Subtyping and Substitutability
or:
How to be a poser
Type checking and Program Execution

• Coherence:
  – *if*
    - a program is statically analyzed and all operations assigned types…
  – *then*
    - when that program runs, the values computed by the program and provided (dynamically) to each operation should be consistent with the results of the type assignment.
What is the nature of this consistency?

– definition time:  \( C \text{ implements } R, \ C \cdot m: S \rightarrow T \)

– message send time:  \( \frac{o: R, v: S}{o \cdot m (v): T} \)

• What values can \( v \) have at run time?

– In other words ...

\( v \) is a variable that may refer to many values. What restrictions are placed on those values by the above type assignment?
What is the nature of this consistency?

- **definition time:** \( C \) implements \( R, \ C \cdot m : S \rightarrow T \)

- **message send time:** \( \frac{\text{o: } R, \ v: S}{\text{o.m (v): } T} \)

• What values can \( v \) have at run time?

- In other words ...

\( v \) is a variable that may refer to many values. What restrictions are placed on those values by the above type assignment?

\( v \) has exactly type \( S \)? Can we make more relaxed?
In an Object-Oriented Language:

What’s the goal of the type system?
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What’s the goal of the type system?

– Type systems attempt to guarantee that all messages will be understood (and that the correct method will be executed).

– So we *can* be more relaxed, provided that we maintain this property
Type Conformance

- If an object understands all the messages defined on a type, we say that it **conforms to** that type.

- If an object O1 has a *bigger* type than object O2 then a client will not care if O1 replaces O2.
  - *bigger* ⇒ more (or the same) operations

- On the other hand, if an object has a *smaller* type, then some messages may not be understood.
  - *smaller* ⇒ fewer (or the same) operations

- So… an instance of a bigger type can **pose** as an instance of a smaller type (type conformance)
Type Conformity — An example

The printerDriver takes an argument an object of type printer.

```plaintext
printer = type p
  method name () → string
  method printText (textStream, username) → status
  method pagesPrinted () → integer
end p
```

or, in Java syntax

```java
interface printer
{
  string name ()
  status printText (textStream, username)
  integer pagesPrinted ()
}
```
Example — continued

postScriptPrinter =

type ps

  method name () → string
  method printText(textStream, username) → status
  method printPostScript (textStream, username) → status
  method pagesPrinted → integer

end ps
Can the user distinguish?

– The *postscriptPrinter* has one additional method. All other operations have exactly the same signature.
If I have a some code:

```java
printer p;
textStream ts;
username u;
...
p.printText(ts, u);
```

Is this OK?

– What if *p* is a *postscriptPrinter*?

– This kind of specialization supports *upward compatibility*. 
The nameserver findLocalPrinter operation is initially defined:

```
nameserverV1 = type n1
  method findLocalPrinter(roomNumber) → (printer, roomNumber)
end n1
```

And is further enhanced to allow us to find a postscriptPrinter:

```
nameserverV2 = type n2
  method findLocalPrinter(roomNumber) → (postscriptPrinter, roomNumber)
end n2
```

Both nameservers support findLocalPrinter, and they take the same argument type, but they return different types.
Can I use nameserverV2 for nameserverV1?

– Consider the method \( n \):

\[
\text{Integer } n \ (\text{nameserverV1 } ns) \\
\{ \\
\quad \text{Integer } roomNumber; \\
\quad \text{textStream } ts; \\
\quad \text{username } u; \\
\quad \ldots \\
\quad (\text{printer, roomNumber}) = ns.\text{findLocalPrinter}(\text{myRm}); \\
\quad \text{printer.printTextStream}(ts, u); \\
\quad \text{return } roomNumber \\
\}
\]
Enhance the print operation:

```
fileV1 = type n1
    method printOn(printer) → status
    method open() → testStream
end n1

fileV2 = type n2
    method printOn(postScriptPrinter) → status
    method open() → textStream
end n2
```
Can I use fileV2 objects in place of fileV1 objects?

```cpp
method I (fileV1 f) {
    printer p;
    ...f.printOn(p);
}
```

- If f refers to a fileV1?
- If f refers to a fileV2?
Contravariance

M.op:  \( A \rightarrow B \)  
N.op:  \( C \rightarrow D \)

- **M can be used in place of N if:**
  - \( C <: A \)  (contravariant) \( C \) has more or the same number of operations
  - \( B <: D \)  (covariant) \( B \) has more or the same number of operations

- In this case M is more applicable than N (we write \( M <: N \))
  - In any context where I have an object N, I can apply its \( op \) to any \( C \).
  - If (instead) I have object M, since a \( C \) may replace an \( A \), M.\( op \) can also apply to any \( C \).
From the client’s side…

M.op: A $\rightarrow$ B     N.op: C $\rightarrow$ D

• In any context where I expect to get a result from N.op, I expect it to be of type D.

