore V^* = 1021 \text{ ft/s}$$

$$\frac{p^*}{p_0} = 0.5283 \implies p^* = (0.5283)(25.51 \text{psia}) \therefore p^* = 13.5 \text{psia}$$