17.9 SLUDGE PRODUCTION IN ACTIVATED SLUDGE SYSTEMS

The sludge produced in an activated sludge process or any biological treatment process is a function of the substrate characteristics, sludge age, and other environmental conditions, particularly temperature of the mixed liquor. The rate of production of biological solids is given by Eqs. (17.4) or (17.27). Note that these equations describe the total production of biological solids, which appear in the effluent from the secondary clarifier and in the waste sludge line from the underflow of the secondary clarifier.

An observed yield factor (a net yield factor as discussed in Section 17.3.2) can be formulated for a process at given operating conditions.

\[ P_{X_b} = (-Yr_s - k_e X_Y) V = -Y_{obs_b} r_s V \]  \hspace{1cm} (17.49)

where

- \( P_{X_b} \) is rate of biological sludge production
- \( Y_{obs_b} \) is the observed yield factor for biological solids

Sludge age affects the concentration of MLVSS and temperature and other environmental factors affect both the yield factor and the endogenous decay coefficient.

The rate of total solids production from a biological treatment process will be higher than the rate calculated from Eq. (17.49) because this equation only describes biological VSS production and there is normally a significant amount of solids in the influent to the aeration basin. Degradation of influent solids has been discussed in Section 17.6.2. The rate of production of solids from a biological process is

\[ P_{X_t} = -\frac{Yr_s + k_e X_Y}{f_b} V + Q_f_i X_{T_0} = -Y_{obs_t} r_s V \]  \hspace{1cm} (17.50)
The other extreme for degradation of the influent VSS is that all influent were degradable suspended substrate that were transformed into biological solids. To solve the equations it is also necessary to know the contribution of these degradable VSS to the influent BOD$_3$. Commensurate with the assumption that all VSS were metabolized in the process it will be assumed that the influent VSS fully contributed to the influent BOD$_3$.

The influent VSS concentration is $0.50(90 \text{ mg/L}) = 45 \text{ mg/L}$.

It is also necessary to know the VSS:TSS ratio of the degradable VSS. It will not be 0.50 and a value of 0.75 will be used (the ratio of degradable VSS to TSS in the influent is not necessarily the same as the ratio of VSS:TSS for biological solids). Now the degradable VSS is associated with $(45 \text{ mg/L})/0.75 = 60 \text{ mg TSS/L}$. Besides the 15 mg/L of suspended inorganic solids associated with the biomass there would be 30 mg/L ISS in the influent. Some of the influent ISS could be solubilized in the process but it will be assumed that no ISS has been solubilized. The factor $f_i$ in Eq. (17.50) can now be determined as

$$f_i = \frac{30}{90} = 0.333$$

The VSS component of influent TSS is implicitly contained in the influent BOD$_3$. The inorganic solids associated with the influent degradable VSS are assumed to be released with metabolism but the formation of biomass from metabolism of the degradable VSS will reassimilate inorganics. The observed yield factor and rate of solids production are

$$Y_{obs,3} = \frac{Y_{s} + k_s X_v}{f_d r_s} - \frac{Q_f X_{T0}}{r_s V}$$

$$= 0.640 \text{ mg TSS/mg BOD}_3 - \frac{(0.333)(90 \text{ mg TSS/L})}{(5 \text{ h})(-624 \text{ mg/L/d})} \left( \frac{24 \text{ h}}{\text{d}} \right)$$

$$= 0.640 \text{ mg TSS/mg BOD}_3 + 0.231 \text{ mg TSS/mg BOD}_3 = 0.871 \text{ mg TSS/mg BOD}_3$$

$$P_{X3} = -Y_{obs,3} r_s V = -\left(0.871 \text{ mg TSS/mg BOD}_3 \right) \left(-624 \frac{\text{mg BOD}_3}{\text{L/d}} \right) \left(5.21 \times 10^6 \text{ L} \right) \left(\frac{1 \text{ kg}}{10^6 \text{ mg}}\right)$$

$$= 2.83 \times 10^3 \text{ kg/d}$$

The actual solids production rate will lie between $2.83 \times 10^3$ and $4.33 \times 10^3$ kg/d. There are numerous factors to be determined in arriving at the actual rate of solids production. Observed yield factors can be calculated from pilot studies and with sufficient measurements the values of the factors can be assessed. Their values are of academic interest because the overall yield factor is known. But this example illustrates the phenomena that are involved.

