EXPERIMENTAL OBSERVATION AND MODELING
OF OVERLAND HEIGHT FINDING

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ABSTRACT

We present airborne experimental observations at UHF and VHF of electromagnetic scattering from terrain and demonstrate how such a signal can be used to determine target height. We describe the time difference of arrival (TDOA) processing and then go on to discuss how doppler information can be used to supplement or replace TDOA processing. We describe two such methods. The first entails observing the doppler separation between the target and its image and inverting to recover height. The second method uses the doppler domain to filter out-of-plane energy and improve the performance of TDOA processing.

1.0 INTRODUCTION

In recent work (Zurk, 1997a; Zurk and Coutts, 1997b) we presented examples where the time difference of arrival (TDOA) between direct and ground reflected pulses can be used to determine target height when the pulses are separated in the time domain. The measured TDOA can be related to target height by using an appropriate propagation model and signal processing techniques such as Matched Field Processing (MFP) (Zurk, 1997c). The choice of the propagation model is driven by the nature of the terrain. Good agreement with experimental data has been demonstrated with a geometrical optics model used with a wideband pulses synthesis (Zurk 1997d).

In this paper we consider using the doppler information to improve the ability to determine target height. We first introduce equations which describe the doppler separation between the target and image sources. Separation between the two depends primarily on the velocity difference between the two platforms and is strongly geometry dependent. Targets which have a vertical velocity component require less doppler resolution to separate from their images and are hence more likely candidates for application of doppler height finding techniques.

We consider two methods whereby the doppler information can be utilized for target height information. The first method is applicable when there is sufficient doppler resolution to separate the target and image in the doppler domain. In this circumstance the doppler difference can be measured and related to the target height. In the second method the target and image are not separable in the doppler domain. The doppler information is instead used to improve the TDOA estimates by filtering the out-of-plane scattering that is not included in the propagation models and which can deteriorate the estimates. We illustrate both of these methods with data obtained in a recent airborne propagation experiment (Zurk, 1997a-d) and show that they provide acceptable height estimates. In practical scenarios the radar platform may not have enough doppler resolution to implement doppler height finding unless the target is ascending or descending.

2.0 TDOA PROCESSING

In a multipath environment pulses will arrive staggered in time such that the pulses interacting both with the target and the terrain arrive later than those returning directly from the target. Whether or not they are separated in time (or separated in range gate) will depend on the bandwidth BW of the radar. Under a spherical earth model the minimum target height at which separation occurs is

\[ z_2' = \frac{cr}{2z_1'}BW \]  

(1)

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where $z_1'$, $z_2'$ are the target and radar heights adjusted for the earth's curvature and mean altitude (Kerr, 1951), $c$ is the speed of light, and $r$ is the target range.

The above equation can be used to determine target height overland if the surface is well approximated by a smooth curved surface, such as the ocean. A recent airborne propagation experiment was designed to investigate whether or not height finding is feasible over variable terrain. A pulsed CW signal from an airborne radar was received and recorded from a second airborne platform. Further details can be found in (Zurk, 1997c). The data analysis shows that the errors in the height estimates from TDOA processing increase for rougher terrain. An example of this data is shown in Figure 1 and 2. The left-hand plots shows the SNR of the data plotted as a function of time with the direct pulse arriving at $t=0$. The terrain for Figure 1 was the fairly uniform desert floor in White Sands Missile Range (WSMR). The reflected pulse is strong and uniform arriving with the time delay predicted by (1). The right-hand plot of the figure shows the histogram of height estimates obtained by applying (1) to each pulse in the data. As expected, the distribution of estimates is fairly narrow with a mean estimate of 18.5 kft (actual target height was 17.5 kft).

Figure 1. VHF H-pol data from desert terrain in WSMR. Left-hand plot shows SNR with direct pulse at $t=0$. Right-hand plot shows height estimates from TDOA processing.

In Figure 2, the terrain was rough, mountainous terrain. For this terrain the reflected pulse is lower in energy and more variable in nature. The mean estimate differs from the true height by 5000 ft and there is a wide variance in the estimated heights. In the following sections we investigate whether doppler information can be used to improve the height estimate for rough terrain or for the situation when the pulses are not separated in time.

