Given: Tabulated data from pitot probe measurements in a boundary layer. Dynamic pressure measurements are made with a water-filled, U-tube manometer.

<table>
<thead>
<tr>
<th>$y$ (mm)</th>
<th>$h_m$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.1</td>
<td>10.6</td>
</tr>
<tr>
<td>4.3</td>
<td>21.1</td>
</tr>
<tr>
<td>6.4</td>
<td>25.6</td>
</tr>
<tr>
<td>10.7</td>
<td>32.5</td>
</tr>
<tr>
<td>15.0</td>
<td>36.9</td>
</tr>
<tr>
<td>19.3</td>
<td>39.4</td>
</tr>
<tr>
<td>23.6</td>
<td>40.5</td>
</tr>
<tr>
<td>26.8</td>
<td>41.0</td>
</tr>
<tr>
<td>29.3</td>
<td>41.0</td>
</tr>
<tr>
<td>32.7</td>
<td>41.0</td>
</tr>
</tbody>
</table>

Schematic: The sketch depicts a pitot probe apparatus for measuring velocity in a boundary layer. The total and dynamic pressure taps of the pitot probe are attached to the two legs of a U-tube manometer.

Find:

- $\delta_{99}$, the boundary layer thickness
- $\delta^*$, the displacement thickness
- $\theta$, the momentum thickness

Assumptions: Assume that the given data corresponds to a boundary layer velocity profile. No other assumptions are necessary.

Properties: Specific weight of water: $\gamma_w = 999 \text{ N/m}^3$, Density of air at standard conditions: $\rho = 1.23 \text{ kg/m}^3$. 

Analysis: The first step of the analysis is to convert the manometer readings $h(y)$ to velocity values $u(y)$. After that, the definitions of $\delta_{99}$, $\delta^*$, and $\theta$ are applied to the $u(y)$ data.

The dynamic pressure of the free stream is

$$\Delta p_{\text{dyn}} = \frac{1}{2} \rho u^2 \tag{*}$$

where $\rho$ is the density of the air, and $u$ is the velocity in the $x$ direction. The pressure measured by the manometer is

$$\Delta p = \gamma_m h_m \tag{**}$$

Use Equation (*) and Equation (**) to eliminate the $\Delta p$ gives

$$\frac{1}{2} \rho u^2 = \gamma_m h_m$$

Solve for $u$ to get

$$u = \sqrt{\frac{2\gamma_m h_m}{\rho}}. \tag{** *}$$

Applying Equation (*** *) to the given data yields the $u(y)$ and $u(y)/U$ values in the following table.

<table>
<thead>
<tr>
<th>$y$ (mm)</th>
<th>$h_m$ (mm)</th>
<th>$u$ (m/s)</th>
<th>$u/U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>2.1</td>
<td>10.6</td>
<td>4.15</td>
<td>0.508</td>
</tr>
<tr>
<td>4.3</td>
<td>21.1</td>
<td>5.85</td>
<td>0.717</td>
</tr>
<tr>
<td>6.4</td>
<td>25.6</td>
<td>6.45</td>
<td>0.790</td>
</tr>
<tr>
<td>10.7</td>
<td>32.5</td>
<td>7.27</td>
<td>0.890</td>
</tr>
<tr>
<td>15.0</td>
<td>36.9</td>
<td>7.74</td>
<td>0.949</td>
</tr>
<tr>
<td>19.3</td>
<td>39.4</td>
<td>8.00</td>
<td>0.980</td>
</tr>
<tr>
<td>23.6</td>
<td>40.5</td>
<td>8.11</td>
<td>0.994</td>
</tr>
<tr>
<td>26.8</td>
<td>41.0</td>
<td>8.16</td>
<td>1.000</td>
</tr>
<tr>
<td>29.3</td>
<td>41.0</td>
<td>8.16</td>
<td>1.000</td>
</tr>
<tr>
<td>32.7</td>
<td>41.0</td>
<td>8.16</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The data shows that $\delta_{99}$ is somewhere between $y = 19.3$ mm and $y = 23.6$ mm. Use linear interpolation to find $\delta_{99}$

$$\delta_{99} = 19.3 + \frac{0.990 - 0.980}{0.994 - 0.980} (23.6 - 19.3) = 22.37$$

Therefore $\boxed{\delta_{99}=22.4 \text{ mm}}$
The displacement thickness is
\[ \delta^* = \int_0^\infty \left( 1 - \frac{u}{U} \right) dy \]
and the momentum thickness is
\[ \theta = \int_0^\infty \frac{u}{U} \left( 1 - \frac{u}{U} \right) dy \]

