Reflection charts relating to impedance matching

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Overview

• Introduces reflection chart (smith chart)

• Hemispherical chart and projections
  – Transformations (rectangular to polar co-ordinates)
  – Impedance ratio – reflection ratio

• Uses and applications of smith chart

• Limitations of smith chart
Formulas

\[ z = \frac{Z}{Z_0} = z \exp j\phi = r + jx = \frac{R}{Z_0} + \frac{jX}{Z_0} \]

\[ w = -w \exp -j2\beta = u + jv = \frac{z-1}{z+1} \]

\[ w = \rho = \frac{S-1}{S+1} \]

\( S = r \) on scale of upper vertical radius of hemisphere chart.

- The negative sign of \( w \) places the angular origin at the origin of impedance ratio \((z = 0)\).
- The angle \(-2\beta\) is the angle of reflection in a line of length \(\beta\) radians or \(\beta/2\pi\) wavelengths.
$z_0 =$ reference impedance (wave impedance of line)  
(constant pure resistance) (ohms)  
$Z = Z \exp j\phi = R + jX =$ complex impedance (ohms)  
$z = Z/Z_0 = z \exp j\phi = r + jx =$ complex impedance ratio  
$Z, \phi =$ polar components of impedance (ohms, radians)  
$R, X =$ series rectangular components of impedance (ohms)  
$w = -w \exp -j2\beta = u + jv =$ complex voltage-reflection ratio  
$w = \rho = \frac{S - 1}{S + 1} =$ scalar reflection ratio (coefficient)  
(radius on hemisphere chart)  
$\beta =$ angle length of line ($Z_0$)  
$d = \beta/2\pi =$ length of line in wavelengths  
$S = \frac{1+\rho}{1-\rho} =$ voltage standing-wave ratio (VSWR > 1)  
$S = r$ on scale of upper radius on vertical axis of hemisphere chart.
• Impedance ratio was previously plotted on rectangular coordinates to display a locus over a range of frequency.
• The same locus is plotted on polar coordinates of the reflection ratio (reflection coefficient, w=u+jv) within a unit circle.
Orientation of the hemisphere charts

- We plot a locus of simultaneous variations of two coordinate-dependent variables with some independent variable.
- Ex-frequency locus of \( z = R(w) + jX(w) \) ------ \( Y = f(x) \)
- \( w \)-angular frequency
Logical coordinates for the map of complex variables

- The axis of imaginary part \(y\) has paired values, so we use left-right orientation. Fig(c)
- Positive-real scale is naturally associated with increasing values, so use from 0 to infinity. Fig(d)
Projections of the Impedance sphere

3(a)-The complex impedance can be projected on sphere. Positive-real half-plane is projected on one hemisphere. Thus the name hemisphere chart.

3(b)Here the circle becomes the hemisphere chart of reflection coefficient, on which we can plot impedance ratio.
Smith & carter chart

- Top is zero and bottom is infinite
Formula

\[ z = \frac{Z_L}{Z_0} = \frac{R + jX}{Z_0} = r + jx \]

\[ \Gamma_L = \Gamma_r + j\Gamma_i = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_L - Z_0)/Z_0}{(Z_L + Z_0)/Z_0} = \frac{z - 1}{z + 1} = \frac{r + jx - 1}{r + jx + 1} \]

\[ r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 - 2\Gamma_r + \Gamma_i^2} \]

\[ x = \frac{2\Gamma_i}{1 + \Gamma_r^2 - 2\Gamma_r + \Gamma_i^2} \]

\[ (\Gamma_r - \frac{r}{r+1})^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \]

\[ (\Gamma_r - 1)^2 + (\Gamma_i - \frac{1}{x})^2 = \frac{1}{x^2} \]
Hemisphere chart – On a circular area, there is an orthogonal grid of circular arcs marked with the real and imaginary components of the impedance ratio.

\[
(r_r - \frac{r}{r+1})^2 + r_i^2 = \left(\frac{1}{1+r}\right)^2
\]

\[
(r_r - 1)^2 + (r_i - \frac{1}{x})^2 = \frac{1}{x^2}
\]

Constant resistance circles

Constant reactance curves
Smith chart

Normalized Impedance and Admittance Coordinates
Lines of constant $r$ in the complex $z$-plane get mapped into circles in the complex $w$-plane. Lines of constant reactance are also mapped onto circles of radius $1/x$ in the $w$-plane.

Note - Rim of chart (Compression)
There’s also a mirror image of the chart that instead of having constant resistance circles, and constant reactance curves, has instead constant conductance circles and constant susceptance curves.
Applications of smith chart

• Reflection chart is a pair of coordinates on which impedance locus can be plotted for over a range of frequency.
• Goal-To match Zin with load impedance
• We do adding or subtracting a complex impedance in series or parallel and even do inversion of impedance. Chart-A (Text-book-116page)
Deducing the impedance of load or Zin when the length of line between load and Zin is given.
Thus this chart is useful in presenting and manipulating impedance computations for various purposes.

Quarter & half wavelength
Z not equal to Y
{0.25 wavelength (or 180 degree)
0.5 wavelength gives same value.
Note

Sliding probe
• Which is used to observe minimum & maximum voltage (or current).
• Moving the probe along length of line we can measure \( S = \frac{V_{\text{max}}}{V_{\text{min}}} \)

Directional coupler
• Which is used to directly measure complex reflection ratio
• The Distance \( d \) is subtracted to give reflection coefficient at the load, which determines the complex load impedance
Diagrams of the two methods of impedance measurement.

(a) By sliding probe on slotted line.

(b) By directional coupler.

Two methods for measurement of impedance with reference to a transmission line.
• Typical use of reflection chart is finding the frequency locus of the complex impedance of an antenna.
Blank chart

- Do not require complete grid of coordinates. (To avoid confusion)

Fig. 14. The scales on the skeleton hemisphere chart.
Frequencies of resonance in a long biconical dipole antenna

- Each frequency of response appears near a minimum or maximum of impedance.
- Each frequency of resonance may be taken at the point of zero angle (vertical line).
- Locus on the chart is spiral tending toward convergence at higher frequencies.
- Convergence is proportional to radiation of power from a travelling wave along the length.
• At frequencies much below the first resonance, the locus approaches the rim of chart, which indicates small radiation power factor of a “small antenna”

• At higher frequencies, the spiral converges toward a point representing the wave resistance of biconical conductor.
Limitations of Smith chart

- It needs the choice of reference value (Zo—reference impedance)
- Display is compressed near the rim of the chart.
Reference

- Internet
- Text books
- Applied Electromanetics-Fawwaz T.Ulaby
- Transmission lines Waveguides,and Smith chart—Richard L.Liboff  G.Conrad Dalman
- Electronic Applications of the smith chart---Phillip H. Smith