• But B $\subset$ D, so a B may be used anywhere a D is…

• so it’s OK for M to return a B, since that B object can pose as a D object
In Words:

Type $A$ conforms to type $B$
a.k.a. ($A <: B$) or ($A$ is a subtype of $B$)
iff:
- for every operation in $B$ there is a corresponding operation in $A$.
  (It is OK for $A$ to have more operations), and
- for every operation shared by $A$ and $B$
  the type of the result in $A$ conforms to the type of the result in $B$, and
  the types of arguments in $B$ conform to the types of the corresponding arguments in $A$. 
Example

• In Smalltalk, we can write:

```
#(1 2 3 5 7 11 13 17 19 23) collect: [ :each | each * 2 ]
```

and get …

• Wouldn’t it be nicer to write

```
#(1 2 3 5 7 11 13 17 19 23) collect * 2
```

• How can we make this work?
Polymorphism Example: The Trampoline

• \( #(1\ 2\ 3\ 4\ 5\ 6) \text{ collect} \ *\ 2 \)
  – sending the array the \text{collect} message builds a \text{Trampoline} with two instance variables:
    - collection: \( #(1\ 2\ 3\ 4\ 5\ 6) \)
    - iterationMessage: \( a\ \text{Message} \)
      with selector \text{collect}; and arguments: \( #(\text{nil}) \)

• When the Trampoline receives \( \star\ 2 \)
  – it replaces the \text{nil} in that arguments array with ... something? ... but what?
  – \textit{sends} that \text{iterationMessage} to \( #(1\ 2\ 3\ 4\ 5\ 6) \)
From the Array’s side...

• It gets the message \textit{collect:} with an array containing a single argument.

• What does it do?
  – it names the argument \textit{aBlock}
  – sends that \textit{aBlock} the \textit{value:} message six times, with 1, 2, 3, 4, 5 and 6 as arguments.
  – collects the results and puts them into a new Array
  – answers that Array.
Collection » collect:

collect: aBlock
"Evaluate aBlock with each of the my elements as the argument. Collect the resulting values into a collection like me. Answer the new collection."

| newCollection |
newCollection ← self species new.
self do: [:each | newCollection add: (aBlock value: each)].
↑newCollection

• Does aBlock really have to be a Block?
What is the constraint on the argument of `collect:`?

- *Not* that the argument is a Block
  - although blocks work just fine
- the argument must understand `value`:
  - and `value:` should expect one of the array elements as its argument.
• If Smalltalk had type declarations, `collect:` would have a signature like

```
Collection »collect: f ∈ BinaryFunction, or
Collection<e> »collect: f ∈ BinaryFunction <e>
```

where `BinaryFunction<e> =`

```
type bf

method value: <e> →<e>
end bf
```

• Blocks like `[: x | x * 2 ]` do indeed have type `BinaryFunction <Integer>`

• Does the Trampoline use a Block as the argument to `collect:`?
Message ∈ BinaryFunction

• No! Trampoline uses a Message

doesNotUnderstand: aMessage
  l args l
  args ← iterationMessage arguments.
  args at:args size put:aMessage.
  ↑iterationMessage sentTo:collection.

• Is this ok?
  – why?

message »value: receiver
  ↑receiver perform: selector withArguments: args
So What?

This is an example of

• Interfaces:
  – Block and Message both implement the *BinaryFunction* interface

• Subtyping
  – Block and Message are both subtypes of *BinaryFunction*

• Conformance
  – Block and Message both conform to *BinaryFunction*
• Polymorphism?
  – Some people say yes, because *collect*: takes arguments of different classes
  – Other people say no, because *collect*: doesn’t do anything with those arguments
    - on the contrary, it *relies* on them all having the *same type*: *BinaryFunction*. 
Where’s the Polymorphism?

String >> **valueOf**: anObject

anObject ifNil: [↑ 'nil'].
↑ anObject printString

- **String>>valueOf**: shows Polymorphism
  - objects of varying types are provided as parameters
  - the single method does the same thing with all of those parameters

- anObject printString and anObject ifNil:
  - polymorphism or genericity?
More Subtyping Examples

sandwich = {bread: BreadType, filling: foodType}
cheesesandwich = {bread:BreadType, filling :CheeseType, sauce: sauceType}

– if

cheeseType <: foodType

– then

cheeseSandwich <: sandwich

Suppose we have

function f: sandwich → integer
function g: cheeseSandwich → integer,

Can f be used in place of g? Or g in place of f?
ANSWER!

We can use \( f \) in place of \( g \) because \( \text{cheeseSandwich} \preceq \text{sandwich} \) and the function subtyping rule is contravariant for the argument.

\[
\begin{align*}
c &: \text{cheeseSandwich}; \\
s &: \text{sandwich} \\
f(s); & \quad \leftarrow \text{cannot use } g \text{ here} \\
g(s); & \quad \leftarrow \textbf{not} \text{ OK, since } s \text{ cannot pose as a cheese} \\
g(c); & \quad \leftarrow \text{can use } f \text{ here OR } g \\
f(c); &
\end{align*}
\]

we can replace this instance of \( g \) with \( f \), since a \( \text{cheeseSandwich} \) can pose as a \( \text{sandwich} \) \((f \preceq g)\)
More examples:

What if

\[
\begin{align*}
f &: \text{Int} \rightarrow \text{Sandwich} \\
g &: \text{Int} \rightarrow \text{CheeseSandwich} \\
\text{cheesesandwich} &= c; \\
c &= g(1);
\end{align*}
\]

- If I replace \( g \) with \( f \), then \( c \) is actually bound to a sandwich.

- Can a sandwich pose as a cheeseSandwich? (NO!)

- Here \( g \ll f \) because \( \text{cheeseSandwich} \ll \text{sandwich} \)
Yet more sandwiches

• Updatable variable:

```plaintext
var x: T
var y: S
S <: T
```

- On the RHS (expressions), \( y: S \) can pose as a \( T \), since \( S <: T \).
- On the LHS (variables) \( y: S \) cannot pose as a \( T \):
  ```plaintext
  x := aT ← OK since x must name a T
  x := aS ← OK since S <: T
  ```
- **But:**
  ```plaintext
  y := aT ← Not OK, since aT cannot pose as an S.
  ```