Microwave Bandpass Filters Containing High-Q Dielectric Resonators.

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Introduction

- Dielectric object resonate in various modes
- If $\varepsilon_r$ is high electric and magnetic fields are confined in and near the resonator so the $Q$ (quality factor) is high.
- Non uniform $\varepsilon_r$ has loss due to radiation and dissipation.
- Sometimes electric losses occur due to the finite loss tangent of the dielectric.
For fundamental mode resonance the dielectric resonator are on the order of one wavelength in dielectric material.

\[ \lambda_d = \lambda / \sqrt{\varepsilon_r} \]

- Where \( \lambda_d \) = wavelength of dielectric
- \( \lambda \) = wavelength of air
- \( \varepsilon_r \) = relative dielectric constant
Dielectric Resonators Configuration

- Rectangular resonators
- Cylindrical resonators (L < D) – most practical
- Cylindrical resonators (L > D)
Fig. 1. Fundamental mode fields for three dielectric-resonator configurations. (a) Rectangular resonator, $a$ and $b > c$. (b) Cylindrical resonator, $L < D$. This case is preferable for most filter applications. (c) Cylindrical resonator, $L > D$. 
First three resonant frequencies versus L with constant D for cylindrical resonators

Fig. 2. First three resonant frequencies versus length of a dielectric cylinder of constant diameter.
The analysis of coupling coefficient between dielectric resonators requires the following basic parameters of a single resonator in its fundamental mode:

1) resonant frequency,
2) field distribution,
3) stored energy, and
4) magnetic-dipole moment.
Dielectric cylinder is a contiguous magnetic wall waveguide => waveguide problem.

Dielectric region – waveguide -above Cut off frequencies

Air regions – waveguide – below cut off frequencies.

At resonance – standing wave in dielectric region and exponentially decreasing waves in air region.
Fig. 3. Dielectric cylinder in magnetic-wall waveguide boundary.
Principal resonance of interest is the lowest order circular electric mode TE_{01}\delta,

Where delta is the non zero ratio \(2L/\lambda g <1\).

The resonant frequency \(f_0 = c/\lambda_0\) is obtained by solving the equations for \(\lambda_0\).

Where

\[
\beta_d \tan \frac{\beta_d L}{2} = \alpha_a \quad (1)
\]

\[
\beta_d = 2\pi \sqrt{\frac{\varepsilon_r}{\lambda_0^2} - \frac{0.586}{D^2}},
\]

\[
\alpha_a = 2\pi \sqrt{\frac{0.586}{D^2} - \frac{1}{\lambda_0^2}}. \quad (2)
\]
Experimental data on four disks with $\varepsilon r = 98$ and $L/D = 0.24$ to 0.62 show that above equation (1) yield resonant frequencies of about 10 percent lower than the measured values.

The circular electric field has $E\theta$ component. The second order model yields $E\theta$ as

$$E_\theta = \begin{cases} E_0 f(z) \frac{J_1(k_c r)}{J_1(p_{01})} & 0 \leq r \leq a_0 \\ 0 & r > a_0 \end{cases}$$ (3)
Where
\[ k_c = \frac{2\pi}{\lambda_c} = \frac{p_{01}}{a_0} = \frac{2p_{01}}{D} \] (4)

and \( p_{01} \) is the first positive root of \( J_{01}(p_{01}) = 0 \), or \( p_{01} = 2.405 \).

The function \( F(z) \) is as follows in the dielectric and air regions

\[ f(z) = \begin{cases} 
\cos \beta_d z & \quad \frac{-L}{2} \leq z \leq \frac{L}{2} \\
\cos \frac{\beta_d L}{2} e^{-\alpha_a |z| - \frac{L}{2}} & \quad |z| \geq \frac{L}{2} 
\end{cases} \] (5)

The magnetic-dipole moment vector \( M \) is defined in terms of an electric current distribution as follows:

\[ M = \frac{1}{2} \int \int \int R \times i dv \] (6)

- \( R \) is the vector distance from an arbitrary fixed reference point,
- \( i \) is the current density,
- and the integration is performed over a volume enclosing the current distribution.
In the above equation I = jωD = jωεE

\[ M = \frac{jωε_0}{2} \int \int \int ε_r R × E dv. \] \hspace{1cm} (7)

Substituting Eq (3) in Eq (7)

\[ M = \frac{j2πωε_0E_0}{J_1(p_{01})} \int_{z=0}^{∞} ε_rf(z)dz \int_{r=0}^{a_0} r^2J_1(k_0r)dr. \]

Integration w.r.t to ‘r’ gives

\[ M = \frac{j4πωa_0^3ε_0E_0}{(p_{01})^2} \int_{0}^{∞} ε_rf(z)dz \]

