1. Wind-driven flow and mixing

Wind velocity, $V_a$, of $\frac{1}{2} m_a V_a^2$

Friction (Drag)

Water velocity, $V_w$ of $\frac{1}{2} m_w V_w^2$

Wind drag, $V_a$, translates to water via friction

At equilibrium, $KE_a = KE_w$

$KE_a = \frac{1}{2} m_a V_a^2$

$KE_w = \frac{1}{2} m_w V_w^2$

$m_a V_a^2 = m_w V_w^2$

$\frac{V_w^2}{V_a^2} = \frac{m_a}{m_w} = \frac{V_a/P_a}{V_w/P_w}$

$\frac{V_w}{V_a} = \sqrt{\frac{P_a}{P_w}} = \left(\frac{1.2 \text{ kg/m}^2}{10^3 \text{ kg/m}^3}\right)^{1/2} \approx 0.03$

$V_w \approx 3\% V_a$

"Three to Rule" for max. wind drift

(2-3%, really)
Propagation of velocity

\[ \vec{V}_0 \rightarrow \vec{V}_a \]

\[ \downarrow \quad \vec{V}_w^\circ \quad \text{SURFACE DRIFT/CURRENT} \]

\[ \downarrow \quad \text{? Function: } U(z) = f(z) \]

1. **Laminar Flow**: No turbulence
   - Drag via SHEAR
   - Molecular Viscosity (fluid 'stickiness')

\[ U_0 = 0 \quad \text{("No slip")} \]

"SHEAR STRESS" = DOWNWARD FLOW OF MOMENTUM

**Laminar**: \[ \mu = \text{constant viscosity} \]

\[ \mu_w = 10^{-3} \text{ Pa.s} \]

\[ \tau = \mu \frac{\partial u}{\partial z} \quad \text{Shear Stress} \quad \text{Pa} \]

\[ u = \frac{\partial u}{\partial z} \quad \text{Pa.s} \quad \text{m/s} \quad \mu \]

\[ \partial u = \frac{\tau}{\mu} \quad u = \left( \frac{\tau}{\mu} \right) z + U_s \]

\[ \sqrt{\text{LINEAR}} \]

**Turbulent**: \[ \mu \neq \text{constant} \quad \mu_e = \text{Eddy Viscosity} \]

\[ \mu_e \gg \mu \quad 10^3 - 10^6 \text{ Pa.s} \]

\[ \text{MOMENTUM} \quad \text{NEW MIXED} \quad \text{down by Eddies} \]

Simplest model: \[ \mu_e = k' z \]

Grows with distance from surface
So use variable viscosity \( \mu_e = k \xi \)

\[
\tau = \mu_e \frac{\partial \xi}{\partial \xi} = -k \xi \frac{\partial \xi}{\partial \xi}
\]

\[
\int \xi = -\left( \frac{c}{k} \right) \frac{1}{\xi} \xi \xi
\]

Rearrange and integrate to solve for \( \xi = f(z) \)

\[
\xi = -\left( \frac{c}{k} \right) \ln|z| + \text{Const}
\]

At \( z = 0 \), \( \xi = \xi_s \) (surface velocity)

Hence \( \text{Const} = \xi_s \)

\[
\xi = -\left( \frac{c}{k} \right) \ln|z| + \xi_s
\]

A

Lecture Error (2017)

So called "velocity defect law"

\[
V_s
\]

Velocity profile decreases as a logarithmic function of depth

\[
V = \xi_s \left( 1 - \frac{c}{k} \ln|z| \right)
\]

Note: In class I accidentally said velocity decreases exponentially with depth. Really it's logarithmic, (still ends up similar shape "caving" downward)
Now combine heating (solar), velocity, and mixing. 

- **Velocity**
  - **Ve**
  - **Vw**
  - **Stronger mixing near surface**
  - **Weak mixing at depth**
  - **Light penetration**
  - **Q(z)**
  - **Heat input drops off with depth**

Water gets most heat input near surface

- Lakes: 50% heat enters top 1 meter.
- But water also mixes most right near surface.

**Heat tends to:**
1. Get mixed down, "levelled out"
2. But warm surface water also resists total mixing because it's buoyant.