Figure 2. VHF H-pol data from desert terrain in WSMR. Left-hand plot shows SNR with direct pulse at $t=0$. Right-hand plot shows height estimates from TDOA processing.
3.0 SEPARATION IN DOPPLER

In this section we develop the equations which describe the target and the image doppler under a flat earth approximation. The scenario of interest is an airborne radar at altitude $z_1$ with a target at a ground range of $r$ and unknown height $z_2$. The radar is at position $P_R=(0,0,z_1)$, the target is at $P_T=(r,0,z_2)$ and the coordinate system is aligned such that the look direction is along the $x$ axis. The radar and target velocities can be written as $\mathbf{V}_R=(V_{Rx},V_{Ry},V_{Rz})$ and $\mathbf{V}_T=(V_{Tx},V_{Ty},V_{Tz})$, respectively. The image point is then located at $P_T'=(r,0,-z_2)$ with velocity $\mathbf{V}_T'=(-V_{Tx},V_{Ty},-V_{Tz})$. The change is direct path range $R_D$ and image range $R_I$ can be written as

$$R_D = \frac{(P_T-P_R) \cdot (P_T-P_R)}{|P_T-P_R|}$$

$$R_I = \frac{1}{\sqrt{r^2 + (z_2-z_1)^2}}[(V_{Tx} - V_{Rx})r + (V_{Tz} - V_{Rz})(z_2-z_1)]$$

$$R_I = \frac{1}{\sqrt{r^2 + (z_2-z_1)^2}}[(V_{Tx} - V_{Rx})r + (V_{Tz} + V_{Rz})(z_2-z_1)]$$

(2)

(3)

and the corresponding two-way doppler difference between the target and its image is

$$\Delta f_D = \frac{2}{\lambda} (R_I - R_D)$$

(4)

As can be seen from the above equations, when there is a non-zero difference in vertical velocity between the radar platform and the target the doppler difference can be large due to the second term in the R.H.S. of (2) and (3). This pertains to a target such as a ballistic missile which has considerable vertical velocity and results in large doppler separation between target and image. For targets of this type the doppler separation can typically be used to determine target height.

It is more difficult to achieve separation when neither the radar platform nor the target are ascending or descending. In this scenario, $V_{Tx}=V_{Rx}=0$ and the expression for the doppler difference reduces to

$$\Delta f_D = f_D \left( \frac{r^2 + (z_2-z_1)^2}{\sqrt{r^2 + (z_2-z_1)^2}} - 1 \right)$$

(5)

$$f_D = \frac{2(V_{Tx} - V_{Rx})r}{\lambda \sqrt{r^2 + (z_2-z_1)^2}}$$

(6)

From (5) it can be seen that the magnitude of the doppler separation depends on the relative heights of the radar and the target. It further depends on the target doppler given in (6). The plots in Figure 3 show this behavior for a radar at an altitude of 30,000 ft with two different velocity regimes. The grey scale indicates the doppler resolution needed to discriminate a target from its image as given by (5). The first plot shows the doppler separation for a “slow” target with $V_{Tx}=V_{Rx} = 60$ m/s. The right-hand plot is for a “fast” target with $V= 200$ m/s. Once again, greater tar-
Target velocity results in increased doppler separation between the target and its image. The plots in Figure 3 also have contour lines showing the point at which the direct and reflected pulses separate in the time domain for system bandwidths of 0.25, 1.0 and 3.0 MHz, and using (1). For example, a target 100 km in range and 10 kft in altitude will require approximately 0.5 MHz of bandwidth to separate the pulses in time. From Figure 3, if that target is flying level with $V=60$ m/s the doppler difference between the target and the image is less than 1.0 Hz. A faster level target with $V=200$ m/s would result in a doppler separation of 3.0 Hz. Finally, the same target flying parallel to the radar but descending at a rate of 100 m/s is separated from its image by more than 30 Hz (from (2) and (3)). The ability of a radar to separate the target and its image in the doppler domain depends on how long the integration time is relative to the target velocity. Whether the target separates in the time or doppler domain depends in a complicated fashion on the target’s velocity, range and height relative to the system bandwidth and doppler resolution.