We used bessels function identities
\[
\int x^2 J_1(x) \, dx = x^2 J_2(x) \quad \text{(10)}
\]

\[
J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x). \quad \text{(11)}
\]

With \( n = 1 \), (11) reduces to

\[
J_2(p_{01}) = \frac{2}{p_{01}} J_1(p_{01}) - J_0(p_{01}) = \frac{2}{p_{01}} J_1(p_{01}) \quad \text{(12)}
\]

- The result with \( \varepsilon r = 1 \) in the \( z = L/2 \) to infinity region,

- \( \eta = \sqrt{\mu_0 / \varepsilon_0} = 120\pi \) and \( \omega \varepsilon_0 = 2\pi / \lambda \eta = 1/60\lambda \)

\[
M = \frac{\pi D^3 L \varepsilon_r E_0}{240(p_{01})^2 \lambda} \left[ \frac{2}{\beta d L} \sin \frac{\beta d L}{2} + \frac{2}{\varepsilon_r \alpha_a L} \cos \frac{\beta d L}{2} \right].
\]

- At resonance peak values of magnetic field energy and electric field energy are equal.

\[
W_m = W_e = \frac{\varepsilon_0}{2} \int \int \int \varepsilon_r E^2 \, dv.
\]
Substituting Eq(3) and Eq(5) and integrating

\[ W_m = \frac{2\pi \varepsilon_0 E_0^2}{[J_1(p_{01})]^2} \int_{z=0}^{\infty} \varepsilon_r [f(z)]^2 dz \int_{r=0}^{a_0} r [J_1(k_c r)]^2 dr. \]  

(15)

\[ \int x [J_1(x)]^2 dx = \frac{x^2}{2} \{ [J_1(\alpha x)]^2 - J_0(\alpha x)J_2(\alpha x) \}. \]  

(16)

\[ \text{When } J_0(P_{01}) = 0; \]

\[ W_m = \pi a_0^2 \varepsilon_0 E_0^2 \int_{0}^{\infty} \varepsilon_r [f(z)]^2 dz. \]

Integration and using Eq(5) gives

\[ W_m = \frac{1}{8} \varepsilon_0 \varepsilon_r \pi D^2 L E_0^2 \]

\[ \cdot \left[ \frac{1}{2} \left( 1 + \frac{\sin \beta_d L}{\beta_d L} \right) + \frac{\cos^2 (\beta_d L/2)}{\alpha_c L \varepsilon_r} \right]. \]  

(18)

\[ \text{When } \beta_d L \text{ is very small compared to } \varepsilon_r \text{ values inside the brackets yields unity.} \]
The formula $F = \mu_0 M_1^2 / 2 W_{m_1}$ is used as expression in the coupling formulas derived later.

Using Eq (7) and (14) gives

$$F = \frac{\mu_0 M_1^2}{2 W_{m_1}} = \frac{2\pi^3}{\lambda^2} \left[ \int \int \epsilon_r r^2 E_\theta dr dz \right]^2.$$  \hspace{1cm} (19)

Substituting M and W and using $\pi^3 / p_0^4 = \pi^3 / 2.405^4 = 0.927$,

$$F = \frac{\mu_0 M_1^2}{2 W_{m_1}} = \frac{0.927 D^4 L \epsilon_r}{\lambda_0^2} \left[ \left\{ \frac{2}{\beta_d L} \sin \frac{\beta_d L}{2} + \frac{2}{\epsilon_r \alpha_a L} \cos \frac{\beta_d L}{2} \right\}^2 \right] \cdot \left\{ \frac{1}{2} \left( 1 + \frac{\sin \beta_d L}{\beta_d L} \right) + \frac{\cos^2 (\beta_d L / 2)}{\epsilon_r \alpha_a L} \right\}. \hspace{1cm} (20)$$
Specific calculation for $er=100$ shows that the quantity decreases from 1.02 to 0.98 as $L/D$ increases from 0.25 to 0.7. Experience has shown that this range of $L/D$ includes the most practical range for design purposes. Over the wider range $L/D=0.15$ to 1.0, the deviation from unity is within $\sim 4$ percent.

\[
F = \frac{\mu_0 M_1^2}{2W_{m1}} = \frac{0.927 D^4 L \epsilon_r}{\lambda_0^2} \quad 0.25 \leq L/D \leq 0.7. \quad (21)
\]
Generalized coupling between Magnetic Dipoles

- Resonator are represented by conducting loops, loops have an inductance $L$ and are resonated at $f_0$ by series capacitors $C$.
- Magnetic dipole moment of loop 1 is
  \[ M_1 = AI_1 \tag{22} \]
- $A$ is the loop area, $I_1$ the current

