Directional Coupler Project

Rick Campbell

Portland State University

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Project Example -- Directional Coupler Design

Pre-CAD era:

Signal Flow Graphs, Even-mode Odd-mode transmission line theory, S-matrix, Linear Algebra....

Post-CAD era:

Go straight from Maxwell’s equations to basic RLC circuit theory, and from there quickly to a complete, buildable design.

Then use 3D EM simulator to implement our basic understanding in guided wave hardware.
While proceeding from the coupled Transmission Line model to the equivalent lumped element circuit model, bear in mind:

**Inductance is a volume in space with Magnetic fields. We calculate inductance by integrating over the volume--not the wire.**

**Capacitance is a volume in space with Electric Fields. We calculate capacitance by integrating over the volume--not the metal plates.**
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Textbook Microstrip Directional Coupler

In the coupled region, the E fields and H fields of the transmission lines interact.
To analyze, terminate all ports in characteristic impedance $R$. 
Now replace the microstrip version with a circuit model:

H field coupling is modeled with a coupled inductor

E field coupling is modeled with a capacitor

note the simplifications in the model

explore later using LTSpice simulation
Now that we have an elementary circuit model, we can use superposition to treat the E and H behavior separately. Start with H:
Start redrawing, using circuit basics and what we know about transformers:
Now replace the coupled inductor model with a simplified model:

Exercise: What would a more complete 1:1 transformer model include?

Note how much more information this simple transformer model provides than the usual rule of thumb $xL > 10R$. 
In this circuit model, voltages and currents are identical on each side of the ideal 1:1 transformer, so it may be omitted.
Finally, invoke Thevenin equivalent to obtain a circuit model for the voltage drop across the inductor and current around the loop.
Again, carefully, with math and carefully keeping track of the numbered ports:
Redraw:
Find current:

\[ i_1 = \frac{2v}{Z} \]

\[ Z = 2R + \frac{2Rj\omega L}{2R + j\omega L} \]

\[ i_1 = \frac{2v}{2R + \frac{2Rj\omega L}{2R + j\omega L}} \]

\[ i_1 = \frac{2v (2R + j\omega L)}{4R^2 + 4Rj\omega L} = \frac{1}{2} \frac{2R + j\omega L}{R^2 + RJ\omega L} \]
volts:

\[ i_1 = \frac{1}{2} \left( \frac{2R + j\omega L}{R^2 + Rj\omega L} \right) \]

\[ v_3 = +1v - i_1R = 1 - \frac{R}{2} \left( \frac{2R + j\omega L}{R^2 + Rj\omega L} \right) \]

\[ = 1 - \frac{1}{2} \left( \frac{2R + j\omega L}{R + j\omega L} \right) \]

\[ v_4 = -1v + i_1R = -1 + \frac{R}{2} \left( \frac{2R + j\omega L}{R^2 + Rj\omega L} \right) \]

\[ = -1 + \frac{1}{2} \left( \frac{2R + j\omega L}{R + j\omega L} \right) \]
Now, the same procedure for Electric field coupling:
Again, invoke Thevenin equivalent circuit:

\[
i_{1C} = \frac{1}{R + \frac{1}{j\omega C}} = \frac{j\omega C}{1 + j\omega RC}
\]
Find the voltages $v_{3c} = v_{4c}$ due to electric coupling:

$$V_{3c} = V_{4c} = i_{1c} \frac{R}{2} = \frac{R}{2} \frac{j\omega C}{1 + j\omega RC}$$
The circuit behaves as a directional coupler when the sum of the voltages at port 4 due to magnetic coupling and electric coupling add to zero:

\[
\frac{R}{2} \frac{j\omega C}{1 + j\omega RC} - 1 + \frac{1}{2} \frac{2R + j\omega L}{R + j\omega L} = 0
\]
some careful complex arithmatic:

\[-2 + \frac{j\omega RC}{1 + j\omega RC} + \frac{2R + j\omega L}{R + j\omega L} = 0\]

\[-2 + \frac{j\omega RC (1 - j\omega RC)}{1 + \omega^2 R^2 C^2} + \frac{(2R + j\omega L)(R - j\omega L)}{R^2 + \omega^2 L^2} = 0\]

\[-2 + \frac{j\omega RC + \omega^2 R^2 C^2}{1 + \omega^2 R^2 C^2} + \frac{2R^2 + \omega^2 L^2 - j\omega RL}{R^2 + \omega^2 L^2} = 0\]

\[
\frac{(j\omega RC + \omega^2 R^2 C^2)(R^2 + \omega^2 L^2) + (1 + \omega^2 R^2 C^2)(2R^2 + \omega^2 L^2 - j\omega RL)}{(1 + \omega^2 R^2 C^2)(R^2 + \omega^2 L^2)}
\]
separating real and imaginary terms:

\[-2 + \frac{2R^2 + \omega^2 L^2 + 3 \omega^2 R^4 C^2 + 2 \omega^4 R^2 L^2 C^2}{R^2 + \omega^2 L^2 + \omega^2 R^4 C^2 + \omega^4 R^2 L^2 C^2} + \frac{j\omega R^3 C - j\omega R L + j\omega^3 R L^2 C - j\omega^3 R^3 L C^2}{R^2 + \omega^2 L^2 + \omega^2 R^4 C^2 + \omega^4 R^2 L^2 C^2} = 0\]

For this expression to = 0, real and imaginary parts have to independently = 0. For imaginary part to = 0 independent of frequency, \(\omega\) terms and \(\omega^3\) terms must = 0 separately.

\[j\omega R^3 C - j\omega R L = 0\]
\[j\omega R^3 C = j\omega R L\]
\[j\omega R^2 C = j\omega L\]
\[R^2 = \frac{j\omega L}{j\omega C}\]
The familiar expression relating $R$, $L$ and $C$ from transmission line theory is sufficient to set the $\omega^3$ and real terms to zero as well. This may be confirmed by substitution.

$$R^2 = \frac{j\omega L}{j\omega C} = \frac{L}{C}$$

Note that this is more general than the most interesting 3dB twisted quadrature hybrid case:

$$R^2 = \frac{j\omega L}{j\omega C} \quad R = \omega L \quad R = \frac{1}{\omega C}$$
Expressions from the prior analysis may be used to find the voltages at each of the four ports:

Solid line is E coupling and dashed line is H coupling. Note the polarity of the resulting voltages across the resistors $R$. 
A surprising result is that cancellation at port 4 is independent of frequency over a very wide range.
Another useful result is that the voltage at port 3 rises at 6 dB per octave when a small amount of signal is coupled.
The coupled port output rise of 6dB/octave allows us to add an RC circuit to obtain flat response over wide bandwidth:

What’s in the black box?

The Bird Wattmeter is the industry standard portable tool for measuring forward and reflected power on transmission lines connected to HF-VHF-UHF transmitters.
Bird Wattmeter measuring forward power into load Z

detector diode

50 uA meter
Bird Wattmeter measuring reflected power from load $Z$

Pretty cool! Now you can make your own custom Bird Wattmeter elements
Null when:  \( L = R^2 C \)

Explore using an LTSpice simulation with \( R = 50 \) ohms. Start with \( X_L = X_C = R \) and then vary \( L \) and \( C \).
Class Exercise:

Design, build, and measure a forward and reflected power meter to optimize an antenna connected to the 50 MHz 10 mW classroom source.