Figure 3. Contours showing the separation in doppler between the target and its image as given in (5) when the radar is at 30,000 ft and both target and radar are flying level. Left-hand plot shows results when $V=V_{TX}-V_{RX} = 60$ m/s. Right-hand is for $V=200$ m/s.

4.0 HEIGHT ESTIMATES USING DOPPLER SEPARATION

As discussed in the previous section, there are scenarios when the target and its image are separated in doppler but not in the time domain. Equations (5) and (6) relate the direct path doppler and the image doppler to the target height. If a system has sufficient resolution it can be used to estimate height. In this section we illustrate this process with UHF overland propagation data obtained a recent airborne propagation experiment (Zurk, 1997c). The flight formation was roughly orthogonal and the range and doppler information served to partition the propagation environment up into roughly orthogonal sectors. A range-doppler map was formed by fitting the data to a 12th order autoregressive model. The result is shown in Figure 4. Energy resulting from the target and from its image is separated in the doppler domain by 1.46 Hz but is not separated in the time domain. Application of (5) yields a height estimate of 14.3 kft compared to the actual height of 12.5 kft. Although the estimate is a couple of thousand of feet off, it provides some measure of height in a region where there is insufficient bandwidth to do any height finding with TDOA methods.
Figure 4. Range-doppler map of received energy for UHF HV data from WSMR. Energy from the target and its image are separated in doppler by 1.46 Hz but are not resolvable in the time domain which had a range resolution of 1500 m.

5.0 HEIGHT ESTIMATES USING DOPPLER FILTERING

In this section we consider the scenario where the target is separated from its image in the time domain but the presence of out-of-plane scatter deteriorates the height estimate. In this situation we use the doppler domain to filter out the energy arriving from other doppler bins and also to produce coherent gain on target.

We use an example VHF VV data from a mode 2 data collection at WSMR where the two airplanes flew in a parallel formation over a mountainous region in WSMR. As shown in Figure 5, the pulses were separated in the time domain but the ground reflections have variable strength due to the rough terrain. There is also energy appearing in the time domain from out-of-plane scatter. When the pulses in this data group are used in a TDOA height estimate, they produce the distribution of target heights shown on the right side of Figure 5. As can be seen from the plot, the out-of-plane scatter can erroneously be interpreted as specular scatter from a target at a higher altitude.

Figure 5. VHF V-pol data obtained with a 3 MHz pulsed CW radar flying over WSMR. Left-hand plot shows the SNR for a CPI in the time domain with the direct pulse shown arriving at $t=0$ followed by the ground reflection. Right-hand plot shows the distribution of estimated target heights for the pulses derived from TDOA analysis. Bi-modal distribution results from out-of-plane scatter.

The left-hand plot in Figure 6 shows the iso-range and iso-doppler contours for the flight which are drawn at the range and doppler resolution of the system. In contrast to contours in Figure 4, the parallel formation results in iso-lines that are roughly parallel in orientation to the range lines but have finer spacing or resolution.
Figure 6. Left-hand plot shows range-doppler map for the data in Figure 5. The iso-range lines are drawn at the range resolution of 100 m and the doppler contours are drawn at 1 Hz. The right-hand plot shows the energy in the range-doppler domain where doppler filtering can be used.

If only the energy arriving at zero doppler is considered (see right-hand plot of Figure 6), the TDOA between the direct and specular return can be accurately estimated from (1) and a height estimate of 15.6 kft is obtained. Thus, the TDOA estimate has been improved by coherent processing and doppler filtering.

6.0 CONCLUSIONS

In this paper we have demonstrated target height estimation from overland multipath. We have presented airborne propagation data to illustrate TDOA height estimates. Two additional techniques involve utilizing the doppler domain: 1) the doppler difference between the target and its image can produce a height estimate and 2) information from the doppler domain can be used to filter out-of-plane scatter to improve the TDOA estimation. Choice of a method depends in a complicated fashion on the flight geometry.

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7.0 REFERENCES


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