![Diagram](image-url)
The magnetic stored energy is

\[ W_{m1} = \frac{1}{2}LI_1^2. \] (23)

If \( L_m \) is the mutual inductance, the voltage induced in loop2 due to the current in loop1 is given as

\[ V_2 = j\omega L_m I_1. \]

which can also give as:

\[ V_2 = -\oint E_2 \cdot dl = j\omega \iint B_2 \cdot ds \]

\[ = j\omega \mu_0 \iint H_2 \cdot ds \] (25)

Integrated over the area of the loop

\[ V_2 = j\omega \mu_0 H_2 \cdot A. \] (26)
Solving above equations for coupling coefficient ‘k’ = \( \frac{L_m}{L} \)

\[
k = \frac{L_m}{L} = \frac{V_2 I_1}{j2\omega W_{m1}} \quad (27)
\]

Substituting the values of \( V_2 \) and \( W_{m1} \) from Eq (22) and (26)

\[
k = \frac{\mu_0 H_2 M_1}{2W_{m1}} = \frac{FH_2}{M_1} \quad (28)
\]

Is particularized to the case of a pair of identical magnetic dipoles in a waveguide.

\[
E_p^{\pm} = (e_p \pm e_{zp})e^{\mp zp} \quad (29)
\]

\[
H_p^{\pm} = (\pm h_p + h_{zp})e^{\mp zp}. \quad (30)
\]

\[
\int\int e_p \times h_p \cdot dS = 1 \quad (31)
\]

So we get in terms of transverse and longitudinal components.
The total fields are given by

$$E^+ = \sum_p a_p E^+_p, \quad H^+ = \sum_p a_p H^+_p \quad (32)$$

$$E^- = \sum_p b_p E^-_p, \quad H^- = \sum_p b_p H^-_p \quad (33)$$

Where $a_p$ and $b_p$ are amplitude factors of waves in $+z$ and $-z$ directions

$$a_p = \frac{j\omega \mu_0}{2} H^-_p \cdot M \quad (34)$$

$$b_p = \frac{j\omega \mu_0}{2} H^+_p \cdot M \quad (35)$$
Transverse Orientation in a Cutoff Rectangular Waveguide

- Modes due to H component in M direction
- TE10 has low cutoff and so low attenuation
- TE10 field to represent field at sufficient longitudinal distance from dipole
Transverse Orientation in a Cutoff Rectangular Waveguide *contd.*

- Coupled dielectric resonators inside a rectangular metal tube – transverse orientation
Transverse Orientation in a Cutoff Rectangular Waveguide *contd.*

- Field due to the moment of first magnetic dipole

\[ H_2 = H_x^+ = \sum_{m,n} a_{mn} h_{xmn} e^{-\alpha_{mn}s} \]

\[ a_{mn} = - j \frac{\omega \mu_0}{2} h_{xmn} M_1. \]

\[ \alpha_{mn} = \frac{2\pi}{\lambda_{cmn}} \sqrt{1 - \left( \frac{\lambda_{cmn}}{\lambda} \right)^2} \text{ Np/m} \]

\[ \lambda_{cmn} = \frac{1}{\sqrt{\left( \frac{m}{2a} \right)^2 + \left( \frac{n}{2b} \right)^2}}. \quad (39) \]
Transverse Orientation in a Cutoff Rectangular Waveguide contd.

- Power normalization relationship in terms of Characteristic wave impedance for TE and TM modes.

\[ e \times h \cdot dS = Z h_z dS \]

\[ Z_{mn} \left[ \iint_S h_{xmn}^2 dS + \iint_S h_{ymn}^2 dS \right] = 1 \]

\[ Z_{mn} = \frac{\eta \lambda_o}{\lambda} = \frac{j2\pi \eta}{\alpha_{mn} \lambda} \quad \text{(TE)} \]

\[ = \frac{\eta \lambda}{\lambda_o} = \frac{\alpha_{mn} \lambda \eta}{j2\pi} \quad \text{(TM)} \]
Transverse Orientation in a Cutoff Rectangular Waveguide contd.

• Field components for TE modes

\[
h_{xmn} = \frac{m}{a} A_{mn} \sin \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right)
\]

\[
h_{yln} = \frac{n}{b} A_{mn} \cos \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right)
\]

• Field components for TM modes

\[
h_{xmn} = \frac{n}{b} B_{mn} \sin \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right)
\]

\[
h_{yln} = - \frac{m}{a} B_{mn} \cos \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right)
\]
Transverse Orientation in a Cutoff Rectangular Waveguide *contd.*

- Coupling coefficient

\[
k = \frac{F}{ab} \left[ \sum_m \alpha_m e^{-\alpha_m s} + 2 \sum_{m,n} \frac{\alpha_m^2}{\alpha_{mn}} e^{-\alpha_{mn} s} \right].
\]

\[
k = \frac{F \alpha_{10} e^{-\alpha_{10} s}}{ab} \quad \text{s large.}
\]
Coupling-coefficient data for configuration

Solid curve – single mode
Dashed curve – multi mode
Coupling-coefficient data for configuration
Solid curve – single mode
Dashed curve – multi mode
Axial Orientation in a cutoff Rectangular Waveguide

- Configuration of Band pass filter series of dielectric disk resonators positioned with their axes along the center line of the rectangular non propagating waveguide.