These formulas must be evaluated numerically. Numerical integration using the trapezoid rule is easy to perform with the built-in MATLAB function called \texttt{trapz}. Assume that the \( u \) and \( y \) vectors contain \( u(y) \) and \( y \), respectively. Assume the free stream velocity is stored in the variable \( U \). The values of \( \delta_{99} \) and \( \theta \) are obtained with the following two statements.

\[
\begin{align*}
\text{delStar} &= \text{trapz}(y, 1-u/U); \\
\text{theta} &= \text{trapz}(y, (u/U).*(1-u/U));
\end{align*}
\]

The \texttt{MYO9_13} function performs all calculations necessary to satisfy the assignment. Running \texttt{MYO9_13} produces the following text output and the plot in Figure 1.

\[
\begin{array}{ccccc}
\text{y (mm)} & \text{h (mm)} & \text{u (m/s)} & \text{u/U} \\
0.0 & 0.0 & 0.00 & 0.000 \\
2.1 & 10.6 & 4.15 & 0.508 \\
4.3 & 21.1 & 5.85 & 0.717 \\
6.4 & 25.6 & 6.45 & 0.790 \\
10.7 & 32.5 & 7.27 & 0.890 \\
15.0 & 36.9 & 7.74 & 0.949 \\
19.3 & 39.4 & 8.00 & 0.980 \\
23.6 & 40.5 & 8.11 & 0.994 \\
26.8 & 41.0 & 8.16 & 1.000 \\
29.3 & 41.0 & 8.16 & 1.000 \\
32.7 & 41.0 & 8.16 & 1.000 \\
\end{array}
\]

\[
\begin{align*}
\text{delta99} &= 22.37 \text{ (mm)} \\
\text{delta*} &= 4.19 \text{ (mm)} \\
\text{theta} &= 2.24 \text{ (mm)}
\end{align*}
\]
Figure 1: Velocity profile and computed values of $\delta_{99}$, $\delta^*$ and $\theta$. 
function MYO9_13
% MYO9_13 Solution to problem 9.13 in Munson, Young, and Okiishi

% Given data
% y = position (mm)
% h = manometer reading, (mm H20)
y = [0 2.1 4.3 6.4 10.7 15.0 19.3 23.6 26.8 29.3 32.7];
h = [0 10.6 21.1 25.6 32.5 36.9 39.4 40.5 41.0 41.0 41.0];
gammaw = 999; % specific weight of water, N/m^3
rhoa = 1.23; % density of air

% Convert manometer readings to velocities
% 0.5*rhoa*u^2 = gammaw*h => u = sqrt(2*gammaw*h/rhoa)
% h/1000 is manometer reading in meters
U = max(u); % Free stream velocity
plot(u,y,'o-'); xlabel('u (m/s)'); ylabel('y (mm)');

% Print a nice table of data
fprintf('
 y (mm) h (mm) u (m/s) u/U
');
for i=1:length(y)
    fprintf('%7.1f %7.1f %9.2f %9.3f
',y(i),h(i),u(i),u(i)/U);
end

% Find del99 by interpolation in the velocity data
iU = min(find(u==U)); % iU is index of first element with u(y) = U
del99 = interp1(u(1:iU)/U,y(1:iU),0.99); % Find y value where u/U = 0.99

% use numerical integration to compute delStar and theta
delStar = trapz(y,1-u/U); % Integral of (1-u/U) w.r.t. y
theta = trapz(y,(u/U).*(1-u/U)); % Integral of (u/U)*(1-u/U) w.r.t. y
fprintf('
\delta_{99} = %6.2f (mm)
\delta^* = %6.2f (mm)
\theta = %6.2f (mm)
',del99,delStar,theta);

% Add horizontal lines at y = delta, delStar, and theta
hold on
plot([0 U],del99*[1 1],'r--'); % horizontal line at y = del99
theText = sprintf('\delta_{99} = %4.2f',del99);
text(0.1*U,1.05*del99,theText,'FontSize',16); % label the line

plot([0 U],[delStar delStar],'r--'); % line at y = delStar
theText = sprintf('\delta^* = %4.2f',delStar);
end

plot([0 U],[theta theta],'r--'); % line at y = theta
theText = sprintf('\theta = %4.2f',theta);
end

hold off
xlabel('u (m/s)'); ylabel('y (mm)');