HW#3 Solutions

Problem #1 - Rotating disk

Given: Disk made up of carbon steel grey cast iron

ID = 1"
OD = 12"
Thickness = 1"

Find: rpm for which $\tau$ exceed $S_u = 31000$ psi and the disk shatters

Solution

$$\tau = \rho w^2 \left( \frac{3+\nu}{8} \right) \left( \frac{r_i^2 + r_0^2}{r^2} + \frac{r_r^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2 \right)$$

Setting $r = r_i$

$$\tau = \rho w^2 \left( \frac{3+\nu}{8} \right) \left( r_i^2 + 2r_0^2 - \frac{1+3\nu}{3+\nu} r_i^2 \right)$$

where

$$\frac{\rho}{g} = \frac{\gamma}{386} = 0.0006736 \text{ uM/V}^3$$

where $\gamma = 0.26 \text{ lbs/ft}^3$

$q = 386$
and
\[ \Gamma_i = 0.5 \]
\[ \Gamma_0 = 6'' \]
\[ \gamma = 0.211 \]

Setting \( \tau_t = 31000 \) Psi

\[ 31000 = (0.0006786) \omega^2 \left( \frac{8 + 0.211}{8} \right) (0.5^2 + 2(6)^2 - \frac{1 + 3(0.211)}{3 + 0.211} (0.5)^2) \]

\[ \Rightarrow \omega^2 = 158.9777.5 \]

and
\[ \omega = 1260.8 \text{ rad/s} \]

or
\[ n = \omega \left( \frac{60}{2\pi} \right) = 12040 \text{ rpm} \rightarrow \text{ANS} \]
Problem #2

Given: Steel pulley hub press fit on a solid shaft

\( \phi \text{ Shaft} = 50.025 \text{ mm} \)
\( \phi \text{ Hub} = 50.000 \text{ mm} \)
\( \phi \text{ Hub OD} = 100 \text{ mm} \)

Find:

a) Interface pressure \( \sigma \) in MPa
b) Tangential stress in the hub at inner radius

c) Radial stress for the hub at inner radius

\[ P = \frac{E \delta}{2R^3} \left[ \frac{(R_0^2 - R^2)(R^2 - R_i^2)}{R_0^2 - R_i^2} \right] \]

Where

\[ E = 207,000 \text{ MPa} \]
\[ \delta = \frac{0.025}{2} = 0.0125 \text{ mm} \]
\[ R_i = 0 \]
\[ R = 25 \text{ mm} \]
\[ R_0 = 50 \text{ mm} \]

Therefore

\[ P = \frac{207,000 \times (0.0125)}{2 \times (25)^3} \left[ \frac{(50^2 - 25^2)(25^2 - 0)}{(50)^2} \right] = 38.75 \text{ MPa} \]
b) For the hub \( p_0 = 0 \)

\[
\sigma_t = \frac{r_i^2 p_i}{r_0^2 - r_i^2} \left( 1 + \frac{r_0^2}{r_i^2} \right)
\]

Where

\( r_i = 25 \text{ mm} \)

\( p_i = 38.75 \text{ MPa} \)

\( r_0 = 50 \text{ mm} \)

\[
\sigma_t = \frac{(25)^2 (38.75)}{(50^2 - 25^2)} \left( 1 + \frac{50^2}{25^2} \right) = 64.6 \text{ MPa}
\]

c) Radial stress at the inner surface is equal to pressure

\( \sigma_r = -38.75 \text{ MPa} \)
ME 313 HW#3 Problem #3

Given
- A steel hook with rectangular cross-section supporting 10000 lbs
- Inner radius = 2 inches
- Outer radius = 4 inches
- Thickness = 1 inch

Find
a) Maximum bending tensile stress in psi at the inner radius using curved beam formula
b) A graph shown the bending stress as the inner radius changes from 2 inches to 20 inches. The outer radius is always 2 inches more than the inner radius.

Solution

The maximum bending stress at the inner radius is

\[ \sigma_i = \frac{Mc_i}{Aer_i} = \frac{30000(0.885)}{(2)(0.1146)(2)} = 57919 \text{ psi} \]

Where
\[ M = 10000(r_i + 1) = 10000(3) = 30000 \text{ in-lb} \]
\[ c_i = (r_a - r_i) = (2.885 - 2) = 0.885 \text{ in} \]
\[ A = 2 \text{ in}^2 \]
\[ e = (r_c - r_a) = (3 - 2.885) = 0.1146 \text{ in} \]

where
\[ r_a = \frac{h}{\ln \frac{r_o}{r_i} + 1} = \frac{2}{\ln \frac{(r_i + 2)}{r_i} - 1} = \frac{2}{\ln \frac{4}{2}} = 2.885 \text{ in} \]

The change in bending stress as a function of the inner radius is shown in the following graph.