Head Loss non-Circular Ducts
ME 322 Lecture Slides, Winter 2007

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Head Loss in a Horizontal Pipe (1)

Consider fully-developed flow (laminar or turbulent) in a horizontal duct with non-circular cross section.

Apply a momentum balance

\[ pA - (p - \Delta p)A - \bar{\tau}_w LP = 0 \]  \hspace{1cm} (1)

where \( \bar{\tau}_w \) is the average wall shear stress around the perimeter, and \( P \) is the perimeter.
Head Loss in a Horizontal Pipe (2)

Rearrange Equation (1)

\[
\frac{\Delta p}{L} = \tau_w \frac{P}{A}
\]  \hspace{1cm} (2)

Consider the ratio \(A/P\) for a round pipe:

\[
\frac{A}{P} = \frac{(\pi/4)D^2}{\pi D} \quad \Rightarrow \quad D = \frac{4A}{P} \quad \text{for a round pipe}
\]

We define the Hydraulic Diameter as

\[
D_h = \frac{4A}{P}
\]  \hspace{1cm} (3)
Head Loss in a Horizontal Pipe (3)

Rectangle

$D_h$ for duct with rectangular cross section

$$D_h = \frac{4A}{P} = \frac{4WH}{2(W + H)} = \frac{2WH}{W + H}$$

$D_h$ for parallel plates is obtained in the limit as $W \to \infty$

$$\lim_{W \to \infty} D_h = \lim_{W \to \infty} \left[ \frac{2WH}{W + H} \right] = \lim_{W \to \infty} \left[ \frac{2H}{1 + \frac{H}{W}} \right] = 2H$$

Annulus

$D_h$ for flow through an annulus

$$D_h = \frac{4\pi(D_2^2 - D_1^2)}{\pi D_1 + \pi D_2} = \frac{D_2^2 - D_1^2}{D_2 + D_1} = D_2 - D_1$$
Head Loss in a Horizontal Pipe (4)

Back to momentum equation: Use $D_h = 4A/P$

$$\frac{\Delta p}{L} = \bar{\tau}_w \frac{P}{A} \implies \frac{\Delta p}{L} = \bar{\tau}_w \frac{4}{D_h}$$

(4)

Compare to the solution for fully-developed flow in a pipe

$$\frac{\Delta p}{L} = \tau_w \frac{P}{A} \implies \frac{\Delta p}{L} = \tau_w \frac{4}{D}$$

(5)

So if we take $\bar{\tau}_w \sim \tau_w$ and $D_H \sim D$ then the two solutions are identical. This suggests that for non-circular ducts we can use the same design data.
Head Loss in a Horizontal Pipe (5)

Engineering calculations for non-circular ducts

1. Use $D_h$ to compute $\text{Re}_{D_h}$ and $\varepsilon/D_h$
2. Use Colebrook Equation or Moody chart to find $f = \mathcal{F}(\text{Re}_{D_h}, \varepsilon/D_h)$
3. All remaining analysis is the same

Note: The average velocity is

$$V = \frac{Q}{A} \quad V \neq \frac{Q}{\frac{\pi}{4}D_h^2}$$
Three types of pipe flow problems (1)

1. Head loss problem
   - Given $L$, $D$, $Q$ (or $V$), and pipe roughness $\varepsilon$
   - Compute $f$, $h_L$, $\Delta p$, etc.

2. Flow rate problem
   - Given $L$, $D$, $h_L$ and $\varepsilon$
   - Compute $V$, (or $Q$)
   - Requires iteration

3. Pipe sizing problem
   - Given $L$, $Q$ (or $V$), and $h_L$
   - Compute $D$ required to provide the desired flow
Basic Head Loss Problem – No Minor Losses

Given $L$, $D$, $Q$ (or $V$), and pipe roughness $\varepsilon$

1. Look up fluid properties $\rho$, $\mu$
2. Compute $Re_D$ to determine whether the flow is laminar or turbulent
3. If turbulent, look up $\varepsilon$ for the pipe material
4. Use the Colebrook equation or the Moody chart to find $f$
5. Use the Darcy-Weisbach equation to compute $h_L$
6. Use the steady-flow energy equation to find other terms, e.g. pressure drop
Basic Flow Rate Problem – No Minor Losses

Given $L$, $D$, $h_L$ and $\varepsilon$

1. Solve the energy equation for $h_L$
2. Guess $f$: use the “wholly turbulent” range to find $f$ for the known value of $\varepsilon/D$.
3. Solve for $V$ with the Darcy-Weisbach equation

$$h_L = f \frac{L V^2}{D 2g} \quad \Rightarrow \quad V = \sqrt{\frac{2gh_L D}{fL}}$$

4. Compute $Re_D$
5. With new $Re_D$, use the Colebrook equation or the Moody chart to find $f$
6. If $f_{new} \approx f_{old}$ stop, otherwise return to step 3
Basic Pipe Sizing Problem – No Minor Losses

Given $L$, $Q$ (or $V$), $h_L$ and $\varepsilon$ compute $D$ for a round pipe

1. Solve energy equation for $h_L$
2. Guess $D$
3. Compute $\varepsilon/D$, $Re\_D$
4. Find $f$ (Colebrook equation or Moody chart)
5. Solve for $D$ by combining Darcy-Weisbach equation and energy equation

$$h_L = \frac{f}{D} \frac{L V^2}{2g} = \frac{f}{D} \frac{L}{2g} \frac{Q^2}{(\pi/4)^2 D^4} = \frac{f}{\pi^2 g} \frac{8LQ^2}{1} \frac{1}{D^5} \implies D = \left[ \frac{8LQ^2 f}{\pi^2 g h_L} \right]^{1/5}$$

6. If $D_{\text{new}} \approx D_{\text{old}}$, stop, otherwise return to step 3

Note: Choose next larger standard pipe size