Observed yield factors for domestic wastewater with primary sedimentation range from 0.33 to 0.88 mg TSS/mg BOD$_3$; without primary sedimentation the range is 0.62–1.18 mg TSS/mg BOD$_3$ for sludge ages up to 30 d and temperatures between 10 and 30°C (WEF and ASCE, 1992a). Processes that operate at high sludge ages such as extended aeration have lower observed yield factors than processes operated at lower sludge ages. Higher temperatures result in lower observed yield factors than lower temperatures.
where

- \( P_{X_l} \) is the rate of sludge production
- \( f_i \) is the ratio of VSS: TSS for biological solids
- \( f_i \) is the fraction of influent TSS that was not degraded
- \( Y_{obs,i} \) is the observed yield factor for total solids

Studies are needed to determine the factor \( f_i \) in Eq. (17.50). It is a function of the degradable portion of the influent VSS and the VSS:TSS ratio for the influent. An example will illustrate the factors on which \( f_i \) depends.

### Example 17.3 Sludge Production in an Activated Sludge Process

An activated sludge process is treating primary settled effluent that has a BOD₅ of 150 mg/L and TSS concentration of 90 mg/L in a flow of 25,000 m³/d (6.61 Mgal/d). The detention time in the aeration basins is 5 h. MLVSS is 2100 mg/L and the effluent contains 20 mg/L of BOD₅. The yield factor is 0.68 mg VSS/mg BOD₅ and the endogenous decay coefficient is 0.05 d⁻¹. The VSS:TSS ratios for biological solids and influent SS are 0.80 and 0.50, respectively. Find the observed biological yield factor and the range of the observed yield factor for total solids. Also find the possible range for rate of total solids production.

The rate of substrate removal (Eq. 17.15) is

\[
 r_s = \frac{(S_a - S_2)}{\theta_d} = -\frac{(150 - 20)}{5 \text{ h}} \left( \frac{24 \text{ h}}{d} \right) = -624 \text{ mg/L/d}
\]

The volume of the aeration basins is

\[
 V = Q \theta_d \left( \frac{25,000 \text{ m}^3}{d} \right) \left( \frac{1 \text{ d}}{24 \text{ h}} \right) = 5208 \text{ m}^3 = 5.21 \times 10^6 \text{ L}
\]

Applying Eq. (17.49),

\[
 Y_{obs,h} = \frac{Y_{r_s} + k_c X_v}{r_s} = \frac{0.68(-624 \text{ mg/L/d}) + 0.05 \text{ d}^{-1}(2100 \text{ mg/L})}{-624 \text{ mg/L/d}} = 0.512 \text{ mg VSS/mg BOD}_5
\]

The rate of biological solids production is

\[
 P_{X_b} = -Y_{obs,h} r_s V = -\left(0.512 \frac{\text{mg VSS}}{\text{mg BOD}_5}\right) \left(-624 \frac{\text{mg BOD}_5}{\text{L}}\right) \left(5.21 \times 10^6 \text{ L}\right) \left(\frac{1 \text{ kg}}{10^9 \text{ mg}}\right) = 1.67 \times 10^3 \text{ kg/d}
\]

If none of the influent VSS were degradable then \( f_i = 1 \) in Eq. (17.50) and the maximum theoretical solids production would occur.

\[
 Y_{obs,t} = \frac{Y_{r_s} + k_c X_v}{f_i r_s} = \frac{0.512 \text{ mg VSS/mg BOD}_5}{0.80 \text{ mg VSS/mg TSS}} = \frac{(1)(90 \text{ mg TSS/L})}{(5 \text{ h})(-624 \text{ mg/L/d})} = 0.640 \text{ mg TSS/mg BOD}_5 + 0.692 \text{ mg TSS/mg BOD}_5 = 1.332 \text{ mg TSS/mg BOD}_5
\]

\[
 P_{X_t} = -Y_{obs,t} r_s V = -\left(1.332 \frac{\text{mg TSS}}{\text{mg BOD}_5}\right) \left(-624 \frac{\text{mg BOD}_5}{\text{L}}\right) \left(5.21 \times 10^6 \text{ L}\right) \left(\frac{1 \text{ kg}}{10^9 \text{ mg}}\right) = 4.33 \times 10^3 \text{ kg/d}
\]