Spring warming

**Temperature**

- **Z**
- **T(z)**
- **Z ~ 4°C**
- **4°C**

**Mar** **Apr** **May**

- **Well mixed (isothermal)**
- **Surface layer**
- **With wind mixing at top**
- **25°C**
- **Warm**
- **Epilimnion** (upper lake)
- **Thermocline**
- **Cold**
- **Hypolimnion** (lower lake)
The period of summer stratification is characterized by an upper stratum of more or less uniformly warm, circulating, and fairly turbulent water, the *epilimnion* (Fig. 6-3). The epilimnion overlies a deep, cold, and relatively undisturbed region, the *hypolimnion*. The stratum between the epilimnion and the hypolimnion exhibits a marked thermal discontinuity, and is termed the *metalimnion*. The metalimnion is defined as the stratum of steep thermal gradient, and can be demarcated by the intersections of a plane along the gradient to the points of maximum curvature from the approximately homothermal conditions of the epilimnion and the hypolimnion (graphically depicted in Fig. 6-3). The term *thermocline* has been defined variously, but correctly refers to the plane or surface of maximum rate of decrease of temperature with respect to depth. An extensive discussion of these terms and their conceptual basis is given by Hutchinson (1957). Terms in wide use that are functionally synonymous with the above definition of the metalimnion include the German *Sprungschicht*, and discontinuity layer as used in the United Kingdom.
Terminology of Lake Mixing ("Mixis")

Dimictic Lakes. These lakes circulate freely twice a year, in the spring and fall, and are directly stratified in summer, inversely stratified in winter. Dimictic lakes represent the most common type of thermal stratification observed in most lakes of the cool temperate regions of the world. Their characteristics were described in detail earlier in this chapter. Such lakes also are found commonly at high elevations in subtropical latitudes.

Warm Monomictic Lakes. In these lakes, temperatures do not drop below 4°C, they circulate freely in the winter at or above 4°C, and they stratify directly in the summer. Warm monomictic lakes are common to warm regions of the temperate zone, particularly those influenced by oceanic climates, and to mountainous areas of subtropical latitudes. Most lakes of the central and eastern portions of North America and the interior of Europe exhibit a distinct continental type of dimictic stratification, whereas a warm monomictic stratification is prevalent in many coastal regions of North America and Northern Europe.

Oligomictic Lakes. These lakes are generally tropical and have rare circulation periods at irregular intervals, and temperatures always well above 4°C. Lakes of small to moderate area or lakes of very great depth in the tropics are often observed to maintain stable stratification, even though only a small temperature difference may exist between the surface and bottom strata. These lakes are common in equatorial regions of high humidities and may circulate only at irregular intervals during periods of abnormally cold weather. Circulation can be quite rare and occur between several years of continuous feeble stratification (Vollenweider, 1964).

Polymictic Lakes. These are lakes with frequent or continuous circulation. Polymictic lakes have been further divided (Ruttner, 1963). Cold polymictic lakes circulate continually at temperatures near or slightly above 4°C. These lakes commonly are of large area and moderate depth, and are found in equatorial regions of high wind and low humidity, where little seasonal change in air temperatures occurs. At very high altitudes in equatorial regions, cold polymictic lakes gain a significant amount of heat during the day, but nocturnal losses are sufficient to permit complete mixing during the night.

Warm polymictic lakes are usually tropical lakes that exhibit frequent periods of circulation at temperatures well above 4°C. Annual temperature variations are small in the equatorial tropics and result in repeated periods of circulation between short intervals of heating and weak stratification, followed by periods of rapid cooling. Under these circumstances convective circulation is sufficient, in combination with wind, to disrupt stratification.

A useful schematic arrangement of these six types of lakes based on thermal and circulation characteristics is presented in Figure 6-6, in which generalizations are drawn on altitudinal and latitudinal distribution. Such generalizations are difficult to maintain; many exceptions exist, particularly at low altitudes, where there is a strong influence of oceanic ameliorations of climate. The diagram does, however, demonstrate the general observed geographical distribution.

The above discussion of thermal lake types refers to lakes with sufficient depth to form a hypolimnion. Numerically speaking, when considering thousands of small, relatively shallow lakes that literally cover the tundra, the major lakes do not stratify during ice-free periods. The depth required to thermally vary so greatly with surface area, basin orientation in relation to prevailing wind, depth-volume relations, protection by surrounding topography and vegetation, and other factors that generalize in this region are misleading.