1. **Kirchhoff's Law**

- Any body $T > 0$ K radiates energy

- Radiation proportional to absorption

\[ W_i = \text{power density of radiation (power/area)} \]
\[ = \text{energy flux (energy/time/area)} \]
\[ \alpha_i = \text{specific absorptivity (} 0 < \alpha_i < 1) \]

\[ \frac{W_1}{\alpha_1} = \frac{W_2}{\alpha_2} = W_B = \text{black body radiator power density} \]

2. **Planck's Law**

Radiation from a black body is distributed w/r/t wavelength

\[ W_B = \int_0^\infty W_B^\lambda d\lambda \]

\[ \log \text{relative emission} \]

\[ E_\lambda = \frac{C_1}{\lambda^5(e^{\frac{C_2}{\lambda T}} - 1)} \]

\[ \lambda_{max} = \frac{0.29 \text{ cm-K}}{T} \]

\[ E = \text{total emissive power} = W_B \cdot \lambda \]
Consider two blackbodies $T_2 \gg T_1$.

Which body is radiating more thermal energy?

\[ W_B = \sigma T^4 \]

\[ \sigma = 8.3 \times 10^{-11} \text{cal cm}^{-2} \text{min}^{-1} \text{K}^{-4} \]
\[ = 5.7 \times 10^{-8} \text{ J m}^{-2} \text{s}^{-1} \text{K}^{-4} \]

\[ W_i = \varepsilon_i \sigma T^4 \]

\[ \varepsilon_i = \text{emissivity} = \sigma \text{ at thermal equilibrium} \]

\[ W_1 = (0.3)(3.3 \times 10^{-11})(311)^4 = 311 \text{ K} \]
\[ = 0.23 \text{ cal cm}^{-2} \text{ min}^{-1} \text{ (langley/min)} \]
\[ = 160 \text{ Watt/m}^2 \]

\[ W_2 = (0.8)(3.3 \times 10^{-11})(311)^4 = 0.62 \text{ langley/s} \]
\[ = 434 \text{ W/m}^2 \]
The figure illustrates the comparison between the Sun and Earth's radiation spectra. The Sun's black-body radiation, with a temperature of 6000 K, emits mostly in the ultraviolet and visible wavelengths. Earth's black-body radiation, with a temperature of ~290 K, emits mainly in the infrared, with a peak around 10 μm.

**Sun's Radiation:**
- W^Sun_\text{min} = 10^7,000 \text{ ly min (at surface of Sun)}

**Earth's Radiation:**
- W^Earth_\text{min} = 0.56 \text{ ly min (at Earth's surface)}

The diagram shows absorption bands of the Earth's atmosphere at different wavelengths:
- O_3
- H_2O
- H_2O/CO_2
- CO_2
- H_2O

The atmosphere is transparent to most incoming solar radiation (visible and near-IR), allowing sunlight to penetrate. However, it absorbs heavily the outgoing far-IR radiation of the Earth, a phenomenon known as the "greenhouse effect."
The spherical shape of the earth is very important.

At higher latitudes, light is spread over greater area; hence, less intense.

\[ W^0_s \approx 2.0 \text{ langleys/min (cal/cm}^2\text{/min) \approx 1400 \text{ W/m}^2 \text{ (in outer space)} \]

\[ I_o = W^0_s \sin \alpha = W^0_s \cos \phi \]

\[ \alpha = [90^\circ - \text{(latitude)}] \]

\[ \alpha = \text{solar angle (See Eagleson's chapter)} \]

Most intense per unit area of surface

Less intense per unit area of surface

\[ \sin \alpha = \sin \delta \sin \phi + \cos \delta \cos \phi \cos \gamma \]

\( \delta = \text{Declination} \)

\( \phi = \text{Local Latitude} \)

\( \gamma = \text{Hour Angle} \)
SOLAR ANGLE CALCULATION

\[ \sin \alpha = \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos \tau \]

\( \delta = \text{declination} \)
(the seasonally varying angle of the plane traversed by the sun across the sky)

\( \Phi = \text{latitude} \)
(the geographic angle along the surface of the earth)

\( \tau = \text{hour angle of the sun} \)
(the angle of the sun along the arc traversed by the sun across the sky)
A. Solar angle for a flat surface can be calculated using the three component angles: $\phi$, $\tau$, $\delta$.

Latitude $\phi$

Hour angle $\tau$
(time of day)

Declination $\delta$
(seasonal)

Daily totals of the undepleted solar radiation received on a horizontal surface for different geographical latitudes as a function of the time of year (solar constant 1.94 cal cm$^{-2}$ min$^{-1}$). (After Gates, 1962.)

B. Solar angle also has local components on the meso-scale or micro-scale:

1. Columbia River Gorge
   - Washington side
   - Oregon side

2. California Hill Country
   - Oak Trees
   - Dry Grass

Absorbance and especially molecular and particulate scattering also affect insolation: smoke, haze, clouds, pollution, etc. So whatever hits the surface is significantly less than ($W_0 \sin \alpha$).
COMPARATIVE LATITUDES
OF FOUR U.S. CITIES
FIGURE 109. Spectral distribution of radiant energy outside the atmosphere and at the earth's surface (air mass 3), with the spectral sensitivity of the human eye.

FIGURE 111. A, mean radiation (cal. cm.$^{-2}$ day.$^{-1}$) received at various times of year at H, Honolulu, Hawaii, lat. 21°18' N.; M, Madison, Wisconsin, lat. 43°05' N.; I, Ithaca, New York, lat. 42°27' N.; F, Fairbanks, Alaska, lat. 64°52' N. (after Hand). Note the slightly lower radiation received at nearly all seasons at Ithaca (Lake Cayuga) as compared with Madison (Lake Mendota).
Beer-Lambert-Bouger Law
Integrate w/r^2 Depth (z)

\[ I_z^t = I_0^t e^{-a_\lambda z} \]

\[ \frac{I_z^t}{I_0^t} = e^{-a_\lambda z} \]

\[ a_\lambda = \text{absorption coeff for wavelength } \lambda \]

Light intensity drops off exponentially w/ depth (z)

2) Extent of light attenuation depends on wavelength of light

- RED LIGHT: \( a_{650 \text{ nm}} \) → LARGE
- GREEN LIGHT \( a_{500} \) → MEDIUM
- BLUE LIGHT \( a_{450} \) → SMALL
- UV LIGHT \( a_{300} \) → VERY LARGE

"a" also known as the "Extinction Coeff."
PRACTICAL APPLICATION
Beer's Law in Natural Water

Define \( K = \int_{a}^{d} a_2 \, da \)

\( K \) = Average absorption of Sunlight for natural water

Often measured empirically with light meter

\( \beta \) = "Extra" absorption often seen in film on surface

\[ \frac{I_{52}}{I_{5}} = (1 - \beta) e^{-Kz} \]
ABSORPTION OF LIGHT BY WATER: Logarithmic Presentation

The exponential equation given by Beer's Law can be linearized by taking the natural log of both sides, or, by plotting light intensity on semi-log scaling.

\[ \frac{I_s}{I_s^*} \]

**DEPTH (m)**

- TURBID OR COLORED WATER
- CLEAR WATER
- VERY CLEAR (Sargasso Sea)

\[ 100 \]

SO-CALLED 10% LIGHT DEPTH: A rough estimate of the maximum depth at which photosynthesis can occur.