Deflection of structures with curved beams for thin members \( \frac{R}{h} > 10 \)

Find deflection of the ends along \( F \) (new diameter)

\[
\delta = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial F} \, dx
\]

\( dx = R \, d\theta \)

\[
\delta = \int_{\theta=0}^\pi \frac{M}{EI} \frac{\partial M}{\partial F} R \, d\theta
\]

\[
M = FR \sin \theta
\]

\[
\frac{\partial M}{\partial F} = R \sin \theta
\]

\[
\delta = \int_{\theta=0}^\pi \frac{FR \sin \theta (R \sin \theta)}{EI} R \, d\theta
\]

\[
\delta = \int_{\theta=0}^\pi \frac{FR^3 \sin^2 \theta}{EI} \, d\theta = \frac{FR^3}{EI} \int_0^\pi \sin^2 \theta \, d\theta
\]

\[
\int_0^\pi \sin^2 \theta \, d\theta = \frac{\pi}{2}
\]

\[ F = 1h < \]
\[ \delta = \frac{FR^3R}{2EI} \]

- \( F \) = lbs
- \( R \) = inches
- \( E \) = PSI
- \( I \) = \( in^4 \)

HW#5

Find deflection if all the load is applied at the tip.

Impact forces
Impact forces

\[ \delta = \text{maximum deflection} \]

\[ (PE)_w = (PE)_{\text{spring}} \quad \text{(no damping)} \]

\[ w (h + \delta) = \frac{1}{2} K \delta^2 \]

\[ \delta = \frac{w}{K} \left( 1 + \sqrt{1 + \left( \frac{2hK}{w} \right)^2} \right) \]

\[ F_{\text{max}} = K\delta = w \left( 1 + \sqrt{1 + \left( \frac{2hK}{w} \right)^2} \right) \]

Note: \( h = 0 \quad F_{\text{max}} = 2w \)

As \( K \to \infty \), then \( F \to \infty \)

Horizontal impact

\[ (KE)_{\text{weight}} = (PE)_{\text{spring}} \quad \text{(no damping)} \]

\[ \frac{1}{2} m v^2 = \frac{1}{2} K \delta^2 \]
\[ \delta = \sqrt{\frac{w}{Kg}} \times \nu \quad \text{(max deflection)} \]

\[ F_{\text{max}} = k \delta = \sqrt{\frac{wk}{g}} \times \nu \]

Example
\[ v = \frac{\text{in}}{\text{sec}} \]
\[ k = \frac{\text{lb}}{\text{in}} \]
\[ w = \text{lb} \]
\[ F_{\text{max}} = \text{lb} \]
\[ g = 386 \quad \frac{\text{in}}{\text{sec}^2} \]

\[ v = \frac{\text{ft}}{\text{sec}} \]
\[ k = \frac{\text{lb}}{\text{ft}} \]
\[ w = \text{lb} \]
\[ F_{\text{max}} = \text{lb} \]
\[ g = 32.2 \quad \frac{\text{ft}}{\text{sec}^2} \]

What happens with column failure

\[ M = FL = \text{Constant} \]

\[ M = F \delta \]

Long columns have a critical load beyond.
Critical load beyond which the column suddenly collapses (Euler) - Euler formula

Similar to

FAILURE Theories

Static loading - Ductile behavior \( \varepsilon > 0.05 \)

- Frames, structural members
- Failure criterion = bulk yielding
  - change of geometry
  - Deformation / distortion
- Failure mechanism: Crystallographic planes sliding against each other
Main Cause: Excessive shear stress

Stress raisers: Ignore them

Theories

\[ \sigma_x = \frac{F}{A} \]

F = tensile test

\[ \sigma = \frac{F}{A} \]

\[ \frac{F}{A} = \sigma \]

\[ \epsilon \]