1. Use the Master theorem to find the order of growth for the following occurrences:
   
   (a) \( T(n) = 4T\left(\frac{n}{2}\right) + n, T(1) = 1 \)
   
   (b) \( T(n) = 4T\left(\frac{n}{2}\right) + n^2, T(1) = 1 \)
   
   (c) \( T(n) = 4T\left(\frac{n}{2}\right) + n^3, T(1) = 1 \)

2. On the midterm you were asked to compare the efficiencies of binary and ternary searches by solving the appropriate recurrences for the number of comparisons. Now that you know it, can you answer this problem using the Master theorem? Explain why, or why not.

3. Consider the DNA sequence
   
   TTATAGATCTCGTATTCTTTTATAGATCTCCTATTCTT

   which might represent part of the human genome. Using the following algorithms, demonstrate how to find gene segment **TCCTATTCTT** in the above DNA sequence.

   (a) Horspool’s algorithm
   
   (b) The Boyer-Moore algorithm

   For each algorithm, show the tables produced in the pre-processing step, and the comparisons made along the way to eventual success.

4. The following hash functions are applied to map arrays \( a[0..4] \), containing 5 integer values, to a hash value in the range \( [0..m] \). For each function, discuss whether it is a “good” hash function. If not, suggest a way in which it might be improved.

   (a) **function** hashA:
      
      \[
      h \leftarrow 0 \\
      \text{for } i \leftarrow 0 \text{ to } 4 \text{ do } h \leftarrow h + a[i] \\
      \text{return } (h \mod m)
      \]

   (b) **function** hashB:
      
      \[
      h \leftarrow 0 \\
      \text{for } i \leftarrow 0 \text{ to } 4 \text{ do } \\
      \hspace{1em} \text{for } j \leftarrow 0 \text{ to } 100 \text{ do } \\
      \hspace{2em} h \leftarrow (h + i \cdot j^2 \cdot a[i]^2) \mod m; \\
      \text{return } h;
      \]

   (c) **function** hashC:
      
      \[
      h \leftarrow 0; \\
      \text{for } i \leftarrow 0 \text{ to } 4 \text{ do } \\
      \hspace{1em} h \leftarrow (a[i] \cdot \text{rand}()); \\
      \text{return } (h \mod m);
      \]