Fig. 7. Dielectric-disk resonators axially oriented in a cutoff square or rectangular waveguide.
The coupling coefficient thus obtained for a pair of axial resonant disks spaced along the center line of a rectangular tube of width \(a\) and height \(b\) is

\[
\kappa = \left( \frac{8\pi^2 F}{ab} \right) \left\{ \sum_{m_0, m} \frac{K_{mn} e^{-\alpha_{mn} s}}{\lambda_{cmn}^2 \alpha_{mn}} \right\} + 2 \sum_{m \geq n > 0} \frac{K_{mn} e^{-\alpha_{mn} s}}{\lambda_{cmn}^2 \alpha_{mn}} \right\} \quad m, n = 2, 4, 6, 8, \ldots. \tag{47}
\]

Let \(m_1 z'\) be the effective \(z\)-directed magnetic dipole moment per unit area concentrated on the central plane of the disk. Then

\[
K_{mn} = \left[ \frac{\iint h_{zmn}(x, y)m_{1z'} dS}{h_{zmn}(a/2, b/2) \iint m_{1z'} dS} \right]^2. \tag{48}
\]

\[
h_{zmn}(x, y) \propto \cos(m\pi x/a) \cos(n\pi y/b)
\]

\[
m_{1z'} \propto B_{1z} \propto H_{1z} \propto J_0 \left( \frac{2p_{01} r}{D} \right) \quad r = 0 \text{ to } D/2
\]

\[
m_{1z'} = 0 \quad r \geq D/2 \tag{49}
\]
\[ K_{mn} = \left[ \frac{\int \int \cos \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right) J_0 \left( \frac{2p_{01}r}{D} \right) r \, dr \, d\theta}{\int \int J_0 \left( \frac{2p_{01}r}{D} \right) r \, dr \, d\theta} \right]^2 \]  

\text{for } m \text{ and } n \text{ even.} \quad (50)

\[ K_{mn} = \left[ \frac{J_0 \left( \frac{\pi D}{\lambda_{cmn}} \right)}{1 - \left( \frac{\pi D}{p_{01}\lambda_{cmn}} \right)^2} \right]^2. \quad (51) \]
Comparison of theoretical and experimental coupling coefficient data—axial orientation in cutoff square waveguide.

Fig. 8. Comparison of theoretical and experimental coupling-coefficient data—axial orientation in cutoff square waveguide.
The third bandpass configuration is a series of dielectric disks positioned with their axes along the axis of a circular nonpropagating waveguide.

This configuration is identical to that of previous case, except that a circular metal tube replaces the rectangular tube.

Because of the cylindrical symmetry, only circular-electric modes, designated TE01, are excited by the equivalent magnetic dipoles.
Substituting the value of F and integrating gives

\[ k = \frac{F}{2\pi a^4} \sum_{n \geq 1} \frac{K_n u_{0n}^2 e^{-\alpha_{0n}s}}{\alpha_{0n} J_0^2(u_{0n})} \]  \hspace{1cm} (52)

\[ K_n = \left[ \frac{J_0 \left( \frac{u_{0n}D}{2a} \right)}{1 - \left( \frac{u_{0n}D}{2p_{01}a} \right)^2} \right]^2 \]  \hspace{1cm} (53)
Coupling coefficient versus center-to-center spacing for a pair of dielectric-disk resonators axially oriented in a cutoff circular tube.

Fig. 10. Coupling coefficient versus center-to-center spacing for a pair of dielectric-disk resonators axially oriented in a cutoff circular tube.
Disadvantages

- The center frequency(1/εr) of the microwave dielectric resonators change with temperature, and this is found to be excessive in most of the applications.
- Because, dielectric resonators vary with temperature.
- Temperature stabilization is one solution.
- Alternatively can use TiO2 ceramics with improved temperature sensitivity
Conclusions

- The analysis of magnetic dipole moment, stored energy and coupling has been confirmed by good agreement between the computed and experimental coupling coefficient values.
- The derivations cover the 3 principal configurations useful for the bandpass filter design.
References

- Microwave Bandpass Filters Containing High-Q Dielectric Resonators SEYMOUR B. COHN, FELLOW, IEEE.
Thank You